According to new syllabus of all intermediate boards of Sindh.

Prof. Rawala's ....

PHYSICS

HELPING BOOK

For
Class XI
(All science students)

UNIT -5

Work, energy and power.

CONTENTS

Thoroughly revised and updated 2026 – 2027

- ***Detailed notes.***
- ***Multiple Choice Questions (M.C.Q's).***
- ***Solution of text book numericals.***
- ***MCQs and solution of numericals from <u>Karachi board</u> past papers.***

Prof.(r) ***Mohammed Hussain Rawala.***
B.Sc. (Hons.), M.Sc.(Physics)
Former head of physics department
Saint Patrick's college, Karachi.

Unit. 5.....

Work, energy and power

Work:

Q.No.1: What is work?

Ans: Work is said to be done by a force when it displaces a body in its own direction.

OR

Work is the scalar or dot product of force "$\vec{F}$" and displacement "$\vec{d}$" Hence it follows laws obeyed by scalar product of vectors.

$$\boxed{Work = \vec{F} \cdot \vec{d}}$$

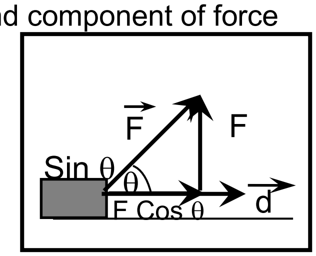
The magnitude of work is given by:

$$Work = F\, d \cos\theta$$

Where "θ" is the angle between force "$\vec{F}$" and displacement "$\vec{d}$".

In other words, work done is equal to the product of displacement and component of force in the direction of displacement. Hence if a force makes some angle with the direction of displacement then:

$$Work = (F \cos\theta)\, d$$



Where $F \cos\theta$ is the component of force in the direction of displacement.

Q.No.2: Is work a vector quantity?

Ans: Work is a ***scalar quantity*** because it does not have a direction, although it is the product of two vectors, force "$\vec{F}$" and displacement "$\vec{d}$".

Q.No.3: When is work done maximum?

Ans: ***Maximum*** amount of work is done when force "$\vec{F}$" is in the direction of displacement "$\vec{d}$" (or direction of motion, when angle between them $\theta = 0°$).

$$Work = F\, d \cos\theta$$

$$Work = F\, d \cos 0°$$

$$\boxed{Work_{max.} = F\, d} \qquad (\cos 0° = 1)$$

Q.No.4: When is work done zero?

Ans: Work done is zero when force "$\vec{F}$" is perpendicular to displacement "$\vec{d}$" i.e. the angle between them is 90°.

$$Work = F\, d \cos\theta$$

$$Work = F\, d \cos 90°$$

$$\boxed{Work_{min.} = 0} \qquad (\cos 90° = 0)$$

- When you walk on a level floor carrying a brick you do no work on the brick because the vertical force supporting the block has no component in the direction of horizontal motion.

- When a body moves along a circular path, force responsible for its circular motion is called centripetal force. work done by centripetal force on the body is **<u>zero</u>**, because centripetal force is always directed **<u>towards center</u>** of circular path and is **<u>perpendicular</u>** to the tangent to circular path which shows the direction of instantaneous displacement along the circle.
- When a body slides along an inclined surface, work done by the normal reaction acting on the body perpendicular to the plane is zero.

In all these examples Force "$\vec{F}$" & displacement "$\vec{d}$" are perpendicular to each other, $\theta = 90°$.

(***<u>Note that</u>*** work may be done by other force/s but not by the forces mentioned above)

<u>Q.No.5</u>: When is work done <u>negative</u>?

***<u>Ans</u>*:** Work done is <u>negative</u> when a force is <u>opposite</u> to the direction of displacement i.e. the angle between them is 180°.

Work = F d Cos θ

Work = F d Cos 180°

$\boxed{\text{Work} = -\text{F d}}$ (Cos 180° = –1)

🗣Work done by force of friction is negative, because force of friction is opposite to the direction of motion (Or direction of displacement) of the body.

<u>Q.No.6</u>: When a body is <u>lifted</u> what is the sign of work done <u>on the body</u> by:

(1) lifting force. (2) gravitational force.

***<u>Ans</u>*:** When a body is lifted:

(1) Work done <u>on</u> the body by lifting force is <u>positive</u> because the direction of lifting force is <u>same</u> as the direction of vertical displacement of the body (both are upward) i.e. the angle "θ" between the direction of force and displacement is 0°.

(2) Work done <u>on</u> the body by the gravitational force of the earth is <u>negative</u> because the direction of gravitational force (downward) is <u>opposite</u> to the direction of vertical displacement (upward) of the body i.e. the angle "θ" between the direction of force and displacement is 180°.

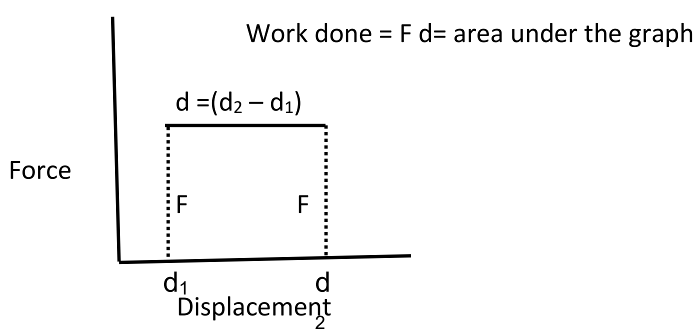
<u>Q.No.7</u>: What is the <u>S.I unit of work</u> and its dimensions?

***<u>Ans</u>*:** The M.K.S or S.I unit of work is "Joule J" (Basically it is N. m).
Work done is said to be "1 J" if a force of "1 N" displaces a body through "1 m" in its own direction.

- Dimensions of work are $[ML^2T^{-2}]$.

<u>Work from Force - displacement graph</u>:

Shape of a graph between force $\vec{F}$ and displacement $\vec{d}$, depends upon whether the force in magnitude and direction throughout the displacement is constant or not.
If magnitude of the force is constant and it is in the direction of displacement then the graph between force and displacement will be a straight line parallel to displacement axis. In this case the amount of work done will be equal to the <u>area under the straight-line</u> graph.



Work done by a variable force:

(1) If the magnitude of force is variable (increasing or decreasing) and the direction of force remains the same throughout motion of the body, then shape of the graph between force and displacement depends upon the changes in the magnitude of force. In such a case the amount of work done can be determined first by dividing the entire displacement into a large number of small displacements, each displacement Δd must be so small that the force has practically a constant value throughout that particular small displacement. Then work done during each small displacement is calculated. Sum of all these will give us the total amount of work done.

In this case amount of work done from the graph will be equal to the area under force-displacement curve (The shape of the curve depends upon the type of variations in the magnitude of force).

(2) If the variable force is such that both its magnitude and direction are continuously changing with time then in that case work done can be calculated by finding the work done during small displacements using magnitudes of force during that small displacement, displacement Δd and angle between them and then finding their sum.

$$\text{Total work done} = \sum \vec{F} \cdot \vec{\Delta d}$$

🗣(***Note that*** when work is done ***on*** a body it gains and when work is done ***by*** a body it loses some form of energy).

