According to new syllabus of all intermediate boards of Sindh.

Prof. Rawala's ....

PHYSICS

HELPING BOOK

For
Class XI
(All science students)

UNIT -4

Rotational and circular motion

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Thoroughly revised and updated 2026 – 2027

- ***Detailed notes.***
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Unit.4**.....**

Rotational and circular motion

Angular motion:

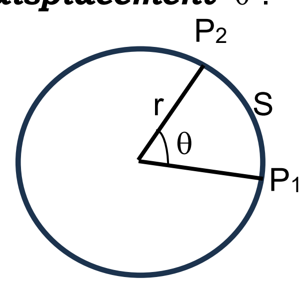
When a body either spins or moves along a circular path its motion is called ***angular motion*** **(*or circular motion*).**

Angular displacement "θ":

***Q.No.1*: What is angular displacement?**

Ans*:** Angle subtended by a body or a point at the center of its circular path is called its ***angular displacement, represented by θ.

Let a body be moving along a circular path of radius "r" and P_1 and P_2 be its instantaneous positions. Length of the arc between points P_1 and P_2 represents the ***linear displacement*** "S" of the body, whereas the angle subtended by the arc at the center of the circular path represents its ***angular displacement*** "θ".



Unit of angular displacement:

***Radian*:**

The S.I unit of angular displacement "θ" is ***radian***.

- Angular displacement "θ" is said to be one radian (1 rad.) if length of the arc "S" is equal to radius "r" of the circular path. Hence this unit of angle can be defined as the ratio of length of the arc S and radius r of the circle.

$$\theta = 1 \text{ radian} \quad \text{when} \quad S = r$$

$$\therefore \qquad \theta = \frac{S}{r} \qquad\qquad \boxed{S = r\,\theta}$$

This equation is valid only when "θ" is measured in radian.

Degree

Most commonly used unit for the measurement of angle is ***degree***.

- If circumference of a circle is divided in 360 equal parts then the angle subtended by each part at the center of the circle is called **one degree 1°**, ***In other words*** for one complete revolution, angle subtended at the center of the circle will be 360°.
- Since "θ" is the ratio of two similar quantities (S and r), therefore, it is a ***dimensionless*** quantity.

Relation between degree and radian:

For one complete revolution of a body, the angle subtended at the center of the circle is 360° whereas the distance covered by the body along the circular path is equal to the circumference 2 π r of the circle. Therefore,

$S = 2\pi r$ & $\theta = 360°$ (for one complete revolution)

But $\theta = \frac{S}{r}$ Therefore, $\theta = \frac{2\pi \cancel{r}}{\cancel{r}}$

$\theta = 2\pi$ radian (for one complete revolution)

Therefore, 2π radian = 360°

1 radian = 57.296° or $\boxed{1 \text{ radian} = \mathbf{57.3°}}$

Similarly, $1° = 2\pi/360° = 0.0175$ radian.

Angular velocity "ω":

***Q.No.2**: What is angular velocity?*

Ans: Rate of change of angle at the center of the circular path is called ***angular velocity*** "ω" of the body.

OR it is the rate of change of angular displacement of a body.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Angular velocity "ω" is a ***vector quantity***. Its direction is determined by ***right hand Rule***. It is always directed ***along the axis of rotation*** and depends upon the direction of angular motion of the body.

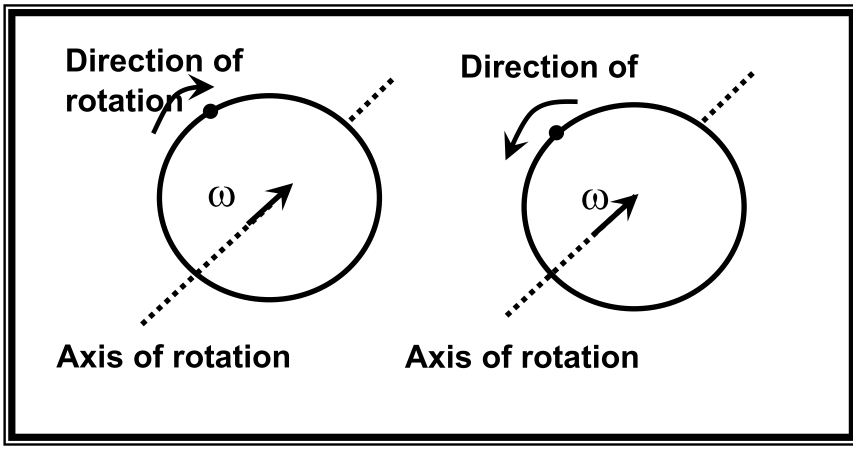
- If a body is moving in ***clockwise*** direction then "ω" will be directed ***into the paper away from the reader*** along the axis of rotation (or revolution).
- If a body is moving in ***anticlockwise*** direction then "ω" will be directed ***out of paper*** or ***towards the reader*** along the axis of rotation (or revolution).

***Q.No.3**: How is direction of angular velocity found?*

Ans: ***To find*** the direction of angular velocity 'ω' of a body, first of all curl the fingers of your right hand in the direction of motion of the body (clockwise or anticlockwise) about the axis of rotation then the stretched thumb will point in the direction of angular velocity. This is also known as ***right hand rule***.

- S.I unit of angular velocity "ω" is ***rad. /sec (radian per second)*** or s^{-1}.
- Angular velocity is sometimes called ***angular frequency***.

Note that every point on a rotating object has the ***same*** angular velocity where as its linear velocity at any point will be along the tangent (also called tangential velocity) to the circular path and its value will be proportional to distance of the point from axis of rotation of the body.



Average angular velocity "ω_{av}":

Q.No.4: What is average angular velocity?

Ans: It is the total angular displacement divided by total time taken

OR It is the angular velocity of a body during any interval of time.

$$\boxed{\omega_{av} = \frac{\Delta\theta}{\Delta t}}$$

Where '$\Delta\theta$' is the total angular displacement of the body during any time 'Δt".

Instantaneous angular velocity "$\omega_{inst.}$":

Q.No.5: What is instantaneous angular velocity?

Ans: It is the ***rate of angular displacement*** of a body at a particular instant or at a particular point.

OR It is the ***angular velocity*** of a body at a particular instant or at a particular point.

Instantaneous angular velocity of a body can be determined by finding the angular displacement of the body during a very short interval of time 'Δt' just before or after the instant, and then dividing it by that short time interval. Time interval 'Δt' is so short that it approaches to zero (it is not exactly zero) Hence:

$$\omega_{inst.} = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta\theta}{\Delta t}$$

Uniform angular velocity:

Q.No.6: What is uniform angular velocity?

Ans: If a body has equal angular displacement in equal intervals of time, however short the time interval may be, then it is said to have ***uniform angular velocity***.

When the angular velocity of a body is uniform its average and instantaneous angular velocities are equal, i.e.

$$\boxed{\omega_{av} = \omega_{inst.}} \qquad \boxed{\omega = \text{uniform}}$$

In other words, if a body is moving with a uniform angular velocity the angular velocity of the body will be equal whether it is measured in a long or a short time interval.

Angular acceleration:

Q.No.7: What is angular acceleration?

