According to new syllabus of all intermediate boards of Sindh.

Prof. Rawala's ....

PHYSICS

HELPING BOOK

For
Class XI
(All science students)

UNIT -2

Kinematics.

Page no.

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Thoroughly revised and updated 2026 – 2027

- ***Detailed notes.***
- ***Multiple Choice Questions (M.C.Q's).***
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Unit. 2**.....**

Kinematics

Scalars:

Q.No.1*: *What are scalars? Give some examples of scalars.

***Ans*:** Scalars are physical quantities which can be described completely without mentioning any direction, they only have **magnitude** along with some suitable **unit**.

OR Scalars are physical quantities which ***do not have direction***.

***Examples*:** Distance, speed, work, power, energy, time, temperature, electric charge, electric and magnetic flux, potential difference, density, volume, area, frequency, wave length, specific heat, etc.

Two or more scalars can be added, subtracted or multiplied like ordinary numbers.

Vectors:

Q.No.2*: *What are vectors? Give some examples of vectors?

Ans*:** Vectors are those physical quantities which have ***magnitude as well as direction and ***obey vector algebra***.

- Hence vectors cannot be described completely without mentioning their direction.

Examples: Velocity, displacement, acceleration, force, momentum, torque, angular displacement, angular Velocity, angular acceleration, angular momentum etc.

Q.No.3*: *How are vectors denoted?

***Ans*:** Vectors are denoted by bold faced letters such as **A, B, C** etc., In hand written material it is a convention to put an arrow above the alphabet such as $\vec{A}$, $\vec{B}$, $\vec{C}$ etc.

Q.No.4*: *How is the magnitude of vector denoted?

Ans*:** The magnitude of a vector $\vec{F}$ is denoted by $|\vec{F}|$ it is read as "modulus of $\vec{F}$" or simply ***"mod. $\vec{F}$". It gives the absolute value of $\vec{F}$. The magnitude of a vector $\vec{F}$ is also denoted by F (without an arrow over the alphabet).

Representation of a vector: (*Graphical representation*)

Q.No.5*: *How is a vector represented graphically?

***Ans*:** Graphically a vector is represented by a directed line segment. The length of this straight line corresponds to the magnitude of the vector according to a suitable scale and it points in the direction of the vector.

Addition of vectors:

Q.No.1*: *How are vectors added?

***Ans*:** Because of their direction, vectors **cannot be** added as ordinary numbers. While adding two or more vectors their direction has to be considered. Hence vectors can be added by head – tail rule (Geometrical method) or by trigonometric method (Rectangular component method).

***Q.No.2*: What is *resultant* vector?**

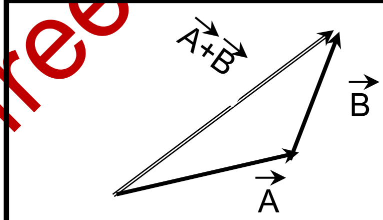
Ans: Resultant of two or more vectors is a ***single vector*** whose ***magnitude*** and ***direction*** are such that it produces the ***same effect*** as the combined effect of given vectors.

Geometrical method (head to rule):

***Q.No.3*: Describe head to tail rule method of vector addition?**

Ans: In head to tail rule of vector addition, tail of second vector is joined with the head of first vector, tail of third is joined with head of second vector and so on, in this manner all the vectors to be added are joined to-gather. Their sum vector or their resultant vector is obtained by joining the head of last vector with the tail of first vector by a straight line. Length of this straight line corresponds to the ***magnitude*** of resultant an arrow is put at the end of this line joined with the head of last vector it shows its ***direction***.

Hence: $\vec{A} + \vec{B} = \vec{R}$



Head to tail rule

Properties of vector addition:

Addition of vectors obeys:

(1) ***Commutative law***.

(2) ***Associative law***.

Commutative law of vector addition:

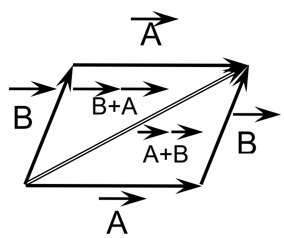
***Q.No.1*: Show that $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.**

OR

Prove commutative law for vector addition?

Ans: To prove that $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. OR To prove commutative law for vector addition, both sides of the above equation must give the same resultant.

To find $\vec{A} + \vec{B}$, according to head to tail rule, head of $\vec{A}$ is joined with the tail of $\vec{B}$, but to find $\vec{B} + \vec{A}$ we will join the head of $\vec{B}$ with the tail of $\vec{A}$. We can see from the following diagram that in both the cases same diagonal of the parallelogram is obtained as their resultant.



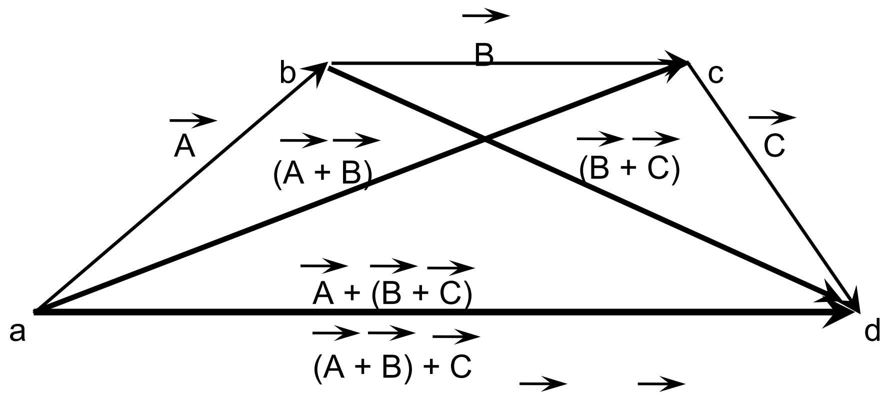
$\therefore$ $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. Proved.

Hence vectors can be added in ***any order***. In other words, we can say that vector addition obeys ***commutative law***.

Associative law:

***Q.No.2*: Show that $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$**

Ans: To prove that $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$. OR To prove <u>associative law</u> for vector addition, first of all we will add vectors "$\vec{A}$" and "$\vec{B}$" their resultant "$\vec{ac}$"will then be added to $\vec{C}$ their resultant "$\vec{ad}$" will be obtained i.e. $(\vec{A} + \vec{B}) + \vec{C}$



Similarly for getting R.H.S, we add vector "$\vec{B}$" and "$\vec{C}$" first and their resultant represented by "$\vec{bd}$" is then added to vector "$\vec{A}$" we will get their resultant represented by "$\vec{ad}$". This will give us the R.H.S of the above equation i.e. $\vec{A} + (\vec{B} + \vec{C})$.

Since the same resultant "$\vec{ad}$" is obtained, therefore, addition of vectors obeys associative law. It means that vectors can be added making any group.

Hence: $\boxed{(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C}).}$ Proved.

- Vectors can be added in any ***grouping***.

***Q.No.3*: What is the resultant of vectors having the same direction?**

***Ans*:** If two or more vectors to be added are in the same direction then the magnitude of their resultant will be equal to the sum of magnitudes of the individual vectors, whereas it's direction is same as the direction of original vectors.

***Q.No.4*: What is the resultant of two opposite vectors?**

***Ans*:** Magnitude of resultant of two opposite vectors will be equal to the difference of magnitudes of individual vectors, whereas the direction of resultant is same as the direction of the larger vector.

- The resultant of two vectors equal in magnitude but opposite in direction is a ***null*** vector (vector whose magnitude is zero).

***Q.No.5*: What is the resultant of a vector and its negative vector?**

***Ans*:** Resultant of a vector and its negative is a null vector.

In other words, resultant of two vectors equal in magnitude but opposite in direction is always a null vector.