Energy:

***Q.No.1**: What is energy?*

***Ans**: **Energy is the ability of doing work**.*

If a body is capable of doing work it has energy.

The amount of energy possessed by a body will be equal to the amount of work which a body can do.

🗣If work is done on a body it gains an equal amount of some form of energy, and when work is done by the body it loses the same amount of energy.

🗣Energy is a ***scalar*** quantity.

🗣Energy has many forms such as mechanical, electrical, magnetic, heat, sound, nuclear, elastic energy etc. These forms of energy are inter-convertible; it means that one form of energy can be converted into another form.

***Q.No.2**: What is the **S.I unit of energy**, and dimensions of energy?*

Ans**:* Unit of energy is same as the unit of work. S.I unit of energy is "Joule***".

- Dimensions of energy are $[ML^2T^{-2}]$

Kinetic energy (K.E):

***Q.No.1**: What is kinetic energy?*

Ans**:* Energy possessed by a body by virtue of its ***motion is called kinetic energy. When a body is in motion it is capable of doing work. This ability of doing work when a body is in motion is called kinetic energy.

***Q.No.2**: On what factors does the kinetic energy of a body depend?*

***Ans**:* K.E possessed by a body is directly proportional to its mass "m" and to the square of its velocity "V^2".

🗣***It means that*** K.E of the body changes in the same way and in the same proportion as its mass is changed i.e. if mass of a body is doubled its K.E will be doubled, if mass is halved its K.E will also be halved, if mass is increased by four times its K.E will also increase by four times and so on.

🗣 K.E of a body depends on square of its velocity, hence K.E of the body changes by square of number of times its velocity is changed. ***In other words***, if velocity of a body is doubled (increased by two times) it's K.E will increase by four times (square of two times), if its velocity is increased by three times (i.e. tripled) then it's K.E will increase by nine times (square of three), If velocity is decreased by two times its K.E will decrease by four times, and so on.

🗣If mass and velocity of a body both are changed then the total number of times kinetic energy changes is equal to the product of change in mass and square of change in velocity. Hence

🗣If mass and velocity of a body both are changed by **two times** (increased or decreased) then it's K.E will change (increase or decrease) by **eight times**, because with mass it will change by **two** times and with velocity it will change by **four** times, hence K.E will change by **eight** times.

If mass m and velocity V both are changed by **three times** then its kinetic energy will change by **twenty seven** times because its mass changes by three times and its velocity changes by nine times ($3^2 = 9$), total number of times K.E changes = 3 × 9 = 27. and so on ….

Derivation:

***Q.No.3*: *Derive a formula for kinetic energy* (*K.E*).**

***Ans*:** Kinetic energy possessed by a body is equal to the amount of work which a moving body can do, or K.E gained by a body is equal to the amount of work done on it in moving it.

Consider a body of mass "m" thrown vertically upward with a certain velocity "V_i". Velocity of the body decreases as the body rises and becomes zero at the highest point.

Initial K.E given to the body at the point of projection is equal to the work done on it to throw it upward, this K.E is used up to do work against gravity as the body rises up. Hence to calculate K.E possessed by the body at the point of projection we will calculate the work done by the body against gravity.

Let "h" be the height reached by the body at which its velocity becomes zero (Or it is the vertical distance through which the body does work against gravity).

Work done by the body is given by:

$$\text{Work done by the body} = \vec{W} \cdot \vec{h}$$

Where "W" is the lifting force.

$\therefore$ Work done by the body = W h Cos θ

$\therefore$ Work done by the body = W h Cos 0°

(θ = 0° because lifting force and displacement are in the same direction)

$\therefore$ Work done by the body = W h (Cos 0° =1)

$\therefore$ Work done by the body = m g h

In the vertical direction : $V_f = 0$ (at the highest point.

$V_i = V$ (at the point of projection)

$a = -g$

$S = h$ (height to which the body rises).

On substituting the above data in the following equation of motion, we get:

$$2\,a\,S = V_f^{\,2} - V_i^{\,2}$$

$$-2\,g\,h = (0)^2 - (V)^2$$

$$h = \frac{V^2}{2\,g}$$

But Work done by the body = m g h

$\therefore$ Work done by the body = m g $\dfrac{V^2}{2\,g}$

$\therefore$ Work done by the body = $\frac{1}{2}$ m V^2

Since work done by the body to rise against gravity through a certain height "h" is equal to the amount of K.E possessed by it at the point of projection, hence:

$$\boxed{K.E = \frac{1}{2} m V^2}$$

Gravitational potential energy (P.E):

Q.No.1: What is potential energy?

Ans: Energy possessed by a body by virtue of its ***position*** or ***configuration*** in the gravitational field is called **gravitational potential energy**.

Potential energy is of many types, such as gravitational P.E, electric P.E, elastic P.E, magnetic P.E, etc.

Gravitational P.E is the energy due to position of a body in the gravitational field. It is equal to the amount of work which the **body can do** while moving in the direction of gravitational field or the amount of work **done on the body** to move it against the direction of gravitational field.

But electric P.E is the energy possessed by a charged body due to its position in the electric field.

A body **gains** P.E when work is done **on** it but it **loses** P.E when work is done **by** it.

Derivation:

Q.No.2: Derive a formula for gravitational potential energy?

Ans: Consider a body of mass "m" lifted through a certain vertical height "h", since the body is moved against gravity hence work has to be done on it. This work done on the body will be stored as the gravitational potential energy.

But work done on the body to lift it is (by the lifting force) = m g h

$\therefore$ $$\boxed{P.E = m\,g\,h}$$

The above formula to calculate P.E of a body is valid only when the force "m g" applied to lift the body is constant throughout the displacement "h". This is possible only when "h" is small (of the order of tens of meters) but if the displacement is large (of the order of thousands of meters) then the force applied no longer remains constant throughout the displacement. Hence the above formula is not a general formula to calculate gravitational P.E it is applicable only to small ordinary vertical heights.

Work done against the gravitational force:

Weight (= mg) of a body is the *force* with which it is attracted by the earth. Work done in the gravitational field of the earth can be positive or negative, depending upon whether the body is allowed to move in the direction of gravity (downward) or it is moved against the direction of gravity (upward).

If a body is allowed to move freely in the direction of gravity (downward, allowed to fall freely) then work is done by the body. It will be positive, as can be seen:

$$\boxed{\text{Work done} = \vec{W} \cdot \vec{h}}$$

Where "$\vec{W}$" is weight of the body and "$\vec{h}$" is the vertical height between initial and final positions of the body, through which the body is allowed to move in the direction of gravity. Since W ∥ h (θ = 0°), the magnitude of work done is given by:

Work done = W h Cos θ

Work done = W h Cos 0° (Cos 0° = 1)

Work done = W h

Work done = m g h

Similarly, if a body is lifted (moved upward) through vertical height "h" without accelerating it, then the work is done on the body and the amount of work done is given by:

Work done = W h Cos θ

Work done = W h Cos 180° (Cos 180°= −1)

Work done = – W h
Work done = – m g h

It shows that work done against gravity (by the gravitational force, weight) is <u>negative</u>.