Ans: Rate of change of angular velocity of a body is called its angular acceleration.

$$\boxed{\vec{\alpha} = \frac{\Delta \vec{\omega}}{\Delta t}}$$

Where '$\Delta \vec{\omega}$' is the change in angular velocity in time 'Δ t'

If 'ω_i' is the initial and 'ω_f' is the angular speed of a body after some time 'Δt", then the magnitude of its angular acceleration 'α' is given by:

$$\boxed{\alpha = \frac{\omega_f - \omega_i}{\Delta t}}$$

- Angular acceleration "α" is a ***vector quantity***.
- S.I unit of angular acceleration is ***rad./s²***.
- Angular acceleration of a body is said to be 1 rad./s² if its angular velocity changes by 1 rad./s in 1 second.

In other words, angular motion of a body is said to be accelerated if its angular velocity is changing (increasing or decreasing).

Average angular acceleration:

***Q.No.8**: What is average angular acceleration?*

Ans: It is the total change in angular velocity divided by total time taken

OR

It is the angular acceleration of a body during any interval of time.

$$\boxed{\alpha_{av} = \frac{\Delta \omega}{\Delta t}}$$

Where '$\Delta \omega$' is the total change in angular velocity of the body during any time 'Δt".

Instantaneous angular acceleration:

***Q.No.9**: What is instantaneous angular acceleration?*

Ans: It is the angular acceleration of a body at a particular instant or at a particular point.

OR it is the rate of change of angular velocity during a very short interval of time or at a particular point.

- Instantaneous angular acceleration of a body can be determined by finding the change in angular velocity '$\Delta \omega$' during a very short interval of time 'Δt' just before or after the instant, and then dividing it by that short time interval. 'Δt' is so short that it approaches to zero (it is not exactly zero) Hence:

$$\alpha_{inst.} = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$$

Uniform angular acceleration:

***Q.No.10**: What is uniform angular acceleration?*

Ans: If there is equal change in the angular velocity of a body in equal intervals of time, however short the time interval may be, then it is said to have ***uniform angular acceleration***.

- When the angular acceleration of a body is uniform its average and instantaneous angular accelerations are equal, i.e.

$$\boxed{\alpha_{av} = \alpha_{inst.}} \qquad \boxed{\alpha = \text{uniform}}$$

In other words, if a body has a uniform angular acceleration then there will be equal change in its angular velocity in equal intervals of time whether it is measured in a long or a short time interval.

Relation between linear and angular quantities:

Consider a point moving along a circular path of radius 'r', such that it covers a linear distance 'ΔS' along an arc which makes an angle '$\Delta\theta$' at the center of circular path in time 'Δt" or ***in other words***, angular displacement of the body is '$\Delta\theta$' in time 'Δt".

Hence: $$\Delta S = r\,\Delta\theta$$

On dividing both sides of this equation by Δt (time taken)

$$\frac{\Delta S}{\Delta t} = r\,\frac{\Delta\theta}{\Delta t}$$

But $\frac{\Delta S}{\Delta t} = V$ (Linear velocity of the body)

And $\frac{\Delta\theta}{\Delta t} = \omega$ (Angular velocity of the body)

Therefore, $$\boxed{V = r\,\omega}$$

It means that, at any instant linear velocity 'V' is equal to 'r' times angular velocity 'ω' of the body.

🗣 ***Note that*** at any given instant angular velocity 'ω' is directed along the axis of rotation (or axis of revolution) and the linear velocity 'V' is directed along the tangent to the circular path at that instant. This linear velocity is also called ***tangential velocity*** (Since it is directed along the tangent to the circular path).

If the angular velocity of a rotating body is changing such that $\Delta\omega$ is change in its angular velocity in time Δt and ΔV is the corresponding change in linear velocity of a point on the rotating body, then: $\Delta V = r\,\Delta\omega$

On dividing both sides of this equation by Δt, we get:

$$\frac{\Delta V}{\Delta t} = r\,\frac{\Delta\omega}{\Delta t}$$

$$\boxed{a = r\,\alpha}$$

It means that, at any instant linear acceleration 'a' (also called tangential acceleration) of a point on a rotating body is equal to 'r' times its angular acceleration 'α'.

<u>Q</u>: Why do points on a disc, rotating with uniform angular speed, lying away from the axis of rotation, move with a higher speed than points lying close to it, although both have the same angular speed?

<u>Ans:</u> Linear speed 'V' of any point on a rotating body at any instant in terms of its angular speed 'ω' is given by: $V = r\,\omega$

This formula shows that if a disk is rotating with a uniform angular speed ω, The angular speed of all points on it will be equal because they will all subtend equal angles at the center in equal intervals of time.

Since their linear speed 'V' depends not only upon their angular speed 'ω' but also upon radius 'r' of their circular path. Therefore, their linear speed will be ***<u>different</u>***.

🗣 Points on a rotating body which are **<u>close</u>** to the axis of rotation will have **<u>smaller</u>** linear speed 'V' because distance 'r' of these points from the axis of rotation (radius of their circular path) is **<u>small</u>**.

🗣Points lying **away** from the axis of rotation of a rotating disk move with a **higher** linear speed.

Equations of angular motion:

Corresponding to equations of uniformly accelerated linear motion there are equations for angular motion when angular acceleration of the body is uniform.

Linear motion	Angular motion
$V_f = V_i + a\,t$	$\omega_f = \omega_i + \alpha\,t$
$S = V_i t + \frac{1}{2} a\,t^2$	$\theta = \omega_i t + \frac{1}{2} \alpha\,t^2$
$2\,a\,S = V_f^2 - V_i^2$	$2\,\alpha\,\theta = \omega_f^2 - \omega_i^2$

Centripetal acceleration a_c:

Q.No.11: What is centripetal acceleration?

Ans: Acceleration of a body directed ***towards the center*** of a circular path is called centripetal acceleration, its magnitude is given by:

$$a_c = \frac{v^2}{r}$$

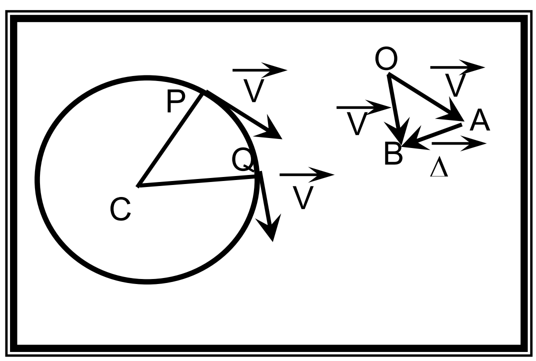
Where "r" is radius of the circular path, and "v" is the linear speed of the body.

Derivation of Centripetal acceleration a_c:

When a body moves along a circular path it's linear velocity at any instant is directed along the ***tangent*** to the circular path at that instant. Hence as the body continues its motion the direction of its linear velocity continuously changes. ***It means that*** motion of the body is accelerated. Acceleration produced in the body is due to a change only in the ***direction of its linear velocity***, it is called ***centripetal acceleration***.

Since K.E of a body depends upon the **magnitude** and **not upon the direction** of linear velocity therefore, ***kinetic energy*** of the body ***does not change***.