***Magnitude of resultant vector*: (*Law of Cosine*)**

If $\vec{A} + \vec{B} = \vec{R}$, where $\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$, and If "θ" is the angle between them, then the magnitude of the resultant vector "$\vec{R}$" or magnitude of sum of two vectors $\vec{A}$ and $\vec{B}$ can be determined by ***Law of Cosine***, according to which:

$$\therefore \quad R^2 = A^2 + B^2 + 2\,A\,B\,\text{Cos}\,\theta$$

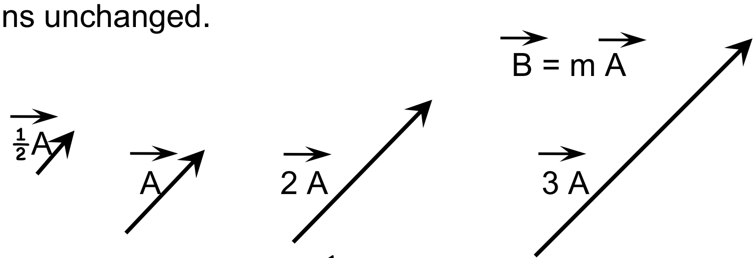
$$\therefore \quad \boxed{\begin{array}{c} R = \sqrt{A^2 + B^2 + 2\,A\,B\,\text{Cos}\,\theta} \\ \text{OR} \\ |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2\,A\,B\,\text{Cos}\,\theta} \end{array}}$$

Note that "θ" is the angle between $\vec{A}$ and $\vec{B}$ when their tails are at the same point.

Above equations give us the magnitude of resultant of two vectors $\vec{A}$ and $\vec{B}$ or they give us the magnitude of sum of two vectors $\vec{A}$ and $\vec{B}$.

Multiplication of a vector by a positive number:

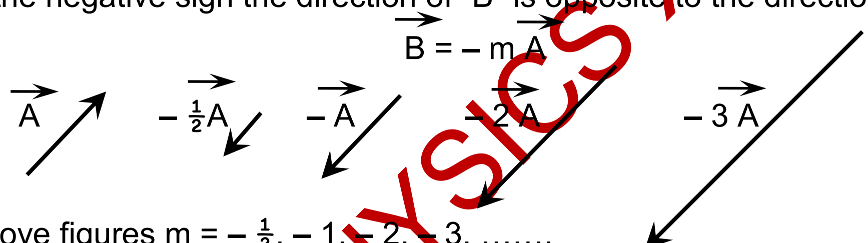
When a vector "$\vec{A}$" is multiplied by a positive number "m" a new vector "$\vec{B}$" is obtained, whose magnitude is equal to "m" times the magnitude of the original vector "$\vec{A}$" it's direction remains unchanged.



In the above figures m = $\frac{1}{2}$, 1, 2, 3, …….

Multiplication of a vector by a negative number:

When a vector "$\vec{A}$" is multiplied by a negative number "– m" the magnitude of new vector "$\vec{B}$" thus obtained is equal to "m" times the magnitude of the original vector "$\vec{A}$" but because of the negative sign the direction of "$\vec{B}$" is opposite to the direction of "$\vec{A}$".



In the above figures m = – $\frac{1}{2}$, – 1, – 2, – 3, …….

- Hence if a negative sign is placed before a vector it's direction ***reverses***, it's direction changes by an angle of ***180***°.

Note that in the above multiplications vectors $\vec{A}$ and $\vec{B}$ are of the same type, having the same unit. For example, if a force '$\vec{F}$' is multiplied by "2" we get "2 $\vec{F}$" which is also a force whose magnitude is twice the magnitude of "$\vec{F}$" its unit is same as the unit of $\vec{F}$.

Division of a vector by a number:

Q.No.1: Describe division of a vector by a number.

Ans: Division a vector "$\vec{A}$" by a non-zero number "n" is equivalent to the multiplication of "$\vec{A}$" by "m" where "m" is the reciprocal of "n" (m = 1/n).

$$\frac{\vec{A}}{n} = m\,\vec{A}$$

The direction of vector "m$\vec{A}$" depends upon the sign of "m".

Unit vectors:

Q.No.1: What is a unit vector?

Ans: A Unit vector is that whose magnitude is ***"1" (one)***.

- A unit vector is used to specify direction.
- Unit vectors are represented by small alphabets with a cap "^" on it.

Q.No.2: *How is a unit vector calculated?*

***Ans*:** Unit vector "$\hat{a}$" parallel to a given vector $\vec{A}$ can be calculated by dividing the given vector "$\vec{A}$" by its magnitude A. Hence

$$\text{Unit vector} = \frac{\text{vector}}{\text{Magnitude of the vector}}$$

$$\boxed{\hat{a} = \frac{\vec{A}}{A}}$$

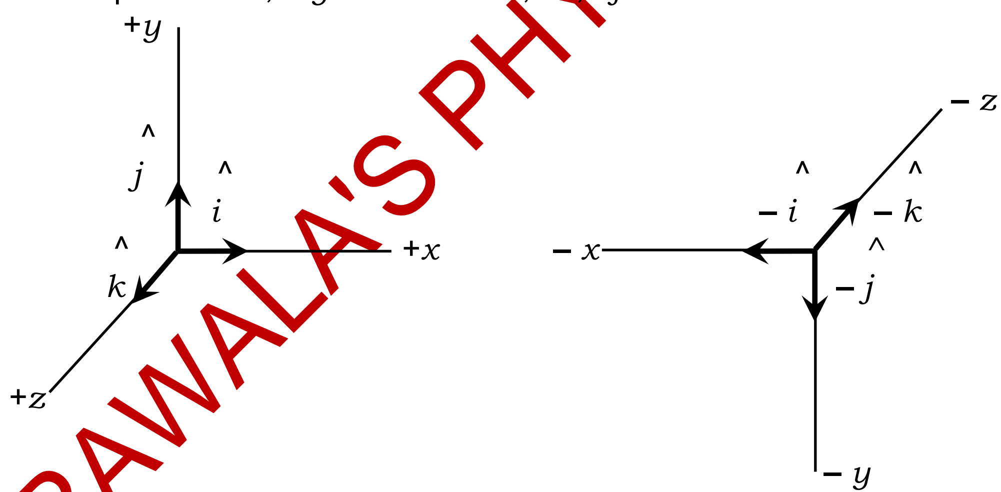
In other words, any vector "$\vec{A}$" can be split into two parts, one part A represents it's magnitude and the other part "$\hat{a}$" represents it's direction.

$$\vec{A} = A\,\hat{a}$$

Q.No.3: *What are rectangular unit vectors or $\hat{i}$, $\hat{j}$ and $\hat{k}$?*

Ans*:** In order to represent the three-dimensional coordinate system three mutually perpendicular vectors are used. These three mutually perpendicular vectors, called ***rectangular unit vectors are $\hat{i}$, $\hat{j}$ and $\hat{k}$. It means that their magnitude is "1" but $\hat{i}$ is along $+x$-axis, $\hat{j}$ is along $+y$-axis and $\hat{k}$ is along $+z$-axis. Hence they are used to represent x, y and z-axes.

To represent $-x$, $-y$ and $-z$-axes, $-\hat{i}$, $-\hat{j}$ and $-\hat{k}$ are used.



Q.No.4: *What is a free vector?*

***Ans*:** A free vector is that vector which can be moved parallel to itself without changing its magnitude and/or direction and applied at any point.

Q.No.5: *What is a position vector?*

Ans*:** A position vector is that vector which is used to give position of a point or anything with respect to a fixed point, such as the origin of a coordinate system (It means that*** it tells you how far and in what direction is a point from the origin of a coordinate system).

Position vector is usually represented by $\vec{r}$. Components of a position vector are called its ***coordinates***. In terms of its coordinates a position vector is given by:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

In terms of magnitude of its coordinates, the magnitude of position vector is given by:

$$|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$$

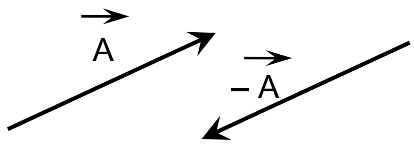
$|\vec{r}| = r$ is the magnitude of position vector.