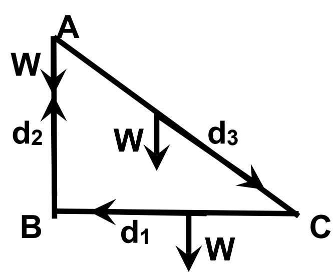
(***<u>Note that</u>*** when work is done ***<u>on</u>*** a body it <u>gains</u> and when work is done ***<u>by</u>*** a body it <u>loses</u> some form of energy).

<u>Q.No.3</u>: Prove that gravitational field is a <u>conservative</u>? (***2016 Karachi Board***)

<u>Ans</u>: <u>Proof</u>:

Any force field is said to be a conservative field if work done in it is independent of path followed and the net amount of work done in a closed path is zero.

To prove that the gravitational field is conservative, we will move a body in a closed path such as CBAC and calculate the net work done. The body is moved from C to B then B to A and finally from A to C. It means that initial and final positions of the body is point C. ***In other words***, the body is moved in a closed path CBAC.



W (= mg) is weight of the body.

Work = W d_1 Cos 90° = 0 (angle between W and d_1 is 90°)
C to B

Work = W d_2 Cos 180° = – W d_2 = – m g d_2 (angle between W & d_2 is 180°, Cos180° = –1)
B to A

Work = W d_3 Cos θ (But in triangle ABC, d_3 Cos θ = d_2)
A to C

Therefore, Work = W d_2 = m g d_2
A to C

Hence network in closed path CBAC will be:

Work = Work + Work + Work
CBAC C to B B to A A to C

Net work = 0 – m g d_2 + m g d_2
CBAC

Net work done = 0
In closed path CBAC

This shows that work done in the gravitational field is ***<u>independent of path followed</u>*** it depends only upon the vertical height d_2 (or h) between initial and final positions, also that ***<u>net work done in a closed path is zero</u>***. Hence gravitational field is a ***conservative field***.

Gravitational and electric fields are <u>conservative fields</u>.

<u>Note that</u> in a conservative field work is always ***<u>reversible</u>***. For example, when we throw a ball up in the air, it slows down as its kinetic energy is used to do work against gravity, it is converted into the gravitational potential energy. But on the way down, the potential energy is used to do the same amount of work in moving the body, hence it is converted back into kinetic energy and the ball speeds up. If there is no air resistance, the ball will land as fast as it was thrown.

General formula for gravitational potential energy.

Q.No.4: Derive a general formula for gravitational potential energy?

Ans: To derive a general formula for gravitational P.E we apply Newton's law of gravitation.

Consider a body of mass "m" lifted from point "1" to point "N" in the gravitational field of the earth through a **long distance**. Force between the body and earth over such a long distance will not be constant over such a long distance.

Hence to calculate the amount of work done to displace the body, we will divide the entire displacement into a large number of small displacements each must be so small that the magnitude of the force through that small displacement practically remains constant.

According to Newton's law of gravitation, the magnitude of force between the body and the earth at points 1 and 2 is given by:

$$F_1 = G\frac{m M_e}{r_1^2} \qquad F_2 = G\frac{m M_e}{r_2^2}$$

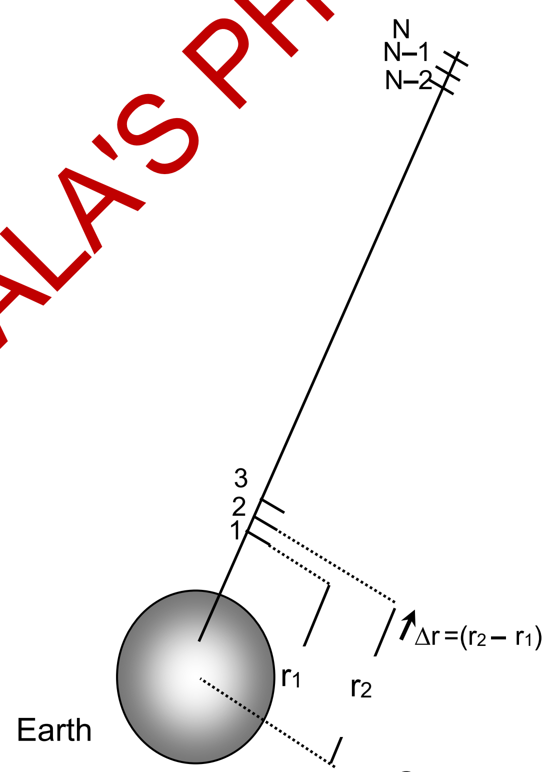
Where "M_e" is mass of the earth and "m" is mass of the body, r_1 and r_2 are the distances of points 1 and 2 respectively from the center of the earth.

Let "Δr" be the magnitude of displacement of the body from 1 to 2, then the average force between earth and the body between these points will be:

$$F_{av.} = \frac{F_1 + F_2}{2}$$

$$F_{av.} = \frac{G\frac{m M_e}{r_1^2} + G\frac{m M_e}{r_2^2}}{2} = \frac{Gm M_e\left\{\frac{1}{r_1^2} + \frac{1}{r_2^2}\right\}}{2}$$

$$F_{av.} = \frac{Gm M_e}{2}\left\{\frac{1}{r_1^2} + \frac{1}{r_2^2}\right\}$$



Work done to displace the body from 1 to 2 through "Δr" is given by:

$$\underset{1\to2}{\text{Work}} = F_{av.}\,\Delta r \quad \text{Where } \Delta r = (r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{1}{r_1^2} + \frac{1}{r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{r_2^2 + r_1^2}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

But $r_2 = r_1 + \Delta r$ $\therefore$

$$\underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{(r_1 + \Delta r)^2 + r_1^2}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$$\therefore \quad \underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{r_1^2 + 2r_1\Delta r + \Delta r^2 + r_1^2}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

Since Δr is small $\therefore \Delta r^2$ can be neglected.

$$\underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{r_1^2 + 2r_1\Delta r + r_1^2}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{2\,r_1^2 + 2r_1\Delta r}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{2\,r_1(r_1 + \Delta r)}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = Gm\,M_e\left\{\frac{r_1(r_1 + \Delta r)}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = Gm\,M_e\left\{\frac{r_1(r_1 + r_2 - r_1)}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = Gm\,M_e\left\{\frac{r_1 r_2}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = Gm\,M_e\left\{\frac{r_2}{r_1\,r_2} - \frac{r_1}{r_1\,r_2}\right\}$$

$$\underset{1\to2}{\text{Work}} = Gm\,M_e\left\{\frac{1}{r_1} - \frac{1}{r_2}\right\}$$

Since work done ***on*** a body to displace it against the gravitational force of the earth is equal to the P.E gained by it, therefore:

$$\underset{1\to2}{\text{P.E}} = Gm\,M_e\left\{\frac{1}{r_1} - \frac{1}{r_2}\right\}$$

Similarly:

$$\underset{2\to3}{\text{P.E}} = Gm\,M_e\left\{\frac{1}{r_2} - \frac{1}{r_3}\right\}$$

$$\underset{3\to4}{\text{P.E}} = Gm\,M_e\left\{\frac{1}{r_3} - \frac{1}{r_4}\right\}$$

$$\vdots \qquad\qquad \vdots$$

$$\underset{N-2\to N-1}{\text{P.E}} = Gm\,M_e\left\{\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}}\right\}$$

$$\underset{N-1\to N}{\text{P.E}} = Gm\,M_e\left\{\frac{1}{r_{N-1}} - \frac{1}{r_N}\right\}$$

The total P.E of the body between point 1 and N will be equal to the sum of the above equations and is given by:

$$\boxed{P.E_{1\to N} = Gm\,M_e\left\{\frac{1}{r_1}-\frac{1}{r_N}\right\}}$$

The above equation actually gives us ***Change in potential energy*** of the body (in this case P.E gained) between any two points such as 1 and N in the gravitational field of the earth. It is equally valid for ***short*** as well as ***long*** distance, hence it is a general formula for P.E gained or lost between any two points in the gravitational field of the earth.