Let $\vec{V_1}$ and $\vec{V_2}$ be the velocities of a point at two successive positions after an interval of time 'Δt' and '$\Delta\theta$' be the angular displacement at the center of circular path of radius 'r' during that time. If the body is moving with a uniform speed then the magnitudes of $\vec{V_1}$ and $\vec{V_2}$ will be equal but the direction of these velocities will be different (each along the tangent at two different points along the same circular path). Hence these two are different velocities. But their magnitude $V_1 = V_2 = V$.



The change in linear velocity $\vec{\Delta V}$ can be determined by joining the tails of $\vec{V_1}$ and $\vec{V_2}$.

Line joining the heads represents '$\vec{\Delta V}$', directed from the head of $\vec{V_1}$ to the head of $\vec{V_2}$ (According to head to tail rule $\vec{V_2}$ is the final velocity and $\vec{V_1}$ is the initial velocity, $\vec{V_2} = \vec{V_1} + \vec{\Delta V}$)

Since the change in velocity '$\vec{\Delta V}$' is directed ***towards the center of the circular path,*** therefore, acceleration is also ***directed towards the center***. This acceleration is called ***centripetal acceleration***.

Triangles OAB and CPQ are similar. Therefore:

$$\frac{AB}{PQ} = \frac{OA}{CP}$$

But $AB = \Delta V$, $PQ = \Delta S$, $OA = V$ & $CP = r$

$$\frac{\Delta V}{\Delta S} = \frac{V}{r}$$

$$\Delta V = \frac{V}{r} \Delta S$$

Divide both sides of the above equation by Δt:

$$\frac{\Delta V}{\Delta t} = \frac{V}{r}\frac{\Delta S}{\Delta t}$$

But $\frac{\Delta V}{\Delta t} = a_c$. & $\frac{\Delta S}{\Delta t} = V$

Therefore, the magnitude of centripetal acceleration a_c is given by:

$$\boxed{a_c = \frac{V^2}{r}}$$

Direction of centripetal acceleration 'a_c' is same as the direction of change in velocity '$\Delta \vec{V}$', Which is directed towards the center of the circular path, therefore, 'a_c' is also directed towards the center of the circular path.

🗣 ***Note that*** the centripetal acceleration is due to a ***change only in the direction of linear velocity***, it is ***not due to any other reason***.

🗣 ***Also note that*** whenever a body moves along a circular path with whatever speed (with a constant or a variable speed) it will always have centripetal acceleration due to a change in direction of its linear velocity.

Centripetal force F_c:

Q.No.12: What is centripetal force?

Ans: Force required to move a body along a circular path is called ***centripetal force*** or ***in other words***, it is the force required to produce centripetal acceleration in a body. This force is called ***centripetal*** because it is always directed ***towards the center*** of circular path followed by a body.

🗣 Centripetal force is always required when a body moves along a circular path. If this force is not provided the body will not move along a circle or part of a circle. If centripetal force is removed during circular motion of a body it will fly off the circular path and will move along the tangent to the circle at the point from where this force is removed.

🗣 In case of our ***solar system,*** our beautiful little earth revolves around the sun because the gravitational force of the sun on earth provides the necessary centripetal force.

🗣 In case of the ***moon,*** moon revolves around the earth because the gravitational force of the earth on moon provides the necessary centripetal force.

🗣 In case of ***an atom,*** negatively charged electron revolves around the positively charged nucleus because the attractive Coulomb force of the nucleus provides the necessary centripetal force.

🗣 In case of ***a stone tied at the end of a string when whirled*** the stone revolves along a circular path because tension in the string provides the necessary centripetal force. Centripetal force acting on a body is given by:

$$\boxed{F_c = \frac{m v^2}{r}}$$

$$\boxed{F_c = m r \omega^2} \qquad (v = r\omega)$$

$$\boxed{F_c = \frac{4\pi^2 m r}{T^2}} \qquad \left(\omega = \frac{2\pi}{T}\right)$$

Where 'r' is radius of the circular path, and 'v' is its linear speed, 'ω' is its angular speed and 'T' is the period of revolution (time to complete one revolution) of the body.

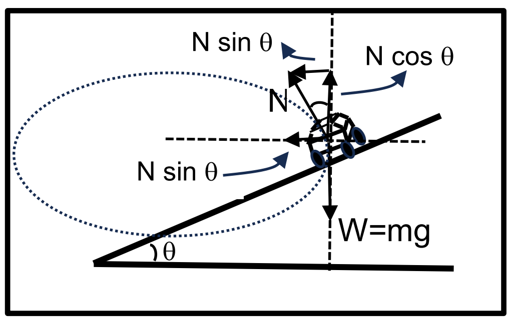
Motion of a car on a curved flat road:

Consider a car taking a turn on a curved flat road. Curved road is a part of a big circular path. We know that when any object moves along a circle or part of a circle ***centripetal force must be provided to it***. In this case friction between tires of the car and road surface provides the necessary centripetal force. Magnitude of this force depends upon many factors such as **road conditions** (i.e. its roughness, icy/ rainy conditions), upon **condition of tires**, **speed of the car** etc. The car cannot travel at high speed and may skid if the road is smooth/icy/rainy and tires are worn out. ***In other words,*** motion of a car on a flat curved road depends mainly upon force of friction. If friction is ***less*** than required the car may skid.

Banked curve: (Karachi Board 2024)

Banked curve is a curved road whose outer side is raised and makes some angle "θ" with the ground.

Purpose of making a banking curve is to decrease dependence of moving vehicles on friction.



Assuming that there is no force of friction between road surface and tires of the car, then forces acting on the car on a banked curve are its weight W = mg (vertically down ward) and normal reaction "N" acting on the car, perpendicular to the road, and makes a certain angle "θ" with the horizontal.

Resolving N into components we see that:

Component N Sin θ provides the centripetal force $\frac{m v^2}{r}$, necessary to move the car on banked curved road without skidding, and

Component N Cos θ balances weight W = mg of the car.

$$N \sin\theta = \frac{m v^2}{r} \;\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

$$N \cos\theta = m g \;\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Dividing equation (*i*) by (*ii*) we get:

$$\frac{\cancel{N}\sin\theta}{\cancel{N}\cos\theta} = \frac{\frac{\cancel{m} v^2}{r}}{\cancel{m} g} \qquad \left(\text{But } \frac{\sin\theta}{\cos\theta} = \tan\theta\right)$$

$$\tan\theta = \frac{v^2}{r g}$$

$$\boxed{\theta = \tan^{-1}\left[\frac{v^2}{r g}\right]}$$

This formula gives us banking angle "θ" for ideal banked road (road free of friction) in terms of speed "v" of the car, radius "r" of the curved road.

- It is use-full for designing and constructing good quality banked curved roads on a high way. This formula is also used in designing banked curve for racing cars and motor bicycles etc.
- For ***high speed*** and ***sharp curved road*** banking angle must be ***large***. Hence for each banking angle and radius of the circular path there is a maximum speed limit with which a car can travel without skidding. If speed of the car is more than this speed limit car will move along a larger circle, on the other hand if speed of the car is below this speed limit it will move along a smaller circular curve.
- ***Note that*** the banking angle "θ" does not depend on mass "m" of the car.
- ***Also note that*** this derivation is for frictionless road surface, if there is friction it will also help to take a sharp turn with high speed on the banked road.