- Position vector is actually the distance of a given point from the origin, in a particular direction (its M.K.S unit is meter).

***Q.No.6*: What is *negative of a vector*?**

***Ans*:** Negative of a vector is another vector whose magnitude is same but the direction is opposite.

***Negative of a vector*:**



***Q.No.7*: What is *null vector*?**

Ans*:** A null vector is that vector whose magnitude is ***zero.

Resolution of vectors:

***Q.No.1*: What is *resolution* of vectors?**

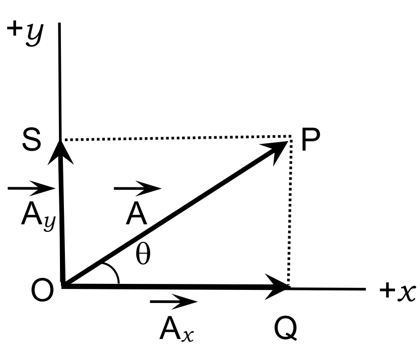
***Ans*:** The process of splitting a vector into two or more components (parts) is called resolution of vectors. Once the process of resolution is complete, the vector is said to have been resolved.

- Two dimensional vectors are resolved into two mutually perpendicular components, one along x- and the other along y-axis. Similarly, a three-dimensional vector can be resolved into three mutually perpendicular components, along x-, y- and z-axes.
- Since these components are mutually perpendicular and along sides of a rectangle, hence these components are also known as the ***rectangular components***.
- ***Note that*** a component along a given axis represents the effective value of the vector in that particular direction.

***Q.No.2*: Derive formulae for rectangular components of a two dimensional vector $\vec{A}$ making an angle "θ" with +x-axis?**

***Ans*:** Let a vector "$\vec{A}$" be represented by a directed line segment "$\vec{OP}$" making an angle "θ" with $+x$-axis as shown.

Angle "θ" is taken in anticlockwise sense starting from +x-axis.



If perpendiculars are drawn from the head of $\vec{A}$ on the two axes then "$\vec{OQ}$" represents $\vec{A_x}$" x-component of vector $\vec{A}$ and "$\vec{OS}$" represents "$\vec{A_y}$" y- component of vector $\vec{A}$.

Since $\vec{QP} = \vec{OS}$, Therefore, $\vec{A_y}$ can also be represented by $\vec{QP}$.

In Δ OQP: $\cos\theta = \frac{OQ}{OP}$ But $OQ = |\vec{A_x}| = A_x$, $OP = |\vec{A}| = A$

$\therefore$ $\cos\theta = \frac{A_x}{A}$

$$\boxed{A_x = A\cos\theta}$$

In terms of its magnitude and direction (using a rectangular unit vector "$\hat{i}$") x-component of any vector "$\vec{A}$" is given by:

$$\boxed{\vec{A_x} = A\cos\theta\ \hat{i}}$$

Similarly, in Δ OQP:

$\sin\theta = \frac{QP}{OP}$ But $QP = |\vec{A_y}| = A_y$, $OP = |\vec{A}| = A$

$\therefore$ $\sin\theta = \frac{A_y}{A}$

$$\boxed{A_y = A\sin\theta}$$

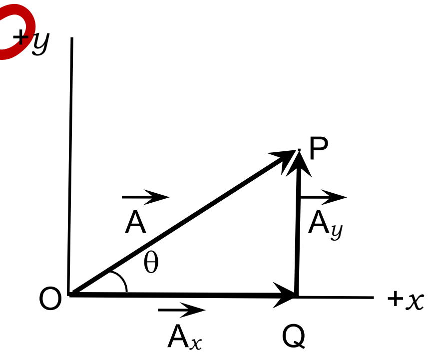
In terms of its magnitude and direction (using a rectangular unit vector "$\hat{j}$") y-component of any vector "$\vec{A}$" is given by:

$$\boxed{\vec{A_y} = A\sin\theta\ \hat{j}}$$

***<u>Composition of a vector with the help of its components</u>*:**

If components of a vector are known then the vector itself can be determined by taking the vector sum of its components.

Hence if $\vec{A_x}$ and $\vec{A_y}$ are the of components of a vector "$\vec{A}$", then by head to tail rule, if head of $\vec{A_x}$ is joined with tail of $\vec{A_y}$ their resultant will be vector "$\vec{A}$", as shown:



***<u>Magnitude of $\vec{A}$</u>*:**

To find the magnitude of vector $\vec{A}$, consider ΔOPQ: In this triangle, according to Pythagoras theorem: $(OP)^2 = (OQ)^2 + (QP)^2$

But $OP = A$ (magnitude of vector $\vec{A}$)

$OQ = A_x$ (magnitude of x-component $\vec{A_x}$)

$QP = A_y$ (magnitude of y-component $\vec{A_y}$)

$\therefore \quad A^2 = A_x^2 + A_y^2 \quad \therefore \quad \boxed{A = \sqrt{A_x^2 + A_y^2}}$

This formula gives us the magnitude of a two-dimensional vector "A" in terms of magnitudes of its components A_x and A_y.

Direction of $\vec{A}$:

To find the direction of the vector $\vec{A}$, or ***in other words*** the angle "θ" which it makes with $+x$-axis, consider ΔOPQ, In this triangle,

$$\tan\theta = \frac{QP}{OQ} \qquad \text{But} \quad OQ = A_x \text{ (magnitude of } x\text{-component } \vec{A}_x) \quad QP = A_y \text{ (magnitude of } y\text{-component } \vec{A}_y)$$

$$\therefore \quad \tan\theta = \frac{A_y}{A_x} \qquad \therefore \quad \boxed{\theta = \tan^{-1}\frac{A_y}{A_x}}$$

Q.No.3: How is magnitude of a three-dimensional vector calculated using its rectangular components?

Ans: If A_x, A_y and A_z are magnitude of components of a vector, then magnitude of the vector $\vec{A}$ can be calculated by:

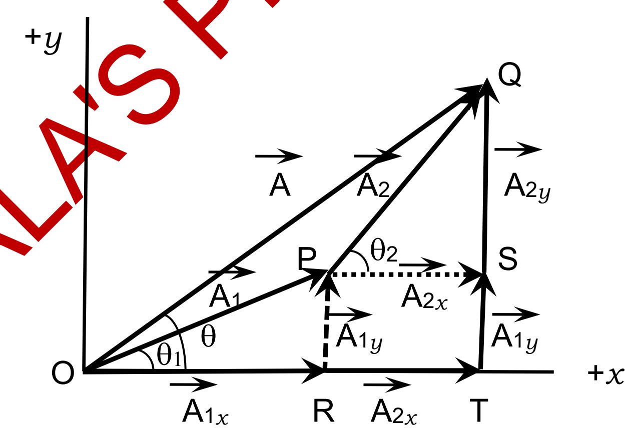
$$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Addition of vectors by rectangular components: (*2026 Karachi Board*)

In order to understand the addition of vectors by rectangular component method, consider two vectors "$\vec{A}_1$" making an angle "θ_1" and "$\vec{A}_2$" making an angle "θ_2" with +x-axis.

First of all, we will follow head to tail rule to add them and then we will add them by their rectangular components. We must get the same resultant (answer vector) by both the methods.

To add these vectors by head to tail rule method, we will join the head of $\vec{A}_1$ with the tail of $\vec{A}_2$. Their resultant "A" will be obtained by joining the tail of $\vec{A}_1$ with the head of $\vec{A}_2$. Hence line $\vec{OQ}$ represents their resultant $\vec{A}$ making an angle "θ" with x-axis.