Absolute potential energy:

***Q.No.5**: What is **absolute potential energy** of a body at a point?*

Ans**:* Absolute potential energy of a body at a point in the gravitational field is ***the amount of work done to move the body from that point to a very far off point.

A point which is very far from the center of the earth may be considered to be outside the gravitational field of the earth, hence the potential energy of a body at such a point will be zero.

Absolute potential energy of a body at point "1" can be calculated by assuming point "N" to be at infinity, hence on substituting $r_N = \infty$ in the following formula we get:

$$P.E_{1\to N} = Gm\,M_e\left\{\frac{1}{r_1}-\frac{1}{r_N}\right\}$$

Absolute potential energy at point "1" is given by:

$$P.E_{abs.} = Gm\,M_e\left\{\frac{1}{r_1}-\frac{1}{\infty}\right\}$$

But $\frac{1}{\infty} = 0$ $\quad\therefore\quad$ $\boxed{P.E_{abs.} = \frac{Gm\,M_e}{r_1}}$

Since the gravitational force is attractive in nature, therefore, absolute potential energy of a body is given by: $\quad\therefore\quad$ $\boxed{P.E_{abs.} = -\frac{Gm\,M_e}{r_1}}$

It means that if a body is moved from a given point to infinite distance its potential energy increases and becomes zero at infinity (It is a negative quantity).

Absolute potential energy of a body of mass "m" at the surface of the earth is given by:

$$P.E_{abs.} = -\frac{Gm\,M_e}{R_e}$$

Similarly absolute potential of the body at any height "h" above the surface of the earth is given by:

$$P.E_{abs.} = -\frac{Gm\,M_e}{R_e + h}$$

$$P.E_{abs.} = -\frac{Gm\,M_e}{R_e\left\{1+\frac{h}{R_e}\right\}}$$

$$P.E_{abs.} = -\frac{Gm\,M_e}{R_e}\left\{1+\frac{h}{R_e}\right\}^{-1}$$

According to Binomial theorem:

$$\left\{1+\frac{h}{R_e}\right\}^{-1} = 1-\frac{h}{R_e}+\frac{h^2}{R_e^2}-\ldots\ldots\ldots$$

Since "h" is small as compared to "R_e", therefore, terms with higher powers of "h" are negligible, $\quad \left\{1+\frac{h}{R_e}\right\}^{-1} = \left\{1-\frac{h}{R_e}\right\}$

Therefore,

$$\boxed{P.E_{abs.} = -\frac{Gm\,M_e}{R_e}\left\{1-\frac{h}{R_e}\right\}}$$

Escape velocity:

***Q.No.1*: What is escape velocity?**

***Ans*:** Escape velocity of a planet is the **minimum velocity** with which a body must be projected up so that it escapes gravity of that planet and never return back.

Explanation:

When we throw a body up, it rises against gravity of the earth and after reaching a certain height it starts falling back. Actually while throwing up we do some work on the body, this work done is stored in the body as kinetic energy. Due to this K.E the body now has the ability to do work against gravity. As the body rises up its K.E is used to do work against gravity. After reaching a certain height this K.E is totally used up. (K.E lost is stored as P.E) At this instant the body comes to rest for a moment and then it starts falling because of the gravitational pull.

Example: Escape velocity for earth.

$V_{esc.} = \sqrt{2\,G\,M_e / R_e}$

$V_{esc.} = \sqrt{2\times 6.67\times10^{-11}\times5.98\times10^{24}/6.38\times10^{6}}$

$V_{esc.}$ = **11182 m/s = 11.182 km/s.**

Escape velocity on the surface of earth at different locations may be slightly different because average radius of the earth is used in this formula. Radius of earth is different at different locations, Earth bulges out at the equator, here radius of earth is more, flattens a little at poles, here radius of the earth is small.

If we keep on throwing the body up with more and more velocity, **in other words**, give it more K.E it rises higher each time it will return back to Earth.

If thrown vertically upward with a certain velocity, called **escape velocity**, the body escapes gravity of the earth. Thrown with such a high velocity body has now enough K.E just to overcome gravitational pull. It now follows a parabolic path and does not return to the earth.

Derivation:

Minimum velocity required to escape the gravity (OR escape velocity) can be derived with the help of law of conservation of energy.

At the surface of earth total energy of the body will be equal to the sum of its initial K.E and Initial P.E. Similarly at the highest point its total energy will be equal to the sum of its final K.E and P.E. Hence according to law of conservation of energy:

Total initial energy = Total final energy

$$K.E_{initial} + P.E_{initial} = K.E_{final} + P.E_{final}$$

$$\tfrac{1}{2}\,m\,V_{initial}^{2} + \left(-\frac{Gm\,M_e}{R_e}\right)_{initial} = \tfrac{1}{2}\,m\,V_{final}^{2} + \left(-\frac{Gm\,M_e}{r}\right)_{final}$$

Where M_e = mass of the earth. R_e = radius of the earth. m = mass of the body

G = universal gravitational constant V = velocity of the body.

r = total distance from center of earth to highest point.

Potential energy is **negative** showing that it is attractive in nature.

If the body is thrown with escape velocity "V_{esc}" then reaching the highest point its K.E is totally used up ($K.E_{final}$ = 0) this will happen at infinite distance where its $P.E_{final}$ will also be zero, here the body will not have the ability to fall back.

Applying law of conservation of energy we get:

$$\tfrac{1}{2}\,m\,V_{esc}^{2} - \frac{G\,m\,M_e}{R_e} = 0 + 0$$

$$V_{esc}^{2} = 2\,\frac{G\,M_e}{R_e}$$

$$V_{esc} = \sqrt{\frac{2\,G\,M_e}{R_e}}$$

This formula gives us escape velocity at the surface of earth (M_e and R_e are mass and radius of earth).

This formula can be used to calculate escape velocity for any planet, using mass of the planet and its radius.

Escape velocities required for some planets using this formula are:

For earth: V_{esc} **= 11.2 km./sec.** For moon: V_{esc} **= 2.38 km./sec.**

For sun: V_{esc} **= 618 km./sec.**

We can see from these values that escape velocity depends upon **mass** and **radius** of a planet. Acceleration due to gravity "g" on the surface of a planet can also be used to calculate its escape velocity. Since $g = \frac{G\,M}{R^2}$

On substituting the value of g in escape velocity formula and then simplifying it we get:

$$V_{esc} = \sqrt{2\,g\,R}$$

- This formula can be used to calculate escape velocity of any planet provided the acceleration due to gravity "g" on its surface and its radius "R" are known.