Using the above formula for banking angle maximum speed with which a car can be driven on a banked curve can also be calculated by rearranging the formula. Speed "v" is given by: $v^2 = r\,g\,\text{Tan}\,\theta$

$$\boxed{v = \sqrt{r\,g\,\text{Tan}\,\theta}}$$

Example:

Calculate the speed at which a car can be driven safely on a 100 m curve banked at 25°. (Assuming the road to be frictionless)

Solution:

$v = \sqrt{r\,g\,\text{Tan}\,\theta}$ OR $v = \sqrt{100\times 9.8\times \text{Tan}\,25°} = \sqrt{456.98} =$ **21.4 m/s= 77 km/h.**

Orbital velocity: (**Karachi Board 2024**)

***Q.No.13**: What is orbital velocity?*

Ans: Velocity with which a satellite orbits (revolves around) a heavenly body is called its **orbital velocity**.

Orbital velocity of a satellite orbiting any heavenly body at the center can be calculated by applying Newton's law of universal gravitation. Heavenly body at the center can be earth, moon, sun etc. and the satellite may be earth (revolving around the sun), moon (revolving around the earth), an artificial satellite (revolving around the earth) etc.

Gravitational force exerted on satellite by the body at the center provides centripetal force necessary to keep the satellite in orbit.

According to Newton's law of gravitation, force F_G with which body at the center attracts the satellite is given by:

$$F_G = G\frac{m\,M}{R^2}$$

Where m is mass of the satellite., M is mass of the central body.

'G' is ***universal gravitational constant*** (G = 6.67×10⁻¹¹ N-m²/kg²).

R radius of the orbit (or distance between centers of satellite and the central body)

Centripetal force F_c required to keep the satellite in orbit is given by:

$$F_c = \frac{m\,V^2}{R}$$

Since $F_c = F_G$ Therefore, $\frac{\cancel{m}\,V^2}{\cancel{R}} = G\frac{\cancel{m}\,M}{R^{\cancel{2}}}$

$$V^2 = \frac{G\,M}{R} \qquad \text{OR} \qquad \boxed{V = \sqrt{\frac{G\,M}{R}}}$$

With the help of above formula orbital speed of any satellite can be calculated, provided we know mass M of body at the center and distance R between centers of the two bodies (G is the universal gravitational constant).

Note that the orbital velocity V of the satellite at any instant is directed along the tangent to its circular path at that instant.

Period of revolution "T":

Q.No.14: What is period of revolution?

Ans: Time in which a satellite completes one revolution around the central body is called its period of revolution T.

In one revolution the satellite covers a linear distance of "$2\pi R$" equal to the circumference of the orbit. Hence

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

Therefore,

$$V = \frac{2\pi R}{T}$$

$$V^2 = \frac{4\pi^2 R^2}{T^2}$$

But

$$V^2 = \frac{G\,M}{R}$$

Therefore,

$$\frac{4\pi^2 R^2}{T^2} = \frac{G\,M}{R}$$

$$\boxed{T^2 = \frac{4\pi^2 R^3}{GM}}$$

With the help of this formula time in which a satellite completes one revolution around the central body (its period of revolution) can be calculated, this is also known as KEPLER'S law.

Inertia:

Q.No.15: What is inertia?

Ans: It is the property of a body to resist a ***change in its state of rest*** or of ***uniform linear motion***. It means that if a body is at rest it tends to remain at rest and if it is in motion it tends to continue its motion without any change in its velocity. Inertia of a body depends only upon its ***mass***.

Hence it is ***difficult*** to move a ***heavy*** body if it is at rest and if it is in motion then it is ***difficult*** to ***speed it up*** or ***slow it down***.

It is ***easy*** to move a ***light*** body if it is at rest, easy to slow it down or speed it up a light body.

Heavy body has ***more inertia*** but light body has ***less inertia***. In other words, due to inertia a body tends to resist linear acceleration.

Hence inertia is for translational or linear motion.

***Moment of inertia*:**

***Q.No.16*: What is moment of inertia?**

Ans*: *Moment of inertia is actually rotational inertia ***it means that*** it is the property of a body to resist its angular acceleration or it shows how difficult or easy it is to change angular velocity of a body.

- If moment of inertia of a rotating body is ***large*** it will be ***difficult*** to change (increase or decrease) its angular velocity or it will strongly resist any change in its angular velocity.
- If moment of inertia of a rotating body is ***small*** it will be ***easy*** to change (increase or decrease) its angular velocity.

Moment of inertia is for angular or circular motion it is also called rotational inertia. It depends upon ***size and shape***, ***mass***, ***distribution of mass*** of a body and upon the ***location of axis of rotation***.

If m_1, m_2, m_3 …. are mass of particles at perpendicular distance r_1, r_2, r_3,… respectively from axis of rotation of the rotating body, then the moment of inertia "I" of the whole body is given by:

$$I = \Sigma\, m_i r_i^2 \qquad (i = 1,2,3 \ldots\ldots)$$

It means that moment of inertia of the whole body is equal to the sum of products of mass and square of perpendicular distance of particles of the body from the axis of rotation.

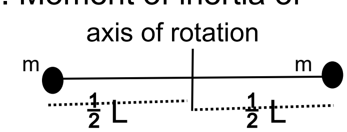
- ❖ Moment of inertia is also known as ***rotational inertia***.
- ❖ MKS unit of moment of inertia is **kg.m²**. It is a ***scalar*** quantity.
- ❖ Dimensions of moment of inertia are $[M^1 L^2 T^0]$.

***Difference between inertia and moment of inertia*;**

- Inertia is for ***linear (or translational) motion*** but moment of inertia is for ***rotational (or angular) motion*** of a body.
- Due to inertia bodies resist ***linear acceleration "a"*** but due to moment of inertia bodies resist ***angular acceleration "α"***.
- Inertia depends only upon ***mass*** of the body but moment of inertia depends not only upon ***mass*** but also upon the ***distribution of mass*** of the body, ***location of axis of rotation*** and the ***perpendicular distance*** of the rotating object from the axis of rotation.
- Bodies of same mass but of different shapes and sizes may have different moment of inertia.
- Momen of inertia is also called ***rotational inertia***.

***Moment of inertia (or rotational inertia) of a two-particle system*:**

Consider two rigid spherical bodies each of mass "m" connected at the ends of a rod of length "L" and of negligible mass. Let the rod rotates about its center so that the axis of rotation passes perpendicularly through its center. Under this condition perpendicular distance of each spherical body from the axis of rotation will be "$\frac{1}{2}$ L". Moment of inertia of this two-body system will be:

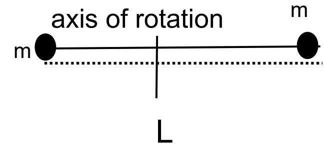


$$I = \Sigma\, m_i r_i^2 = m \times \left(\tfrac{1}{2} L\right)^2 + m \times \left(\tfrac{1}{2} L\right)^2$$

$$I = \tfrac{1}{2}\, m\, L^2$$

If the axis of rotation passes perpendicularly through one of rods ends i.e. if the rod rotates about one of its ends then perpendicular distance from axis of rotation of mass at this end will be zero whereas for the mass at the other end it will be equal to length "L" of the rod. Hence

$$I = \Sigma\, m_i r_i^2 = m \times (0)^2 + m \times L^2$$



$$I = m L^2$$

This example shows that moment of inertia depends also upon the location of axis of rotation.