Now if we resolve $\vec{A}_1$ and $\vec{A}_2$ into x- and y-components, we get:

$\vec{OR} = A_{1x}\,\hat{i} = A_1 \cos\theta_1\,\hat{i} \qquad \vec{PS} = A_{2x}\,\hat{i} = A_2 \cos\theta_2\,\hat{i}$

$\vec{RP} = A_{1y}\,\hat{j} = A_1 \sin\theta_1\,\hat{j} \qquad \vec{SQ} = A_{2y}\,\hat{j} = A_2 \sin\theta_2\,\hat{j}$

$\therefore \quad OT = OR + RT$

But: $RT = PS = A_{2x}$ and $OT = A_x$

$\therefore$ $OT = OR + RT$

$\therefore$ $A_x = A_{1x} + A_{2x}$

OR $A_x = A_1 \cos\theta_1 + A_2 \cos\theta_2$.

Similarly: $TQ = TS + SQ$

But $TS = RP = A_{1y}$ and $SQ = A_{2y}$

$\therefore$ $A_y = A_{1y} + A_{2y}$

OR $A_y = A_1 \sin\theta_1 + A_2 \sin\theta_2$

A_x and A_y represent the magnitudes of rectangular components of the resultant vector "$\vec{A}$". Hence the ***magnitude*** of the resultant vector "$\vec{A}$" is determined by:

$$\boxed{A = \sqrt{A_x^2 + A_y^2}}$$

Similarly, the ***direction*** of the resultant vector $\vec{A}$, or ***in other words*** the angle which it makes with +x-axis, is given by:

$$\therefore \quad \boxed{\theta = \tan^{-1}\frac{A_y}{A_x}}$$

The resultant vector obtained by this method is represented by $\vec{OQ}$, making an angle "θ" with $+x$-axis.

- Hence whether vectors are added by head to tail rule or with the help of their rectangular components, we get the same resultant.
- Addition of vectors by rectangular component method is more accurate, it is especially use full in adding three dimensional vectors.
- By this method ***any number*** of vectors can be added at a time, in this method of addition of vectors diagrams are not needed, values of trigonometric ratios at various angles to calculate values of components are needed.

Steps to be taken for addition of vectors by rectangular component method:

1. Resolve each given vector into its x- and y-components.
2. Add magnitude of all x-components, with their + or – signs, to get the magnitude of x-component A_x of the resultant:

$\therefore$ $A_x = A_{1x} + A_{2x} + A_{3x} + \ldots\ldots\ldots\ldots\ldots\ldots$

OR $A_x = A_1 \cos\theta_1 + A_2 \cos\theta_2 + A_3 \cos\theta_3 + \ldots\ldots\ldots\ldots$

3. Add magnitude of all the y -components, with their + or – signs, to get the magnitude of y-component A_y of the resultant:

$\therefore$ $A_y = A_{1y} + A_{2y} + A_{3y} + \ldots\ldots\ldots\ldots\ldots\ldots$

OR $A_y = A_1 \cos\theta_1 + A_2 \cos\theta_2 + A_3 \cos\theta_3 + \ldots\ldots\ldots\ldots$

Magnitude of the resultant:

Now calculate the magnitude of resultant A by:

$$A = \sqrt{A_x^2 + A_y^2}$$

OR

$$\boxed{A = \sqrt{A_x^2 + A_y^2}}$$

Direction of the resultant:

To find the direction of resultant $\vec{A}$, calculate the angle "θ" which the resultant makes with $+x$-axis by:

$$\boxed{\theta = \tan^{-1} \frac{A_y}{A_x}}$$

Note that the above formula will give us the angle which the resultant makes with $+x$-axis in the first quadrant in anticlockwise sense.

Also note that the positive or negative sign of a component of the resultant depends upon the location of the resultant vector in first, second, third or fourth quadrants. If the resultant vector is in any quadrant, other than the first quadrant, then it's direction must be corrected.

Product of vectors:

Scalar or dot product:

Q.No.1: What is scalar or dot product of vectors? Give some examples.

Ans: If multiplication of two vectors results in a ***scalar quantity*** then such type of multiplication of vectors is known as ***scalar product***.

OR

When two parallel vectors are multiplied then the quantity obtained will be a ***scalar***, this type of multiplication is known as ***scalar product***.
Since this type of product of vectors is represented by a dot placed between vectors to be multiplied, hence it is also called ***dot product***.

Examples.1:

Work is the scalar or dot product of force "$\vec{F}$" and the resulting displacement "$\vec{d}$". It is a scalar quantity, although work is the product of two vectors.

$$\text{Work} = \vec{F} \cdot \vec{d}$$

Examples.2:

Electric flux "φ" is the number of lines of electric force passing normally through a surface, it is a scalar quantity. It is equal to the scalar or dot product of electric intensity (or strength of electric field) "$\vec{E}$" and vector area "$\Delta\vec{A}$" of the surface.

$$\text{Electric flux } \phi = \vec{E} \cdot \Delta\vec{A}$$

Examples.3:

Power is the scalar product of force "$\vec{F}$" and velocity "$\vec{v}$". Power is a scalar quantity.

$$\text{Power} = \vec{F} \cdot \vec{v}$$

Q.No.2: What is the formula for finding the magnitude of scalar quantity obtained in scalar or dot product.

Ans: Formula for finding the magnitude of scalar quantity obtained in dot product of two vectors is:

$$\boxed{\vec{A} \cdot \vec{B} = A\,B \cos\theta}$$

Where "θ" is the angle **between** vectors $\vec{A}$ and $\vec{B}$.

Q.No.3: Show that dot product of two non-zero vectors is zero when vectors are perpendicular to each other.

Ans: Magnitude of scalar or dot product of any two vectors is given by:

$$\vec{A} \cdot \vec{B} = A\,B\,\text{Cos}\,\theta$$

If given vectors are perpendicular to each other, angle "θ" between them will be <u>90°</u>:

$$\vec{A} \cdot \vec{B} = A\,B\,\text{Cos}\,90° \qquad (\text{Cos}\,90° = 0)$$

$$\vec{A} \cdot \vec{B} = 0$$

Hence if scalar product of two non-zero vectors is zero then the vectors are **<u>perpendicular</u>** to each other (θ = 90°).

<u>Q.No.4</u>: Show that $\hat{i} \cdot \hat{j} = 0$.

<u>Ans</u>: According to the definition of dot product:

$$\hat{i} \cdot \hat{j} = i\,j\,\text{Cos}\,\theta$$

Since "$\hat{i}$" is along x-axis and "$\hat{j}$" is along y–axis, hence the vectors to be multiplied are ***<u>perpendicular</u>*** to each other and angle "θ" between them is 90°. Therefore,

$$\hat{i} \cdot \hat{j} = i\,j\,\text{Cos}\,90° \quad (\text{Cos}\,90° = 0) \qquad \text{OR} \qquad \hat{i} \cdot \hat{j} = 1 \times 1 \times 0$$

$$\therefore \qquad \boxed{\hat{i} \cdot \hat{j} = 0}$$

Similarly: $\hat{j} \cdot \hat{k} = 0$ and $\hat{k} \cdot \hat{i} = 0$.

<u>Q.No.5</u>: At what angle is value of scalar or dot product of two vectors of given magnitude <u>maximum</u>.

<u>Ans</u>: Magnitude of scalar or dot product of any two vectors is given by:

$$\vec{A} \cdot \vec{B} = A\,B\,\text{Cos}\,\theta$$

Maximum value of Cos θ is "1" when "θ" is 0° or when vectors are ***<u>parallel</u>*** to each other. Hence:

$$\vec{A} \cdot \vec{B} = A\,B\,\text{Cos}\,0° \qquad (\text{Cos}\,0° = 1)$$

$$\vec{A} \cdot \vec{B} = A\,B$$

Hence scalar product of two vectors is **<u>maximum</u>** when vectors are ***<u>parallel</u>*** to each other or the angle between them is <u>"0°"</u>.