Both the above formulae give us the same value of escape velocity for a planet.

If velocity of a body is less than the escape velocity the body will return back to the given planet. Hence escape velocity is the **minimum velocity required to escape the gravity of a planet**.

Power:

Q.No.1: What is power? Give its dimension.

Ans: ***"Rate of doing work or rate of transferring energy is called power".***

(or it is also equal to the rate of conversion of energy from one form to another).

OR

Power is the **scalar** or **dot product** of force $\vec{F}$ and velocity $\vec{v}$.

- Dimensions of power are $[M\,L^2\,T^{-3}]$.
- Power is a **scalar** quantity.

$$\text{Power} = \frac{\text{work done or energy transferred}}{\text{Time taken}}$$

$$\text{Power} = \frac{\vec{F}\cdot\vec{d}}{t}$$

But $\frac{\vec{d}}{t} = \vec{v}$

$$\boxed{\text{Power} = \vec{F}\cdot\vec{v}}$$

Average power is given by:

$$\text{Power}_{Av.} = \frac{\text{Work done}}{\text{Total time taken}}$$

$$\text{Power}_{Av.} = \frac{\vec{F}\cdot\vec{d}}{\Delta t}$$

OR $\text{Power}_{Av.} = \vec{F}\cdot\vec{V}_{Av.}$

Instantaneous power "Power $_{Inst.}$" (power at a particular instant or at a particular time) can be calculated by imposing a limit on time taken, in this case time taken must be very short and must approach to zero (not equal to zero).

$$Power_{Inst.} = \vec{F} \cdot \lim_{\Delta t \to 0} \frac{\vec{d}}{\Delta t}$$

OR $$Power_{Inst} = \vec{F} \cdot \vec{V}_{inst.}$$

Power is a ***scalar*** quantity.

Q.No.2*: *What is the S.I unit of power?

Ans*:** Unit of power is "watt***".

Power is said to be 1 watt when 1 joule of work is done in one second.

$$1 \text{ watt} = \frac{1 \text{ Joule}}{1 \text{ second}} \qquad 1 \text{ W} = 1\text{J/s}$$

1 kilowatt (kw) = 10^3 w & 1 megawatt (Mw) = 10^6 w

Since Power = $\frac{\text{Work or energy}}{\text{Time}}$ ∴ Work (Or energy) = Power × time

🗣Hence the unit of work (or energy) can also be expressed as a product of unit of power and the unit of time.

If power is expressed in kilowatt (kw) and time in hours (h) then the unit of work (Or energy) will be ***kilowatt-hour (kwh)***:

1 kilowatt-hour (kwh) = 1 kw ×1h

1 kwh = 1000 w × 3600 s (1w = 1j/s)

1 kwh = 3.6 ×10^6 Joules.

1 kwh = 3.6 MJ (1 unit of electrical energy)

🗣On commercial scale electrical energy is measured in "kwh".

(We can define 1 kwh (Or one-unit electrical energy) as the power delivered by an electrical machine of one kilowatt running continuously for one hour). As an example. If a pressing iron of 1 kw is continuously on for 1 h it will convert 3.6×10^6 Joules of electrical energy into heat energy.

***Law of conservation of energy*:**

Q.No.1*: *State law on conservation of energy?

***Ans*:** Energy can neither be created nor destroyed.

One form of energy can be converted into another and so on, but the conversion from one form to another is always such that the total amount of energy before and after the conversion is equal.

🗣***Note that*** when a piece of paper is burnt we will get heat, light along with ashes and carbon dioxide. If we study this simple chemical reaction we will find that during this process ***energy is not created nor is the matter destroyed***.

We know that paper is obtained after chemically processing a certain plant. This plant grows using water and fertilizers from the soil and solar energy. (solar energy during photosynthesis, chemical energy during the formation of new bonds etc. are stored). **In other words**, energy was already stored in paper in different forms. This stored energy is converted into heat and light etc. when we burn it. Experiments show that the total energy stored comes out to be exactly equal to the energy obtained after burning process is complete. Similarly, matter is also converted from one state to another, such as from paper and oxygen of air into carbon dioxide and ashes etc. During this process new bonds are formed. (In general, combustion is an ordinary exothermic chemical process).

🗣***Also note that*** energy can neither be created nor destroyed similarly matter can neither be created nor destroyed. ***It means that*** matter and energy cannot be converted into each other. These are old concepts and are valid under normal conditions.

According to Einstein, matter and energy are **interconvertible**. It means that matter can be converted into energy and vice versa, this conversion takes place according to Einstein's famous equation: $E = m\,c^2$

In **fission and fusion reactions**, applied in atom bomb and hydrogen bomb, tremendous amount of energy is released due to the **conversion of mass into energy**. The amount of energy released reactions (fission or fusion) is enough to destroy a whole city. This energy can also be used for peaceful purpose by converting into some useful form such as electricity as is done in nuclear reactors.

In **pair production** electron - positron pair is produced by **converting energy into matter**. Both electron and positron are matter particles.

Interconversion of K.E and P.E:

Consider a body of mass "m" held at a certain height "h" above the ground. At this point velocity of the body is zero, as it is at rest, hence its kinetic energy will also be zero. Relative to the ground it will have potential energy "mgh".

- Now if the body is released and allowed to fall freely, its velocity keeps on increasing as it falls, ***in other words,*** its kinetic energy will **increase**. But since it falls down its height "h" above the ground decreases and hence its potential energy also **decreases**. It is found that in the absence of any opposing force (such as air resistance), while falling down body will lose P.E and at the same time gain an equal amount of K.E. Hence P.E possessed by the body at the starting point is totally converted into K.E on reaching the ground. ***In other words***, it will have maximum velocity just before hitting the ground. Hence we can write:

P.E lost = K.E gained

If "V" is the velocity with which the body reaches the ground then:

$$m\,g\,h = \frac{1}{2}\,m\,V^2$$

- If a body is thrown vertically upward with velocity "V" (or thrown upward with some K.E) then while going up against gravity its velocity decreases i.e. its K.E energy decreases, it is used to do work against gravity due to which its P.E will increase. After reaching the highest point its velocity reduces to zero. Here it has maximum P.E and zero K.E. While going up body keeps on loosing its K.E and at the same time gains an equal amount of P.E. In this case"

K.E lost = P.E gained

$$\frac{1}{2}\,m\,V^2 = m\,g\,h$$

While falling down **if there is an opposing force** (any frictional force, such as air resistance) then a part of P.E lost by the body will be used to do work against the opposing force. In this case:

P.E lost = K.E gained + work done against frictional force

$$m\,g\,h = \frac{1}{2}\,m\,V^2 + f\,h$$

Where "f" is the force of friction.