Moment of inertia of different bodies:

- Moment of inertia of a ***solid cylinder*** of mass "M" and radius "R", rotating about its axis is: $I = \frac{1}{2} M R^2$
- Moment of inertia of a ***hollow cylinder*** of mass "M", inner radius "a" and outer radius "b" rotating about axis passing through its center is:
  $$I = \frac{1}{2} M (a^2 + b^2)$$
- Moment of inertia of a ***solid sphere*** of mass "M" and radius "R" rotating about axis its center is: $I = \frac{2}{5} M R^2$
- Moment of inertia of a ***solid rod*** of mass "M" and length "L" rotating about one of its ends is: $I = \frac{1}{3} M R^2$

Angular momentum:

***Q.No.17**: What is angular momentum?*

Ans: Angular momentum is for rotational motion; it tells you how difficult it is to change angular velocity of a body.

Angular momentum "$\vec{L}$" is the cross product of position vector "$\vec{r}$" and linear momentum "$\vec{p}$".

$$\vec{L} = \vec{r} \times \vec{p}$$

- It is a ***vector quantity***. Its direction is always perpendicular to the plane containing $\vec{r}$ and linear momentum $\vec{p}$ and can be determined by right hand rule.
- Its MKS unit is **kg m²/s**.
- Its dimensions are **[M L² T⁻¹]**

Since linear momentum of a body is the product of its mass "m" and linear velocity "v" hence magnitude of angular momentum of a body about a certain axis of is given by:

$$L = m\, v\, r \sin\theta$$

"θ" is the angle between the position vector "$\vec{r}$" and linear momentum "$\vec{p}$" of the body.

If $\theta = 90°$ then, $L = m\, v\, r \sin 90°$

$$L = m\, v\, r$$

But $v = r\,\omega$, Where ω is the angular velocity of the body. Hence,

$$L = m\, r^2\, \omega$$

But $m r^2 = I$, where "I" is moment of inertia of the body. In other words,

$$L = I\,\omega$$

We know that angular velocity ω of all the points on a rotating body is same whatever their location may be. Angular momentum, therefore, depends upon its moment of inertia. Higher the moment of inertia of a body higher will be its angular momentum and more difficult it will be to stop a rotating/spinning/rolling body.

Law of conservation of Angular momentum:

Q.No.18: State law of conservation of angular momentum?

Ans: Law of conservation of angular momentum states that:

"*If net torque acting on a body is zero the angular momentum of the body remains constant*".

Torque or moment of force:

Q.No.19: What is torque or moment of force?

Ans: Torque "τ" or is moment of force is the turning effect of a force produced in a body about a certain axis. It is a ***vector*** quantity. It is equal to the vector or cross product of position vector "$\vec{r}$" relative to a certain axis of rotation and force "$\vec{F}$".

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

- Direction of torque is determined by ***right hand rule***. Its direction is same as the direction of $\vec{r} \times \vec{F}$.
- Torques which produce anticlockwise turning in a body with respect to the observer are directed outward (by right hand rule) are taken as ***positive*** whereas torques which produce clockwise turning (directed inward) are taken as ***negative***.

Magnitude of torque acting on a body is given by:

$$\tau = r\,F\,\text{Sin}\,\theta$$

Where "θ" is the angle between position $\vec{r}$ and force $\vec{F}$.

Magnitude of torque acting on a body is ***maximum*** when applied force $\vec{F}$ is perpendicular to the position vector $\vec{r}$, In other words angle θ is 90°.

Relation between torque and moment of inertia:

According to Newton's second law of motion:

$$F = m\,a$$

But linear acceleration "a" and angular acceleration "α" are related by:

$$a = r\,\alpha$$

Note that this formula is valid when the body is moving along a circular path with angular acceleration "α" in this case linear acceleration "a" will be directed along tangent to the circular path. In other words, linear acceleration in the above formula is also called tangential acceleration. Substituting the relation between linear and angular accelerations in second law we get:

$$F = m\,r\,\alpha$$

Multiplying both sides by "r" we get:

$$r\,F = m\,r^2\,\alpha$$

But: $r\,F = \tau$ (torque acting on the body)

and $m\,r^2 = I$ (moment of inertia of the body)

Hence: $\tau = I\,\alpha$

If "I" is constant, then $\tau \propto \alpha$

This is the form of Newton's second law for angular/circular motion. It shows that when a net torque "τ" acts on a body it produces an angular acceleration "α" in it such that torque is directly proportional to the angular acceleration produced. In this case the constant of proportionality is "I" moment of inertia of the body.

Section–A Multiple-Choice Questions MCQs:

Q.No.1: One radian is about:
(A) 25° (B) 45° (C) 37° (D) 57°

Q.No.2: Wheel turns with constant angular speed then:
(A) each point on its rim moves with constant velocity.
(B) each point on its rim moves with constant acceleration
(C) the wheel turns through equal angles in equal times.
(D) the angle through which wheel turns in each second increases as time goes on.
(E) the angle through which wheel turns in each second decreases as time goes on.

Q.No.3: The rotational inertia of a wheel about its axle does not depend upon its:
(A) diameter. (B) distribution of mass.
(C) mass. (D) speed of rotation.

Q.No.4: A force with a given magnitude is to be applied to a wheel. The torque can be maximized by:
(A) applying force near the axle radially outward from the axle.
(B) applying force near the rim radially outward.
(C) applying force near the axle parallel to a tangent to the wheel.
(D) applying force at the rim tangent to the rim.

Q.No.5: An object rotating about a fixed axis, "I" is its rotational inertia and "α" is its angular acceleration. It
(A) is the definition of torque.
(B) is the definition of rotational inertia.
(C)is the definition of angular acceleration
(D) follows directly from Newton's second law

Q.No.6: The angular momentum vector of earth about its rotation axis, due to its daily rotation is directed:
(A) tangent to the equator towards east.
(B) tangent to the equator towards west.
(C) north. (D) towards the sun.

Q.No.7: A stone of 2 kg is tied to a 0.50 m long string and swung around a circle at angular velocity of 12 rad/s. The net torque on the stone about the center of the circle is:
(A) 0 N.m (B) 6 N.m (C) 12 N.m (D) 72 N.m

Q.No.8: A man with his arms at his sides, is spinning on a light frictionless turntable. When he extends his arms:
(A) his angular velocity increases.
(B) his angular velocity remains same.
(C) his angular velocity decreases.
(D) his angular momentum remains same.

Q.No.9: A space station revolves around the earth as a satellite, 100 km above the earth surface. What is the net force on an astronaut at rest inside the space station?
(A) equal to her weight on earth.
(B) a little less than her weight on earth.
(C) less than half her weight on earth.
(D) Zero (she is weightless)

Q.No.10: If the external torque acting on a body is zero, then it
(A) angular momentum is zero.
(B) angular momentum is conserved.
(C) angular acceleration is maximum.
(D) rotational motion is maximum.