<u>Q.No.6</u>: Show that $\vec{A} \cdot \vec{A} = |\vec{A}|^2$ OR $\vec{A} \cdot \vec{A} = A^2$.

<u>Ans</u>: Since the angle between vectors to be multiplied $\vec{A}$ and $\vec{A}$ is zero, therefore,

$$\vec{A} \cdot \vec{A} = A\,A\,\text{Cos}\,0° \qquad (\text{Sine}\ \vec{A} \parallel \vec{A})$$

$$\vec{A} \cdot \vec{A} = A^2 \qquad (\text{Cos}\,0° = 1)$$

- Hence dot product of a vector by itself is equal to the square of magnitude of the vector.

<u>Q.No.7</u>: Show that $\hat{i} \cdot \hat{i} = 1$.

<u>Ans</u>: According to the definition of dot product:

$$\hat{i} \cdot \hat{i} = i\,i\,\text{Cos}\,\theta \qquad (\theta = 0°,\ \hat{i} \parallel \hat{i},\ i = 1)$$

$\hat{i} \cdot \hat{i} = 1\times1\times \text{Cos } 0°$ OR $\hat{i} \cdot \hat{i} = 1\times1\times1$ $\quad$ (Cos 0°= 1)

$$\therefore \quad \boxed{\hat{i} \cdot \hat{i} = 1}$$

Similarly, $\hat{j} \cdot \hat{j} = 1$ and $\hat{k} \cdot \hat{k} = 1$.

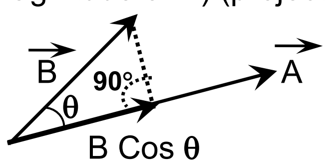
Laws obeyed by scalar or dot product:

Commutative law:

Q.No.1: Show that $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$. OR
Show that scalar product obeys commutative law.

Ans: Scalar or dot product of two vectors is basically equal to the magnitude of first vector multiplied by the projection of second vector on the first vector.

i.e. $\vec{A} \cdot \vec{B}$ = (magnitude of $\vec{A}$) (projection of $\vec{B}$ on $\vec{A}$)



To find $\vec{A} \cdot \vec{B}$ we will multiply the magnitude of $\vec{A}$ by the projection of $\vec{B}$ on $\vec{A}$. But the projection of $\vec{B}$ on $\vec{A}$ is B Cos θ, where "θ" is the angle between vectors $\vec{A}$ and $\vec{B}$, hence:

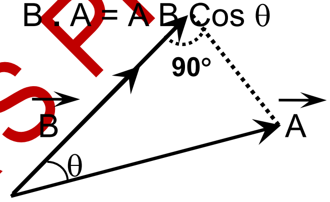
$$\vec{A} \cdot \vec{B} = A\,B \text{ Cos } \theta$$

Similarly, $\vec{B} \cdot \vec{A}$ = (magnitude of $\vec{B}$)(projection of $\vec{A}$ on $\vec{B}$)

In the following diagram we can see that the projection of $\vec{A}$ on $\vec{B}$ is A Cos θ. Hence:

$$\vec{B} \cdot \vec{A} = (B)(A \text{ Cos } \theta)$$

$$\vec{B} \cdot \vec{A} = A\,B \text{ Cos } \theta$$



Projection of $\vec{A}$ on $\vec{B}$ is taken by dropping a perpendicular from the head of $\vec{A}$ in the direction of $\vec{B}$

Right hand side of both equations is same, therefore, $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$.

- Scalar or dot product **obeys commutative law** i.e. $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$.
- It means that the scalar product of vectors can be taken in any ***order***.

Q.No.2: What is the scalar product of vectors opposite to each other?

Ans: By definition magnitude of scalar product of two vectors is given by:

$$\vec{A} \cdot \vec{B} = A\,B \text{ Cos } \theta.$$

In this case the angle between $\vec{A}$ and $\vec{B}$ is 180°

$$\vec{A} \cdot \vec{B} = A\,B \text{ Cos } 180° \qquad (\text{Cos } 180° = -1)$$

$$\vec{A} \cdot \vec{B} = -A\,B$$

Scalar or dot product of vectors opposite to each other is ***negative***.

***Q.No.3**: Give scalar or dot product of vectors in terms of their rectangular components.*

Ans: In terms of their rectangular components, dot product of any two vectors is given by:

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_k \hat{k}) . (B_x \hat{i} + B_y \hat{j} + B_k \hat{k})$$

$$\vec{A} \cdot \vec{B} = (A_x B_x + A_y B_y + A_k B_k)$$

(This result is obtained by taking $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$ and $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$)

Vector or cross product:

***Q.No.1**: What is vector or cross product?*

Ans: If multiplication of two vectors results in a ***vector quantity*** then such type of multiplication of vectors is known as ***vector product***.

OR

When two perpendicular vectors are multiplied then the quantity obtained will be a ***vector***, this type of multiplication is known as ***vector product***.

Since this type of product of vectors is represented by a cross placed between vectors to be multiplied hence it is also called ***cross product***.

$$\therefore \qquad \boxed{\vec{A} \times \vec{B} = \vec{C}}$$

- The new vector "$\vec{C}$" thus obtained is known as ***product vector***.

***Q.No.2**: What is the magnitude of vector product?*

Ans: The magnitude of vector which is obtained as a result of vector product of vectors $\vec{A}$ and $\vec{B}$ (It is also called product vector) is given by:

$$\text{OR} \qquad \boxed{\vec{A} \times \vec{B} = A\,B \sin\theta\ \hat{n}}$$

Where "$\hat{n}$" is a unit vector it shows the direction of product vector, "θ" is the angle between vectors $\vec{A}$ and $\vec{B}$.

***Q.No.3**: How is the direction of product vector found?*

Ans: Direction of the product vector is found with the help of **right-hand rule**.

***Q.No.4**: State **right hand rule**.*

Ans: According to right hand rule, first of all tails of representative lines of vectors to be multiplied is joined to gather. Then rotate the vector which appears first in the product towards the second through smaller of the two possible angles. Then curl the fingers of right hand in the direction of rotation, stretch the thumb it will point in the direction of the product vector.

The product vector is always ***perpendicular*** to the plane formed by the original vectors $\vec{A}$ and $\vec{B}$, it is always **perpendicular** to both $\vec{A}$ as well as $\vec{B}$.

***Q.No.5**: Show that cross product of two vectors is zero when vectors are **parallel** to each other.*

Ans: Vector or cross product of any two vectors is given by:

$$\vec{A} \times \vec{B} = A\,B \sin\theta\ \hat{n}$$

If given vectors are **parallel** to each other, angle "θ" between them will be 0°:

$$\vec{A} \times \vec{B} = A\,B\,\text{Sin}\,0°\,\hat{n} \qquad (\text{Sin}\,0° = 0)$$

$$\vec{A} \times \vec{B} = 0$$

Hence if vector product of two non-zero vectors is zero then the vectors are ***parallel*** or ***opposite*** to each other (θ = 0°, θ = 180°).

***Q.No.6*: Show that $\vec{A} \times \vec{A} = 0$**

***Ans*:** By definition: $\vec{A} \times \vec{A} = A\,A\,\text{Sin}\,\theta\,\hat{n}$

$\therefore \quad \vec{A} \times \vec{A} = A\,A\,\text{Sin}\,0°$ ($\vec{A}$ and $\vec{A}$ are parallel to each other)

$\therefore \quad \boxed{\vec{A} \times \vec{A} = 0}$ (Since Sin 0° = 0)

- Hence vector product of a vector $\vec{A}$ by itself will be zero.

Similarly, $\hat{i} \times \hat{i} = 0$, $\hat{j} \times \hat{j} = 0$ and $\hat{k} \times \hat{k} = 0$.