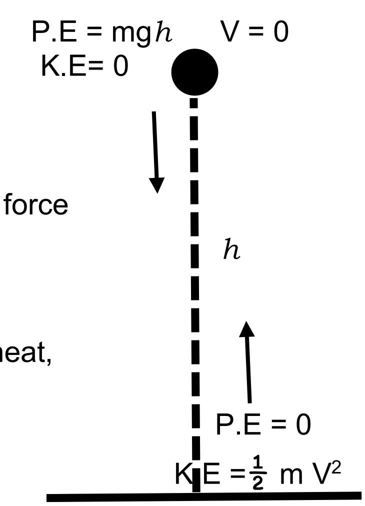
In this case the body will come down with **lesser speed**. (lesser K.E)

While going up **if there is an opposing force** (any frictional force) then a part of K.E of the body will be used to do work against the opposing force. In this case:

K.E lost = P.E gained + work done against frictional force

$$\frac{1}{2}\,m\,V^2 = m\,g\,h + f\,h$$

In this case the body will rise to a **smaller height**.



Note that: energy used to do work in both the cases appears as heat, This heat is immediately taken away by the surrounding atmosphere.

Section–A Multiple-Choice Questions MCQs:

Q.No.1: Work done by centripetal force is always:
(A) Maximum. (B) Minimum.
(C) Zero. (D) None of these.

Q.No.2: A body of mass 5 kg. is moving with a momentum of 10 kg m/s. A force of 0.2 N acts on it in the direction of motion of the body for 10 sec. The increase in kinetic energy is:
(A) 2.8 J (B) 3.2 J (C) 3.8 J (D) 4.4 J

Q.No.3: Kinetic energy of a light body and a heavy body is same. Which body has maximum momentum?
(A) Light body. (B) Heavy body.
(C) Both have same momentum.
(D) none of them.

Q.No.4: Two bodies of masses 1 kg and 2 kg have equal momentum. The ratio of their kinetic energies is:
(A) 2:1 (B) 3:1 (C) 1:3 (D) 1:1

Q.No.5: A body falls from height "h". After it has fallen a height h/2, it will possess:
(A) Only potential energy.
(B) Only kinetic energy.
(C) half potential half kinetic.
(D) more kinetic less potential.

Q.No.6: Which of the following quantity can be multiplying force and velocity?
(A) acceleration. (B) power.
(C) torque. (D) work.

Q.No.7: The minimum velocity given to an object so that it emerges out from the gravitational field of earth is about:
(A)11.2 km/s. (B)15.3 km/s.
(C) 5 km/s. (D) 9.8 km/s

Q.No.8: When one joule of work is done on a body in one second, power of body is said to be: (A) One watt. (B) 0.5 watt.
(C) zero. (D) 100 watts

Q.No.9: The absolute potential energy of an object depends on:
(A) Object's mass and height.
(B) Object's mass and speed
(C) Object's shape and size.
(D) Object's color and temperature.

Q.No.10: The escape velocity of a planet depends on which of the following factors?
(A) Mass of the planet only.
(B) Radius of the planet only.
(C) Both mass and radius of the planet.
(D) Density of the planet.

Q.No.11: When a body moves vertically, the work done will be:
(A)Positive (B)Negative (C)Zero (D) Maximum

Q.No.12: If speed of a body is halved, its kinetic energy becomes:
(2011 Karachi Board)
(A)One fourth (B)Half(C)Three times(D)None of these

Q.No.13: The work done by a conservative force along a closed path is:
(2008, 2011 Karachi Board)
(A)Positive (B)Negative (C) Zero (D)None of these

Q.No.14: A bucket of mass 10 kg. is moved downward in the gravitational field through a distance of 1 m. The work done in this case is equal to: ***(2012 Karachi Board)***
(A)10 J (B) 98 J (C) – 98 J (D) 0.1 J

Q.No.15: The rate of doing work is zero when the angle between force and velocity is: ***(2012 Karachi Board)***
(A) 0°. (B) 45°. (C)180°. (D) 90°.

Q.No.16: A weight lifter consumes 500 J of energy to lift a load in 2 second. The power used by him is: ***(2013 Karachi Board)***
(A)125 watt. (B) 250 watt.
(C) 500 watts. (D) 1600 watt

Q.No.17: If mass and speed both are doubled, the kinetic energy will be:
(2014,05Karachi board)
(A) Double. (B) Four times.
(C) Six times. (D) Eight times.

Q.No.18: Kilowatt hour is the unit of:
(2014 Karachi board)
(A) Energy (B) Power (C) Time (D) Force

Q.No.19: This one of the following is not the unit of power: (***2015 Karachi Board***)

(A) horse-power. (B) joule/sec.
(C) kilowatt-hour. (D) foot-pound/sec

Q.No.20: Both kilowatt hour and electron volt are the units of: (***2017 Karachi Board***)

(A) power. (B) energy.
(C) charge. (D) angular momentum

Q.No.21: A 600N man runs up a stair of 4 m height in 3 seconds. The power needed is: (***2018 Karachi Board***)

(A) 240 watts. (B) 350 watts.
(C) 450 watts. (D) 800 watts.

Q.No.22: If velocity a body is doubled and its mass is reduced to one fourth, its kinetic energy will be: (***2022, 18,12 Karachi Board***)

(A) doubled (B) unchanged (C) halved (D)four-fold

Q.No.23: one kilowatt-hour is equal to: (***2022, 2016 Karachi Board***)

(A) 3.6×10^6 J. (B) 3.9×10^6 J.
(C) 3.6×10^8 J. (D) 3.6×10^9 J.

Q.No.24: The dot product of force and velocity is: (***2022 Karachi Board***)

(A) Work, (B) Power.
(C) Momentum. (D) Energy.

Q.No.25: If $\vec{F} = 3\,\hat{i}$ and $\vec{d} = 6\,\hat{j}$, then work done will be: (***2022 Karachi Board***)

(A) $2\,\hat{j}$ (B) $3\,\hat{j}$ (C) $6\,\hat{j}$ (D) $0\,\hat{j}$

Q.No.26: A boy pushes a toy car on a horizontal floor with a force of 10N up to a displacement of 2m, work done by gravity on the car will be: (***2022 Karachi Board***)

(A) 20J (B) 10J (C) 5J (D) 0

Q.No.27: A force acting on a body is perpendicular to its displacement, the work done is equal to:(***2022 Karachi Board***)

(A) Positive. (B) Negative.
(C) Zero. (D) Infinite.

Q.No.28: A field of force in which the work is independent of path is called a … field. (***2014 Hyderabad Board***)

(A) Conservative. (B) Non conservative.
(C) Energetic. (D) None of these.

Q.No.29: The work done on a body is zero, then angle between the applied force and displacement is: (***2015 Hyderabad Board***)

(A) 0° (B) 45° (C) 90° (D) 180°

Q.No.30: The dot product of force and velocity is …(***2016 Hyderabad Board***)

(A) Work. (B) Power.
(C) Energy. (D) Torque.

Q.No.31: The rate of change of energy is called ………. (***2019 Hyderabad Board***)

(A) Work (B) Power
(C) Force (D) None of these

Q.No.32: Work done by centripetal force is: (***2025 Karachi Board***)

(A) Maximum. (B) Negative.
(C) Zero. (D) Infinite.

Q.No.33: The minimum velocity required for an object to escape earth's gravitational field is approximately: (***2025 Karachi Board***)

(A) 11.2 km/s. (B) 5 km/s.
(C) 15.3 km/s. (D) 9.8 km./s.

Q.No.34: Work done in lifting 30 kg. mass to a height of 20 m is: (when g = 9.8 m/s^2) (***2026 Karachi Board***)

(A) 2800 J. (B) 600 J.
(C) 5880J. (D) 2910 J.