Q.No.11: If ω is the angular speed of a particle moving in a circle of radius 'r', the centripetal acceleration will be:
(***2004 Karachi Board***)
(A) ωr (B) ωr^2 (C) $\omega^2 r$

Q.No.12: Every point on a rotating body has the same: (***2004 Karachi Board***)
(A)Linear velocity. (B) Angular velocity.
(C) Linear acceleration.

Q.No.13: When a body moves with a constant speed in a circle:
(***2005 Karachi Board***)
(A) Its velocity is changing.
(B) Its acceleration is zero.
(C) Its acceleration is increasing.
(D) Its velocity is uniform.

Q.No.14: The rate of change of angular momentum with respect to time:
(***2007 Karachi Board***)
(A) Force. (B) Angular velocity
(C) Angular acceleration. (D) Torque

Q.No.15: Angle subtended at its center by an arc whose length is equal to its radius is:
(***2008 Karachi Board***)
(A) 37.3° (B) 47.3° (C) 57.3° (D) 67.3°

Q.No.16: The unit of angular velocity is:
(***2015 Karachi Board***)
(A) radian/cm. (B) meter/sec.
(C) radian/sec. (D) radian/sec^2.

Q.No.17: One radian is equal:
(***2025 Karachi Board***)

(A) 0.017° (B) 35.7° (C) 57.3° (D) 0.117°

Q.No.18: The turning effect of a force about the axis of rotation is called:

(***2025 Karachi Board***)

(A) Momentum. (B) Inertia.
(C) Orbital velocity. (D) Torque.

Q.No.19: Every point on a rotating body has the same: (***2026 Karachi Board***)

(A) Linear velocity. (B) Angular velocity.
(C) Angular momentum. (D) Torque.

Answers:

(1) (D) 57°
(2) (C) The wheel turns through equal angles in equal time.
(3) (D) Speed of rotation.
(4) (D) Applying force at the rim tangent to the rim.
(5) (D) follows directly from Newton's second law.
(6) (D) towards the sun.
(7) (B) 6 N.m
(8) (C) his angular velocity decreases.
(9) (D) Zero (she is weightless)
(10) (B) angular momentum is conserved.
(11) (C) $\omega^2 r$
(12) (B) Angular velocity.
(13) (A) Its velocity is changing.
(14) (D) Torque.
(15) (C) 57.3°
(16) (C) radian/sec.
(17) (C) 57.3°
(18) (D) Torque.
(19) (B) Angular velocity.

Numericals:

Q.No.1: **A car mechanic applies a force of 800 N to a wrench for the purpose of loosening a bolt. He applies the force which is perpendicular to the arm of the wrench. The distance from the bolt to the mechanic's hand is 0.40 m.**

Find out the magnitude of torque applied?

Data:

F = 800 N r= 0.40 m τ = ?

Solution:

$\tau = F\ r\ \text{Sin}\ \theta$ $(\theta = 90°)$

$\tau = 800 \times 0.40 \times \text{Sin}\ 90°$ **= 320 Nm**

Magnitude of torque applied is **320 Nm**.

Q.No.2: **A car accelerates uniformly from rest and reaches a speed of 22 m/s in 9 s. If the diameter of a tire is 58 cm. Find:**
(a) the number of revolutions the tire makes during this motion, assuming no slipping.
(b) the final rotational speed of the tire in revolution per second.

Data:

$V_i = 0$, $V_f = 22$ m/s, t = 9 s,
d = 58 cm= 0.58m r = 0.29 m
(a) No. of revolutions =?
(b) Final rotational speed ω_f =?

Solution:

(a) Circumference of a wheel
$= 2\ \pi\ r = 2\times22/7\times0.29 = 1.82$ m

But $v_f - v_i = a\ t$ $22 - 0 = a\times9$

$a = 22/9 = 2.44\ m/s^2$

Distance travelled:

$S = v_i\ t + \frac{1}{2}\ a\ t^2$

$S = 0 + \frac{1}{2} \times 2.44\ (9)^2 = 99$ m.

No. of revolutions = $\dfrac{\text{distance traveled}}{\text{Circumference of a wheel}}$

No. of revolutions = 99/1.82=***55 revolutions***.

(b) $v_f = r\ \omega_f$, $\omega_f = \dfrac{v_f}{r} = \dfrac{22}{0.29}$ =***75.86 rad/s***

There are 2 π radians in one revolution.
Hence No. of revolutions/s in 75.86 rad/s

$\omega_f = \dfrac{75.86}{2\ \pi}$ = ***12 revolutions/s***

(a) Number of revolutions are ***55 revolutions***
(b) Final rotational speed is ***75.86 rad/s***
= ***12 revolutions/s***

Q.No.3: **An ordinary workshop grindstone has a radius of 7.5 cm. and rotates 6500 rev/min.**

(a) Calculate the magnitude of centripetal acceleration of its edge in m/s² and convert it into multiples of g.

(b) What is the linear speed of a point on its edge?

Data:

Radius r = 7.5 cm. = 0.075 m
Rotational speed = 6500 rev/min.
$= \dfrac{6500}{60}$ rev/s.

(a) Centripetal acceleration a_c = ?
(b) Linear speed V =?

Solution:

(a) Centripetal acceleration in terms of angular speed is given by:

$a_c = r\ \omega^2$

$\therefore$ $a_c = 0.075\times \dfrac{(6500)^2}{(60)^2}$ = **880.2 m/s².**

Centripetal acceleration of the edge is **880.2 m/s².**

(b) Linear speed of a point on the edge is given by: $V = r\ \omega$

$\therefore$ $V = 0.075\times \dfrac{6500}{60}$ = **8.125 m/s.**

Linear speed of point on the edge is **8.125 m/s.**

Q.No.4: **A satellite is orbiting the Earth with an orbital velocity of 3200 m/s.**

What is the orbital radius?

Data:

Orbital velocity V = 3200 m/s
Orbital radius R = ?
Mass of the earth $M = 5.98\times10^{24}$ kg
Universal gravitational constant
$G = 6.67\times10^{-11}\ N\text{-}m^2/kg^2$

Solution:

Since: $V^2 = \dfrac{G\ M}{R}$ OR $R = \dfrac{G\ M}{V^2}$

$\therefore$ $R = \dfrac{6.67\times10^{-11} \times 5.98\times10^{24}}{(3200)^2}$

$\mathbf{R = 3.895 \times 10^7\ m}$

Orbital radius **3.895 ×10⁷ m.**

Q.No.5: **A satellite is to orbit the earth at a height of 100 km (approximately 60 miles) above the surface of the earth. Determine the speed, acceleration and orbital period of the satellite.**

(Given M_{earth}= 5.98×10^{24}kg, R_{earth} = 6.37×10^{6}m)

Data:

Height $h = 100$ km. $= 10^5$ m

$R_{earth} = 6.37\times10^6$ m.

Radius of orbit $R = R_{earth} + h$

$= 6.37\times10^6+10^5= 6.47\times10^6$ m

$M_{earth} = 5.98\times10^{24}$ kg.