***Q.No.7*: If vectors are *perpendicular* to each other? (Or the angle between them is 90°) their vector or cross product is *maximum*.**

***Ans*:** Vector product of any two vectors is given by:

$$\vec{A} \times \vec{B} = A\,B\,\text{Sin}\,\theta\,\hat{n}$$

When θ = 90°or when vectors $\vec{A}$ and $\vec{B}$ are ***perpendicular*** to each other.

$$\vec{A} \times \vec{B} = A\,B\,\text{Sin}\,90°\,\hat{n} \qquad (\vec{A} \perp \vec{B},\ \theta = 90°)$$

$$\boxed{\vec{A} \times \vec{B} = A\,B\,\hat{n}} \qquad (\text{Sin}\,90° = 1)$$

- Vector product has a ***maximum*** magnitude if vectors to be multiplied are ***perpendicular*** to each other.

***Q.No.8*: Show that $\hat{i} \times \hat{j} = \hat{k}$.**

***Ans*:** By definition:

$$\hat{i} \times \hat{j} = \hat{i}\ \hat{j}\ \text{Sin}\,90°\,\hat{n}$$

(θ = 90° because $\hat{i}$ is along x-axis and $\hat{j}$ is along y-axis, $\hat{i} \perp \hat{j}$.)

$\therefore \quad \hat{i} \times \hat{j} = 1 \times 1 \times 1\,\hat{n} \qquad (\hat{i} = \hat{j} = 1,\ \text{Sin}\,90° = 1)$

$\therefore \quad \hat{i} \times \hat{j} = 1\,\hat{n}$

Direction of $\hat{i} \times \hat{j}$, on applying right hand rule, comes out to be along + z-axis. ***In other words***, the vector obtained is of magnitude "1" and its direction is along + z-axis (hence it is a unit vector along +z-axis). The unit vector along + z-axis is $\hat{k}$. Hence:

$$\boxed{\hat{i} \times \hat{j} = \hat{k}}$$

Similarly, $\hat{j} \times \hat{k} = \hat{i}$ and $\hat{k} \times \hat{i} = \hat{j}$

Commutative law:

Q.No.9: Show that $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ OR $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

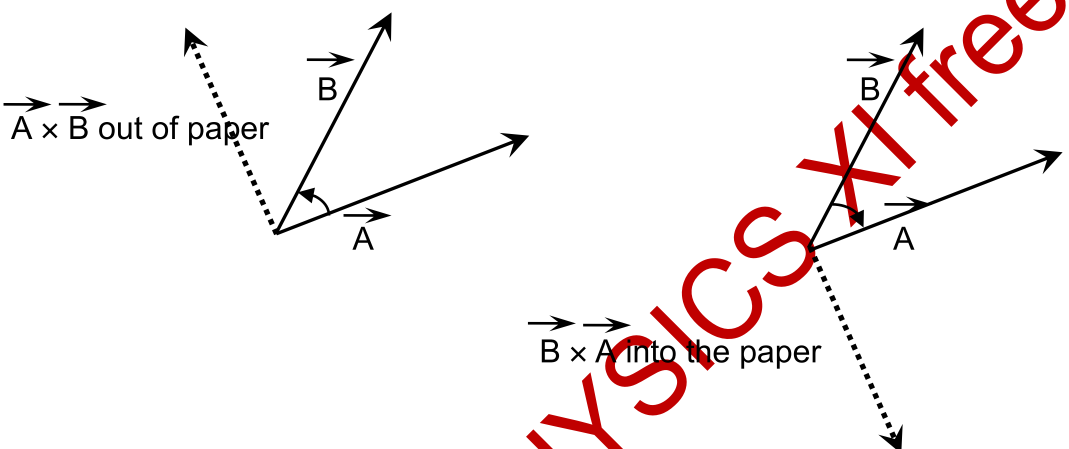
OR

Show that vector product *does not obey commutative law*.

Ans: By definition:

$\vec{A} \times \vec{B} = A\,B\,\text{Sin}\,\theta\,\hat{n}$ and $\vec{B} \times \vec{A} = B\,A\,\text{Sin}\,\theta\,\hat{n} = A\,B\,\text{Sin}\,\theta\,\hat{n}$

These equations show that the magnitude of $\vec{A} \times \vec{B}$ is same as the magnitude of $\vec{B} \times \vec{A}$. But since both products give vector quantities, hence applying right hand rule to the following figure we see that $\vec{A} \times \vec{B}$ is directed ***out of paper*** whereas $\vec{B} \times \vec{A}$ is directed ***into the paper***.



Two vectors are said to be different if either their magnitude, direction or magnitude as well as their direction is different. Since the directions of $\vec{A} \times \vec{B}$ and that of $\vec{B} \times \vec{A}$ is different, therefore, they are ***not equal***.

- ***In other words***, vector or cross product ***does not obey*** commutative law, because on changing the order of vectors to be multiplied the direction of product vector comes out to be opposite. It means that:

$$\boxed{\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}}$$

If a negative sign is placed before a vector then it's direction reverses, Since $\vec{B} \times \vec{A}$ gives a vector quantity, therefore, the direction of $-\vec{B} \times \vec{A}$ after reversing becomes same as the direction of $\vec{A} \times \vec{B}$ and since their magnitude is also equal therefore,

$$\boxed{\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}}$$

- Hence $\vec{B} \times \vec{A}$ is **negative** of $\vec{A} \times \vec{B}$.

But $\vec{A} \times \vec{B} = A\,B\,\text{Sin}\,\theta\,\hat{n}$ $\therefore$ $\boxed{\vec{B} \times \vec{A} = -A\,B\,\text{Sin}\,\theta\,\hat{n}}$

Q.No.10: Show that $\hat{j} \times \hat{i} = -\hat{k}$.

Ans: Vector product ***does not obey*** commutative law because on changing the order of vectors to be multiplied the direction of product vector reverses. In the present case $\hat{j} \times \hat{i}$ comes out to be a unit vector along –z-axis, which is $-\hat{k}$. Hence:

$$\boxed{\hat{j} \times \hat{i} = -\hat{k}}$$

Similarly, $\hat{k} \times \hat{j} = -\hat{i}$ and $\hat{i} \times \hat{k} = -\hat{j}$

Q.No.11: How is a unit vector perpendicular to $\vec{A}$ as well as $\vec{B}$ and to the plane formed by $\vec{A}$ and $\vec{B}$ determined?

Ans: Since the direction of $\vec{A} \times \vec{B}$, on applying right hand rule, comes out to be perpendicular to $\vec{A}$ as well as $\vec{B}$ and to the plane formed by $\vec{A}$ and $\vec{B}$, therefore, if $\hat{n}$ represents the direction of $\vec{A} \times \vec{B}$ then:

$$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n} \qquad \text{or} \qquad \vec{A} \times \vec{B} = |\vec{A} \times \vec{B}|\, \hat{n}$$

Hence a unit vector perpendicular to any two vectors $\vec{A}$, $\vec{B}$ and to the plane formed by them is given by:

$$\boxed{\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}}$$

$\hat{n}$ is a unit vector perpendicular to $\vec{A}$, $\vec{B}$ and to the plane formed by $\vec{A}$ and $\vec{B}$.

Q.No.12: Give the determinant form in which cross product may be written?

Ans: If vectors $\vec{A}$ and $\vec{B}$ are expressed in terms of their rectangular components then their vector product can be found by solving their determinant i.e.

if $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ Then:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- This is the determinant form of vector or cross product.

Q.No.13: Give some examples of vector product.

Ans: ***Example.1***: **Torque "τ"** is the turning effect of a force produced in a body about a certain axis. It is a vector quantity. It is equal to the vector or cross product of position vector "$\vec{r}$" and force "$\vec{F}$".

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

- Direction of torque is determined by right hand rule. Its direction is same as the direction of $\vec{r} \times \vec{F}$.