Answers:

(1) (C) Zero.
(2) (D) 4.4 J
(3) (B) Heavy body.
(4) (A) 2:1
(5) (C) half potential half kinetic.
(6) (B) power.
(7) (A)11.2 km/s.
(8) (A) One watt.
(9) (A) Object's mass and height.
(10)(C)Both mass and radius of the planet
(11) (B)Negative.
(12) (A) One fourth.
(13) (C) Zero
(14) (B) 98 J
(15) (D) 90°
(16) (B) 250 watt.
(17) (D) Eight times.
(18) (A) Energy
(19) (C) kilowatt-hour.
(20) (B) energy.
(21) (D) 800 watts.
(22) (B) unchanged
(23) (A) 3.6×10^6 J.
(24) (B) Power.
(25) (D) 0 j
(26) (D) 0
(27) (C) Zero.
(28) (A) Conservative.
(29) (C) 90°
(30) (B) Power.
(31) (B) Power
(32) (C) Zero.
(33) (A) 11.2 km/s.
(34) (C) 5880J.

Numericals:

Q.No.1: A man pulls a trolley through 10 m by applying a force of 50 N which makes an angle of 60° with the horizontal. Calculate the work done by the man.

Data:

Force applied $F = 50\ N$
Distance covered $d = 10\ m$
Angle between force and displacement $\theta = 60°$
Work done $W = ?$

Solution:

Work done is given by Work $= F\ d\ \cos\theta$

Work $= 50 \times 10 \times \cos 60°$

Work $= 500 \times 0.5$ $\boxed{\text{Work} = 250\ J}$

Q.No.2: A 100 kg man runs up a long stair in 9.8 second. The vertical height of the stairs is 10 m. Calculate its power.

Data:

Mass of man $m = 100\ kg$
Time taken $t = 9.8$ second
Vertical height $h = 10\ m$
Power $P = ?$

Solution:

As the man runs up the stairs he does work against gravity, rate of doing work is called power. But work done against gravity is given by:

Work against gravity $= m\ g\ h$

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{m\ g\ h}{t}$$

$$\text{Power} = \frac{100 \times 9.8 \times 10}{9.8} \qquad \boxed{\text{Power} = 1000\ \text{Watt}}$$

Q.No.3: When an object is thrown upward it rises to a height "h". How high is the object in terms of "h" when it has lost one third of its original kinetic energy?

Data:

Total height reached $= h$
Height at which the object looses one third it's K.E $x = ?$

Solution:

When an object is thrown upward it looses K.E and at the same time gains an equal amount of P.E, provided there is no air resistance.

$\therefore$ K.E lost = P.E gained

$$\therefore \tfrac{1}{2} m v^2 = m g h \qquad \text{OR} \qquad v^2 = 2 g h$$

Where "v" is the velocity with which the object was originally thrown upward.

Let "x" be the height at which the object losses one third its original K.E.

$\therefore$ Amount of K.E lost = P.E gained

$\tfrac{1}{3} \times$ (original K.E) = P.E gained

$$\tfrac{1}{3} \times \left(\tfrac{1}{2} m v^2\right) = m g x$$

On substituting the expression for v^2 in the above equation we get:

$$\tfrac{1}{3} \times \left(\tfrac{1}{2} m \times 2 g h\right) = m g x \qquad \boxed{x = \tfrac{1}{3} h}$$

Q.No.4: A 70 kg man runs up a hill through a height of 3 m in 2 second.

(a) How much work is done against gravitational field =?

(b) What is the average power output?

Data:

Mass $m = 70$ kg. Height $h = 3$ m Time $t = 2$ s.

(a) work done against gravity = ?

(b) power $P = ?$

Solution:

(a) Work done against gravity $= W\ h\ \cos\theta$

$W = mg$ is man's weight (downward)

Work done $= m\ g\ h\ \cos 180°$

(Force W is opposite to displacement h)

Work done $= 70 \times 9.8 \times 3 \times -1 =$ **– 2058 Joules,**

(b) Power $= \dfrac{\text{work}}{\text{Time}} = \dfrac{2058}{2} =$ **1029 watt**

Q.No.5: A neutron travels a distance of 12 m in a time interval of 3.6×10^{-4} second. Assuming its speed to be constant, find its K.E.

Mass of neutron = 1.7×10^{-27} kg.

Data:

Distance covered $S = 12\ m$
Time taken $t = 3.6 \times 10^{-4}$ second
Mass of neutron $m = 1.7 \times 10^{-27}$ kg.
Kinetic energy K.E =?

Solution:

$$\text{Speed } v = \frac{\text{distance}}{\text{Time}} = \frac{12}{3.6 \times 10^{-4}} = 3.33 \times 10^4\ m/s$$

$$K.E = \tfrac{1}{2} m v^2 = \tfrac{1}{2} \times 1.7 \times 10^{-27} \times (3.33 \times 10^4)^2$$

K.E = 9.44×10^{-19} Joules.

Q.No.6: A stone is thrown vertically upward so that it can reach a height of 10 m. Find the speed of stone when it is just 2 m above the ground.

Data:

Height $h = 10\ m$
Height above the ground $= 2\ m$
Speed 2 m above ground $v = ?$

***Solution*:**

K.E with which stone must be thrown so that it reaches 10 m height can be found by:

K.E at the ground = P.E at the highest point

$$\frac{1}{2} m v^2 = m g h$$

$$v = \sqrt{2 g h} = \sqrt{2 \times 9.8 \times 10} = \sqrt{196} = 14 \text{ m/s}$$

Hence speed with which stone is thrown up to reach 10 m height is **14 m/s.**

Considering the downward motion of stone:

$v_i = 0$ at 10 m height, $v_f = ?$ at 2m above ground

$a = g$, $S = 10 - 2 = 8$ m

Applying equation of motion: $2 a S = v_f^2 - v_i^2$

$$2 \times 9.8 \times 8 = v_f^2 - (0)^2$$

$$156.8 = v_f^2 \quad v_f = \sqrt{156.8} = \mathbf{12.5 \text{ m/s.}}$$

Speed 2 m above the ground will be **12.5 m/s.**

***Q.No.7*: The potential energy of a body at the top of a building is 200 Joule. When it is dropped its K.E just before striking the ground is160 Joule. Find work done against the air resistance.**

***Data*:**

P.E at the top = 200 J
K.E just before striking ground = 160 J
Work done against air resistance W =?

***Solution*:**

While falling:

P.E lost by the body = K.E gained by it + work done against air resistance.

200 = 160 + Work done

Work done against air resistance = 200 – 160 = **40 J**

***Q.No.8*:Find the energy equivalent of 1 gram.**

***Data*:**

Mass m = 1 gram. = 0.001 kg
Energy E =?

***Solution*:**

According to Einstein's matter can be converted into energy and vice versa, according to following equation:

$$E = m c^2$$

$$E = 0.001 \times (3\times10^8)^2$$

$$E = \mathbf{9 \times 10^{13} \text{ Joules}}$$

Energy equivalence of 1 gram is **9×10^{13} Joules.**

***Q.No.9*: A 1 kilowatt motor pump pumps water from ground to a height of 10 m. Find how much liters of water it can pump in one hour.**

(Density ρ of water = 1000 kg/m^3)

***Data*:**

Power	P = 1 Kw = 1000 watt
Time	t = 1 h = 3600 sec.
Height	h = 10 m
Density of water	ρ = 1000 kg/m^3)
Number of liters pumped= ?	