Speed $V =?$

Acceleration $a =?$

Orbital period $T =?$

Universal gravitational constant

$G = 6.67\times10^{-11}$ N-m^2/kg^2

Solution:

$$V^2 = \frac{G\,M}{R} \quad \text{OR} \quad V = \sqrt{\frac{G\,M}{R}}$$

$$V = \sqrt{\frac{6.67\times10^{-11}\times 5.98\times10^{24}}{6.47\times10^6}} = \sqrt{61648.563}$$

V = <u>7851.7 m/s</u>. OR <u>7.851×10^3 m/s</u>.

Since the satellite is moving along a circular path therefore, it must have centripetal acceleration, given by:

$$a = \frac{V^2}{R} = \frac{(7.851\times10^3)^2}{6.47\times10^6} = \mathbf{9.53\ m/s^2}.$$

Orbital period can be found by:

$$T^2 = \frac{4\,\pi^2\,R^3}{GM} = \frac{4\pi^2\times(6.47\times10^6)^3}{6.67\times10^{-11}\times 5.98\times10^{24}}$$

$$T^2 = 26{,}806{,}836$$

$$T=\sqrt{26{,}806{,}836}=5177.53\text{ s}=\frac{5177.53}{3600}\text{h}=\mathbf{1.44h}$$

Orbital period is **<u>1.44 h</u>**.

Q.No.6: **A thin disk with a 0.3 m diameter and a total moment of inertia of 0.45 kg.m^2 is rotating about its center of mass. There are three rocks with masses of 0.2 kg on the outer part of the disk. Find the total moment of inertia of the system.**

Data:

Diameter d=0.3 m, Radius r= d/2 =0.15 m

Moment of inertia of disk I=0.45 kg.m^2

Mass of each rock $m = 0.2$ kg.

Total number of rocks $n = 3$

Total moment of inertia =?

Solution:

Moment of inertia of each rock = m r^2

$= 0.2\,(0.15)^2 = 4.5\times10^{-3}$ kg.m^2.

Total moment of inertia of rocks=3(4.5×10^{-3})

Total moment of inertia of rocks=1.35×10^{-2}kg.m^2

∴ Total moment of inertia of the disk = moment of inertia of disk + moment of inertia of rocks

Total moment of inertia of disk = 0.45+1.35×10^{-2}

= **<u>0.4635 kg.m^2</u>**

Total moment of inertia of disk is **<u>0.4635 kg.m^2</u>**.

Q.No.7: **What is the ideal banking angle for a gentle turn of 1.20 km radius on a highway with a 105 km/h speed limit (about 65 mil/h) assuming everyone travels at the limit?**

Data:

Radius of highway r=1.20 km=1200 m

Speed limit v=105 km/h $=\frac{105\times1000}{60\times60}$ m/s

= 29.17 m/s.

Ideal banking angle θ = ?

Solution:

Banking angle is given by:

$$\theta = \text{Tan}^{-1}\ \frac{v^2}{r\,g}$$

$$\theta = \text{Tan}^{-1}\ \frac{(29.17)^2}{1200\times9.8} = \text{Tan}^{-1}\ 0.0723$$

θ = **4.14°**

Ideal banking angle is **<u>4.14°</u>**.

Q.No.8: **A 1500 kg car moving on a flat, horizontal road negotiates a curve as shown in fig. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.**

Data:

Mass of the car $m = 1500$ kg

Radius of the curve $r = 35.0$ m

Coefficient of static friction $\mu_s = 0.523$

Max. safe speed $v =?$

Solution:

In this case the centripetal force is provided by force of friction between road and tires of the car, therefor,

Force of friction = centripetal force

$$\mu_s R = \frac{m v^2}{r}$$

But normal reaction R = m g (weight of car)

$$\mu_s \cancel{m} g = \frac{\cancel{m} v^2}{r}$$

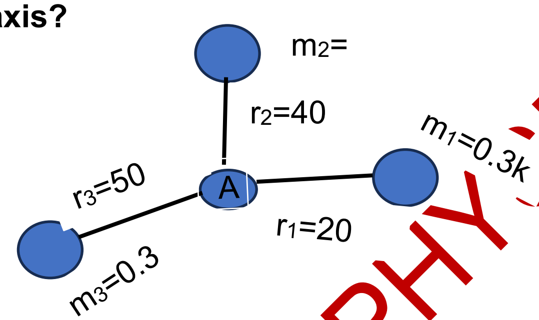
$$v^2 = \mu_s g r = 0.523 \times 9.8\times 35$$

$$v^2 = 179.389$$

$$v = \sqrt{179.389} = \underline{\mathbf{13.394\ m/s}}.$$

🗣 Maximum safe speed the car can have is **<u>13.394 m/s</u>**.

***<u>Q.No.9</u>*: A system of points shown in fig. Each particle has same mass of 0.3 kg and they all lie in the same plane. What is the moment of inertia of the system about given axis?**



Data:

Mass $m_1 = m_2 = m_3 = 0.3$ kg.

Distance $r_1 = 20$ cm. $= 0.20$ m.

$r_2 = 40$ cm. $= 0.40$ m.

$r_3 = 50$ cm. $= 0.50$ m.

Moment of inertia of the system = ?

Solution:

Let the axis of rotation be passing through "A", perpendicular to the plane of paper. Hence:

Total moment of inertia of the system

$= m_1 (r_1)^2 + m_2 (r_2)^2 + m_3 (r_3)^2$

$= 0.3 (0.2)^2 + 0.3 (0.4)^2 + 0.3 (0.5)^2$

$= \underline{\mathbf{0.135\ kg.m^2}}$.

🗣 Total moment of inertia of the system is **<u>0.135 kg.m²</u>**.

***<u>Q.No.10</u>*:(a) What is the angular momentum of a 2.9 kg uniform cylindrical grinding wheel of radius 20 cm when rotating 1550 rpm?**

(b) How much torque is required to stop it in 6 s?

Data:

(a) Angular momentum L = ?

Mass of the wheel M = 2.9 kg.

Radius of the wheel R=20cm =0.20 m

Angular speed ω =1550 rpm.

$= 1550 \times 2\pi/60$ rad./s = 162.32 rad./s

(Angular speed "ω" is given in rpm "revolution per minute", angle subtended in each revolution is 2π radian, ∴ 1 rpm = 2π radian in 60 second)

(b) Torque required τ =?

Time to stop t = 6 s.

Solution:

(a) Angular momentum is given L= I ω

But $I = \frac{1}{2} M R^2$ For cylindrical wheel

Therefore, $L = \frac{1}{2} M R^2 \omega$

$L = \frac{1}{2}\, 2.9 \times (0.20)^2 \times 162.32 = \underline{\mathbf{9.414\ kg\text{-}m^2/s}}$.

(b) Since torque "τ" is the rate of change of angular momentum, and we have to stop the rotating grinding wheel, therefore, its final angular momentum "L_f" must be taken as zero.

$$\tau = \frac{\Delta L}{\Delta t} = \frac{L_f - L_i}{\Delta t} = \frac{0 - 9.414}{6} = \underline{\mathbf{-1.569\ N\text{-}m}}$$

🗣 Angular momentum is **<u>9.414 kg-m²/s</u>** and **<u>–1.569 N-m</u>** toque will be required to stop it.