Example.2: When a charged particle carrying charge "q" moving with some velocity

"$\vec{v}$" enters a magnetic field of strength (called **magnetic flux density**)"$\vec{B}$" it experiences a force "$\vec{F}$", given by:

$$\boxed{\vec{F} = q\,(\vec{v} \times \vec{B})}$$

Magnitude of this force is "q v B Sin θ", where "θ" is the angle at which the charged particle enters a magnetic field or it is the angle between the direction of magnetic field "$\vec{B}$" and the direction of "$\vec{v}$". The direction of this force is determined by right hand rule (It's direction is same as the direction of $\vec{v} \times \vec{B}$) it is always perpendicular to "$\vec{v}$", "$\vec{B}$" and to the plane in which $\vec{v}$ and $\vec{B}$ are.

Example.3: **Angular momentum** is a **vector quantity**, basically It is the momentum of a body whose motion is angular. Angular momentum of a body about a fixed point (such as origin of a coordinate system or axis of rotation) is the vector or cross product of its position vector "$\vec{r}$" and its linear momentum "$\vec{p}$".

$$\boxed{\vec{l} = \vec{r} \times \vec{p}}$$

Where "$\vec{r}$" gives position of the body (or a point) with respect to the fixed point (such as axis of rotation of the body or origin of a coordinate system).

Q.No.14: Give properties of vector or cross product.

Ans: Vector or cross product shows following properties:

(*i*) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

It shows that cross product does not obey commutative law.

(*ii*) $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

(*iii*) $(\vec{A} + \vec{B}) \times \vec{C} = \vec{A} \times \vec{C} + \vec{B} \times \vec{C}$

(*iv*) $\hat{i} \times \hat{i} = 0,\quad \hat{j} \times \hat{j} = 0$ and $\hat{k} \times \hat{k} = 0$

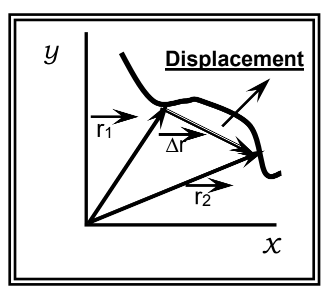
$\hat{i} \times \hat{j} = \hat{k},\quad \hat{j} \times \hat{k} = \hat{i},$ and $\hat{k} \times \hat{i} = \hat{j},$

$\hat{j} \times \hat{i} = -\hat{k},\quad \hat{k} \times \hat{j} = -\hat{i}$ and $\hat{i} \times \hat{k} = -\hat{j}$

Displacement:

Q.No.1: What is displacement?

Ans: It is the shortest straight-line distance between initial and final positions of a body. Or it is the change in position of a body.



Position of a body with respect to a fixed point (origin of axes) is given by a position vector; hence its displacement will be the difference of final and initial position vectors i.e.

$$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$$

• Displacement is a ***vector quantity***; its **magnitude** is equal to the distance between the two positions and it is **directed** towards the **final position** of the body.

• It's S.I unit is ***meter "m"***.

<u>Speed</u>:

<u>Q.No.2</u>: What is <u>speed</u> of a body?
<u>Ans</u>: It is the distance covered by a body in unit time.
Or ***It is the rate of distance covered.***

$$\text{Speed} = \frac{\text{Distance covered.}}{\text{Time taken}}$$

$$\boxed{v = \frac{s}{t}}$$

- Speed is a ***<u>scalar</u>*** quantity.
- S.I unit of speed is ***m/s*** or ***ms*** $^{-1}$. Its dimensional formula is $[LT^{-1}]$

<u>Velocity</u>:

<u>Q.No.3</u>: What is <u>velocity</u>?
<u>Ans</u>: It is the distance covered by a body in unit time in a particular direction.
OR ***it is speed of a body in a particular direction.***
OR ***It is the rate of displacement of the body.***

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$\boxed{\vec{v} = \frac{\vec{s}}{t}}$$

- Velocity is a ***<u>vector</u>*** quantity. Its direction is **<u>same</u>** as the ***direction of displacement*** (or direction of motion).
- S.I unit of velocity is ***m/s*** or ***ms*** $^{-1}$ (same as the unit of speed).
- Its dimensional formula is $[LT^{-1}]$
- In terms of its rectangular components, it is given by:

$$\vec{v} = v_x\,\hat{i} + v_y\,\hat{j}$$

<u>Q.No.4</u>: What is average velocity?
<u>Ans</u>: It the total displacement of the body divided by total time taken.

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

- It is determined by dividing the total displacement of the body by the time taken.

<u>Q.No.5</u>: What is instantaneous velocity?
<u>Ans</u>: Velocity of a body at a ***<u>particular instant</u>*** (or **during a very <u>short interval of time</u>**, or **at a particular point**) is called its ***<u>instantaneous velocity</u>***.

- It is determined by first finding the displacement of the body during a very short interval of time and then dividing that displacement by the short time interval.

$$\boxed{\vec{v}_{inst.} = \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t}}$$

" $\lim_{\Delta t \to 0}$ " is read as limit Δt approaches to zero, it means that the time interval is so short that it is close to zero (but not equal to zero).

<u>Q.No.6</u>: When does velocity become variable?

<u>Ans</u>: Velocity is a ***<u>vector quantity</u>***, hence like any vector quantity it becomes variable when either there is a <u>change in its magnitude</u> (or ***in other words***, there is a change in speed of the body), <u>direction</u> or there is a <u>change in magnitude and direction</u> both.

<u>Q.No.7</u>: What is uniform velocity?

<u>Ans</u>: Uniform velocity is that which ***<u>does not change</u>*** in magnitude and/or in direction.

Or it is the equal distance covered by a body in equal intervals of time in a particular direction (however short the time interval may be).

- When a body moves with a uniform velocity it's instantaneous and average velocities are <u>equal</u>.

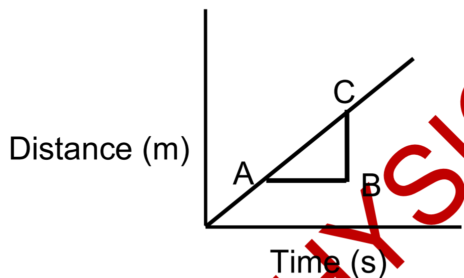
$\vec{v}_{av.} = \vec{v}_{inst.}$ $\qquad$ $\vec{v}$ = uniform.

<u>Q.No.8</u>: How is velocity of a body calculated from its distance-time graph?

<u>Ans</u>: If the direction of motion is not changing then the velocity of a body can be calculated from its distance-time graph by finding **<u>slop</u>** of the graph.

<u>Q.No.9</u>: What is the shape of distance-time graph when a body moves with a uniform velocity?

<u>Ans</u>: When a body moves with a uniform velocity it's distance-time graph will be a **<u>straight line</u>**, passing through origin (as shown in diagram).



(Distance-time graph)

Slope (velocity) = $\dfrac{BC}{AB}$

$\left\{\begin{matrix} BC = S \\ AB = \Delta t \end{matrix}\right\}$

- Slope of this type of graph is ***<u>same</u>*** at all points, which shows that velocity of the body is same everywhere, ***In other words*** this shape of distance-time graph is for a body moving with a ***uniform velocity***.

<u>Q.No.10</u>: What is the shape of distance-time graph of a body moving with a variable speed, in the same direction?

<u>Ans</u>: For a body moving with a variable speed the distance-time graph will be a curve. The shape of the curve depends upon the variation in the speed of body (velocity of a body may also be variable due to a change in its direction).

- In this case, slope of the curve at a given point on the curve is found, which gives instantaneous speed of the body at that instant. The slope of the curve will ***<u>not be</u>*** the same at all points (***in other words***, its speed will not be the same at different instants).

<u>Acceleration</u>:

<u>Q.No.11</u>: What is <u>acceleration</u>?

<u>Ans</u>: Rate of change of velocity of a body is called its acceleration.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$\boxed{\vec{a} = \frac{\Delta \vec{v}}{\Delta t}} \qquad \text{Or} \qquad \boxed{\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}}$$

• Acceleration is a ***vector*** quantity; its direction is ***same*** as the direction of ***change in velocity "$\Delta\vec{v}$"*** of the body.