***Solution*:**

$$\text{Power } P = \frac{\text{work to pump water for 1 hour}}{\text{Time}}$$

If "m" is mass of water pumped, then

OR $P t = m g h$

$$1000 \times 3600 = m \times 9.8 \times 10$$

$$m = \frac{1000\times3600}{9.8 \times 10} = 36{,}735 \text{ kg.}$$

But density of water $\rho = \dfrac{\text{mass of water m}}{\text{Volume of water V}}$

$$V = \frac{m}{\rho} = \frac{36735}{1000} = \mathbf{36.735 \text{ m}^3.}$$

But 1 m^3 = 1000 liter

Volume of water that can be pumped in one hour will be V = 36.735× 1000 = 36735 lit.

= 3.67×10^4 liters

***Q.No.10*: A rocket of mass 2 kg. is launched in air, when it attains height of 15 m the 400 Joules of its chemical fuel burns. Find speed of the rocket at maximum height.**

***Data*:**

Mass of the rocket	m = 2 kg.
Vertical height attained	*h* = 15 m
Fuel burnt = Work done	W = 425 J
Velocity at maximum height	v = ?

***Solution*:**

Due to the burning fuel work is done on the rocket and it moves upward against gravity with an increasing velocity. Hence it gains K.E as well as P.E as it rises:

Work done = K.E gained+ P.E gained

$$\text{Work done} = \frac{1}{2} m v^2 + m g h$$

$$400 = \frac{1}{2}\, 2 \times v^2 + 2 \times 9.8 \times 15$$

$$400 = v^2 + 294$$

$$v^2 = 400 - 294 = 106$$

$$v = \sqrt{106} \qquad \boxed{v = 10.3 \text{ m/s}}$$

Velocity of rocket at the highest point (15 m above the ground) is ***10.3 m/s***.

***Q.No.11*: A motor pumps water at the rate of 500 gm. /min to a height of 120 m. If the motor is 50% efficient then how much input electric power is needed?**

Data:

Mass of water pumped = 500 gm/min = 0.500 kg/60 sec.

Height h = 120 m. Efficiency E = 50%

Input power =?

Solution:

Output power $= \frac{m\,g\,h}{t} = \frac{0.500\times9.8\times120}{60}$

Output Power = 9.8 watt.

Efficiency $= \frac{\text{Output power}}{\text{Input power}} \times 100\%$

Input power $= \frac{\text{output power}}{\text{Efficiency}} \times 100\%$

Input power $= \frac{9.8}{50}\times100\%=$ **19.6 Watt**

19.6 Watt (or approximately **20 Watt**) Input power will be needed.

Q.No.12:**Calculate the work done by a force given by $\vec{F} = 6\hat{i} + 8\hat{j} + 10\hat{k}$ in displacing a body from the position A to the position B. The position vectors of A and B are:**

$$\vec{r}_A = 4\hat{i} + 7\hat{j} + 4\hat{k}$$

$$\vec{r}_B = 9\hat{i} + 5\hat{j} + 7\hat{k}$$

(***1994 Karachi Board***)

Solution:

Displacement of the body is given by:

$\Delta\vec{r} = \vec{r}_B - \vec{r}_A$

$\therefore\quad \Delta\vec{r} = (9\hat{i}+5\hat{j}+7\hat{k}) - (4\hat{i}+7\hat{j}+4\hat{k})$

$\Delta\vec{r} = 9\hat{i}+5\hat{j}+7\hat{k}-4\hat{i}-7\hat{j}-4\hat{k}$

$\Delta\vec{r} = 5\hat{i}-2\hat{j}+3\hat{k}$

But Work done $= \vec{F}\cdot\Delta\vec{r}$

$\therefore$ Work done $= (6\hat{i}+8\hat{j}+10\hat{k}).(5\hat{i}-2\hat{j}+3\hat{k})$

Since $\hat{i}\cdot\hat{j} = 0$, $\hat{j}\cdot\hat{k} = 0$ and $\hat{k}\cdot\hat{i} = 0$

$\therefore$ Work done $= 30\,\hat{i}\cdot\hat{i} - 16\,\hat{j}\cdot\hat{j} + 30\,\hat{k}\cdot\hat{k}$

But $\hat{i}\cdot\hat{i}=1$, $\hat{j}\cdot\hat{j}=1$ and $\hat{k}\cdot\hat{k}=1$

$\therefore$ Work done = 30 –16 + 30

Work done = 44 Units

(If $\vec{F}$ and displacement $\Delta\vec{r}$ are expressed in M.K.S units then the unit of work done will be in ***Joules***.

Q.No.13: **An object weighing 98 N is dropped from a height of 10 m. It is found to be moving with a velocity of 12 m/s just before it hits the ground. How large was the frictional force acting upon it?**

(***2012 Karachi Board***)

Data:

Weight W = mg = 98 N

Mass m = W/g = 98/9.8 = 10 kg.

Height h = 10 m.

Velocity V = 10 m/s.

Frictional force f = ?

Solution:

Loss of P.E=K.E gained + work against friction

$m\,g\,h = \frac{1}{2} m V^2 + f h$

$98 \times 10 = \frac{1}{2}\times10\times(12)^2 + f\times 10$

$980 = 720 + 10f$

$980 - 720 = 10f$

$10f = 260$

$f = \frac{260}{10}$

$\boxed{f = 26\text{ N}}$

Q.No.14: **A force $\vec{F} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ displaces a body through $\vec{r} = 2\hat{i} - \hat{k}$. Calculate the work done.** (***2012 Hyderabad Board***)

Solution:

Work $= \vec{F}\cdot\vec{r}$

OR Work $= (3\hat{i} + 4\hat{j} - 5\hat{k}) . (2\hat{i} - \hat{k})$

Since $\hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{i} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

Therefore,

Work $= (6\hat{i}\cdot\hat{i} + 5\,\hat{k}\cdot\hat{k})$

But $\hat{i}\cdot\hat{i} = \hat{k}\cdot\hat{k} = 1$

Work = 6 + 5

$\boxed{\textit{Work = 11 Units}}$

Q.No.15: **A pump is needed to lift water through a height of 3 m at the rate of 600 gm/min. What must the minimum power of the pump be?** (***2015 Hyderabad Board***)

Data:

Height through which water is to be lifted h = 3 m

Rate at which water is to be lifted = 600 gm/min.

Power P = ?

Solution:

Since the rate of lifting water is 600 g/min. Mass of water to be lifted in one min.is 600 g.

$\therefore$ Mass of water to be lifted in one second is 600/60 gm = 10 gm = 0.010 kg.

Rate of doing work to lift water = Rate at which it gains P.E.

$$= \frac{m\,g\,h}{t} \qquad (t = 1 \text{ sec})$$

$$= \frac{0.01 \times 9.8 \times 3}{1} \text{ J/s} = 0.294 \text{ J/s.}$$

But the rate of doing work is called power.

Power needed to lift water = 0.294W