***<u>Q.No.11</u>*: Determine the angular momentum of the earth**

(a) about its rotation axis (Assuming the earth as a uniform sphere).

(b) in its orbit around the sun (Take Earth as a particle orbiting the sun). The earth has mass 6×10^{24} kg and radius 6.4×10^{6} m. and is 1.5×10^{8} km from the sun

Data:

Angular momentum of the earth L =?

(a) about its rotation axis =?

(b) about its orbit around the sun =?

Mass of the earth $M = 6\times10^{24}$ kg

Radius of the earth $R_e = 6.4\times10^{6}$ m.

Distance between earth and the sun

$R = 1.5\times10^8$ km.= $1.5\times10^8\times10^3$ m=1.5×10^{11}m

***Solution*:**

(a) Angular momentum (in its rotation axis): $L = I\,\omega$

But $I = \frac{2}{5} M R^2$ (For sphere) and $\omega = \frac{2\pi}{T}$

Where T is the time in which earth completes one rotation about its axis (24 hours)

$= 24 \times 60 \times 60$ s = 86400 s.

$$L = \frac{2}{5} M R_e^2 \times \frac{2\pi}{T}$$

$$L = \frac{2 \times 6\times10^{24} \times (6.4\times10^6)^2 \times 2\pi}{5 \times 86400}$$

$L = 7.149 \times10^{33}$ kg-m²/s.

(b) Angular momentum of earth in its orbit around the sun $L = I\,\omega$

But $I = MR^2$ and $\omega = \frac{2\pi}{T}$

Where "T" is the time in which earth completes one revolution around the sun

$= 365 \times 24 \times 60 \times 60 = 31{,}536{,}000$ s.

and "R" is radius of earth orbit (distance between sun and center of earth)

$= R_e + R = 6.4\times10^6 + 1.5\times10^{11} = 1.5\times10^{11}$ m.

$$L = I\,\omega = MR^2 \times \frac{2\pi}{T} = \frac{6\times10^{24}\,(1.5\times10^{11})^2 \times 2\pi}{31{,}536{,}000}$$

$L = \underline{2.69\times10^{40}\text{ kg-m}^2\text{/s}}$.

🗣 Angular momentum of earth in its rotation axis is **<u>7.149 ×10³³ kg-m²/s</u>** and in its orbit around the sun is **<u>2.69×10⁴⁰ kg-m²/s</u>.**

***Q.No.12*: A car is traveling on a flat circular tack of radius 200 m at 30 m/s and has a centripetal acceleration a_c= 4.5 m/s².**

(a) If mass of the car is 1000 kg what frictional force is required to provide the acceleration?

(b) If the coefficient of static friction 'μ_s' is 0.8, What is the maximum speed at which the car can circle the track?

***Data*:**

Radius of the track $r = 200$ m
Speed of the car $v = 30$ m/s
Centripetal acceleration $a_c = 4.5$ m/s²
Mass of the car $m = 1000$ kg
(a) Frictional force f =?
(b) Coefficient of static friction μ_s=0.8
Maximum speed v =?

***Solution*:**

(a) The car is travelling on a circular track, centripetal force required is provided by force of friction between car tires and the road, therefore:

$f = F_c$ but $F_c = m\,a_c$

$\therefore\ f = m\,a_c$

$f = 1000 \times 4.5$ $\boxed{f = 4500\text{ N}}$

***Alternate method*:**

$$f = \frac{m v^2}{r} = \frac{1000\,(30)^2}{200} = \frac{1000 \times 900}{200}$$

$\boxed{f = 4500\text{ N}}$

(b) Maximum force of friction between tires and the road when coefficient of static friction is 'μ_s' is given by:

$f = \mu_s R = \mu_s m g = 0.8 \times 1000 \times 9.8 = 7840$ N

This max. force of friction provides centripetal force required for moving the car along the circular track with maximum speed, hence:

$$f_{max.} = \frac{m\,v_{max.}^2}{r}$$

$$7849 = \frac{1000 \times v_{max.}^2}{200}$$

$$v_{max.}^2 = \frac{7849 \times 200}{1000} = 1568$$

$v_{max.} = \sqrt{1568}$ $\boxed{v_{max.} = 39.6\text{ m/s}}$

🗣 The car can circle with a maximum speed of ***<u>39.6 m/s</u>***. If the car exceeds this speed it will skid out of the circular track.

***Q.No.13*: Tarzan swinging on a win of length 4 m in a vertical circle under the influence of gravity. When the win makes an angle of 20° with the vertical, Tarzan has a speed of 5 m/s.**

Find his centripetal acceleration at this instant. (*2016, 2013 Karachi Board*)

***Data*:**

Length of the wine = Radius of the circular path $r = 4$ m.
Angle with vertical $\theta = 20°$.
Tangential speed $v = 5$ m/s.
Centripetal acceleration a_c =?

***Solution*:**
Centripetal acceleration of Tarzan is given by: $a_c = \frac{v^2}{r} = \frac{(5)^2}{4} = \frac{25}{4}$

$$\boxed{a_c = 6.25\ m/s^2.}$$

🗣 Centripetal acceleration of Tarzan is ***6.25 m/s²***.

***Q.No.14*: A string 1 m long would break when its tension is 69.6 N.**
Find the greatest speed at which a ball of mass 2 kg. can be whirled with the string in a vertical circle.
(*1996 Karachi Board*)

***Data*:**
Length of the string = radius of the circle
r = 1m.
Tension in the string T = 69.6N
Mass of the ball m = 2 kg.
Maximum speed of the ball V =?

***Solution*:**
When the ball is whirled in a vertical circle tension in the string will be maximum when the ball is at the lowest level from the ground, in this position tension 'T' in the string will be vertically upward and weight 'mg' of the ball will be vertically downward. The unbalanced force (upward, towards the center) which provides the necessary centripetal force to move the ball in circle will be T– mg, therefore: $T - mg = \frac{m V^2}{r}$

$$69.6 - 2 \times 9.8 = \frac{2 V^2}{1}$$

$$69.6 - 19.6 = 2 V^2$$

$$50 = 2 V^2$$

$$V^2 = 25$$

$$V = \sqrt{25} \qquad \boxed{V = 5\ m/s.}$$

🗣 The greatest speed with which the ball can be whirled is ***5 m/s***.

***Q.No.15*: A car having mass of 800 kg. is moving on a curve of radius 500 m at a speed of 40 km/h. Find the magnitude of centripetal acceleration and centripetal force of the car. (*2026 Karachi Board*)**

***Data*:**
Mass of the car m = 800 kg.
Radius of curved path r = 500 m.
Speed of the car v = 40 km./h

$$= \frac{40 \times 1000}{60 \times 60}\frac{m}{s} = 11.11\ m/s$$

Centripetal acceleration a_c =?
Centripetal Force F_c =?

***Solution*:**
Centripetal acceleration:

$$a_c = \frac{v^2}{r} = \frac{(11.11)^2}{500} = \frac{123.4321}{500} \boxed{= 0.247\ m/s^2}$$

Centripetal Force:

$$F_c = \frac{m v^2}{r} = \frac{800\ (11.11)^2}{500} \boxed{= 197.49\ N.}$$