• If $v_f > v_i$ i.e. if velocity of a body ***increases*** then its acceleration will be ***positive***, the direction of acceleration and the direction of motion will be ***same***.

• If $v_f < v_i$ i.e. if velocity of a body ***decreases*** then its acceleration will be ***negative***, It is also called ***retardation*** or ***deceleration***, in this case the direction of acceleration and the direction of motion will be ***opposite***.

- S.I unit of acceleration is ***"m/s^2"*** or ***"$m\ s^{-2}$"***
- Its dimensional formula is $[LT^{-2}]$.

Q.No.12: What is instantaneous acceleration?

Ans: It is the acceleration of a body at a **particular instant** or during a very short interval of time.

• It can be calculated by finding the change in velocity during a very short interval of time and then dividing this change in velocity by the short interval of time.

$$\boxed{\vec{a}_{ints.} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t}}$$

" $\lim_{\Delta t \to 0}$ " is read as "limit Δt approaches to zero", it means that the time interval is so short that it is close to zero (but not equal to zero).

Q.No.13: What is uniform acceleration?

Ans: When there is an equal change in velocity in equal intervals of time (whether the time interval is short or long) then the acceleration of the body is said to be uniform.

• When a body has uniform acceleration it's average and instantaneous accelerations will be equal (It means that the rate of change of velocity of the body will be equal whether it is measured during a short or a long interval of time).

Q.No.14: How is acceleration of a body calculated from its velocity-time graph?

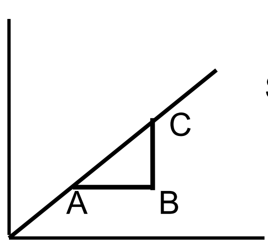
Ans: Acceleration of a body is equal to the ***slop of its velocity-time graph***.

- The slope of velocity-time graph for uniform acceleration is same at all instants.
- The slope of velocity-time graph for non- uniform acceleration is different at different instants.
- The slope of velocity-time graph for retardation is negative.
- The slope of velocity-time graph for uniform velocity is zero (because motion of the body is non-accelerated).

Q.No.15: What is the shape of velocity-time graph if acceleration of the body is uniform?

Ans: If a body moves with a uniform acceleration the shape of its velocity-time graph will be a **straight line**, passing through origin.

Velocity (m/s)



Time (s)

(Velocity-time graph)

Slope (acceleration) = $\frac{BC}{AB}$

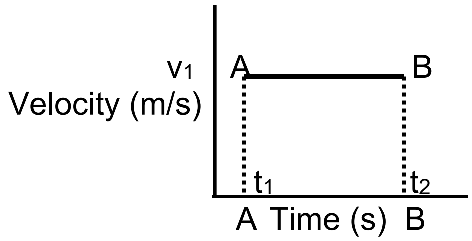
$$\left\{\begin{array}{l} BC = \Delta v \\ AB = \Delta t \end{array}\right\}$$

<u>Q.No.16</u>: What is the acceleration of a body moving with uniform velocity?

<u>Ans</u>*:** When a body moves with a **<u>uniform velocity</u>** i.e. its velocity does not change due To any reason then it's acceleration will be ***<u>zero</u>.

<u>Q.No.17</u>: What is the shape of velocity-time graph if <u>velocity</u> of the body is <u>uniform</u>?

***<u>Ans</u>*:** If a body moves with a <u>uniform velocity</u> the shape of its velocity-time graph will be a **<u>straight line</u>**, parallel to time axis.



Slope (acceleration) = 0

$\Delta v = v_1 - v_1 = 0$

$\Delta t = AB = t_2 - t_1$

<u>Q.No.18</u>: When does motion of a body become accelerated?

<u>Ans</u>*:** Motion of a body becomes accelerated when it moves with a ***<u>variable velocity</u>.

It means that motion of a body will be accelerated only when either there is a <u>change In magnitude</u>, <u>change in direction</u> or there is a <u>change in magnitude as well as direction</u> of its velocity.

<u>Q.No.19</u>: What type of acceleration is produced when only the direction of velocity changes?

<u>Ans</u>*:** When there is a <u>change only in the direction</u> of velocity of a body the resulting acceleration is called ***<u>centripetal acceleration</u>.

- ***In this case*** the path followed by the body will be <u>circular</u> in shape, and the direction of acceleration will be <u>towards the center</u> of the circular path.

🗣 ***<u>Important note</u>*:** *The angle between velocity $\vec{v}$ and centripetal acceleration $\vec{a_c}$ will be 90°, Similarly the angle between velocity $\vec{v}$ and the force (centripetal force) will be 90°.

<u>Q.No.20</u>: What is non-uniform acceleration?

***<u>Ans</u>*:** Acceleration of a body will be non-uniform if there is an unequal change in its velocity in equal intervals of time.

<u>Q.No.21</u>: Is it possible for a body to have acceleration when moving with a:

(i) Constant velocity. ***(ii) Constant speed.***

***<u>Ans</u>*:** A body can have acceleration only when it's velocity changes. Velocity of a body is said to have changed when there is either a change in its magnitude, direction or a change in magnitude as well as direction.

(*i*) A body <u>cannot</u> have acceleration when it moves with a constant velocity.

(*ii*) A body <u>can</u> have acceleration when it moves with a constant speed, because speed represents only the magnitude of velocity of the body. Direction of its velocity may be changing. Hence the body can have acceleration because of the change in direction of velocity although its speed may be uniform.

Equations of motion:

Q.No.1: Give the equations of motion.

Ans: If "S, v_i, v_f" and "a" are the distance covered, initial and final velocities and uniform acceleration of a body, the time taken t are related by the following equations of motion:

$$v_f = v_i + at$$
$$S = v_i t + \tfrac{1}{2} a t^2$$
$$2aS = v_f^2 - v_i^2$$

- While applying these equations, "v_i" is taken as zero (0) when the body starts moving from rest. "v_f" is zero when a moving body finally comes to rest. "a" is positive when $v_f > v_i$. Similarly, "a" is negative when there is retardation (or deceleration) or when $v_f < v_i$.

Important note: these equations ***can only*** be applied to a body if it moves with a uniform acceleration along a straight line. Also, that the direction of initial velocity "v_i" is taken as a reference, any vector quantity whose direction is opposite to the direction of "v_i" is taken as negative. Based on this idea acceleration due to gravity is taken negative "–g" throughout projectile motion. Because in vertical direction acceleration is "g" directed downward, whereas initial velocity in vertical direction at the point of projection "v_{oy}" is upward. All equations of projectile motion are based on this idea.

Motion in two dimensions:

Q.No.1: What is two-dimensional motion?
Give some examples.

Ans: If a body moves in two directions simultaneously (along x- and y-axes), (or it moves in a plane) then its motion is called two-dimensional motion.

Motion along a ***circular path*** and ***projectile motion*** are common examples of two-dimensional motion.

Projectile motion:

Q.No.1: What is projectile motion?
Give examples of projectile motion.

Ans: If a body moves with a constant velocity along horizontal direction and at the same time falls freely under the action of gravity (i.e. it has a constant vertical acceleration "g"), then its motion is called projectile motion.

Examples:

Motion of a shell fired from a gun, motion of a kicked football, jumping of a frog or a locust, motion of a shuttle cock in badminton etc.

Projectile motion is two dimensional, in one plane. Projectile motion in vertical and horizontal directions can be treated independent of each other.

Q.No.2: On what assumptions is the study of projectile motion based?

Ans: Following assumptions are made to make the study of projectile motion simple and easy to understand:

(i) There is no air resistance.

(Air resistance basically affects the projectile motion in horizontal direction, ***in other words***, no force is assumed to act on a projectile in horizontal direction).

(ii) Value of "g" throughout the projectile motion is assumed to be constant.

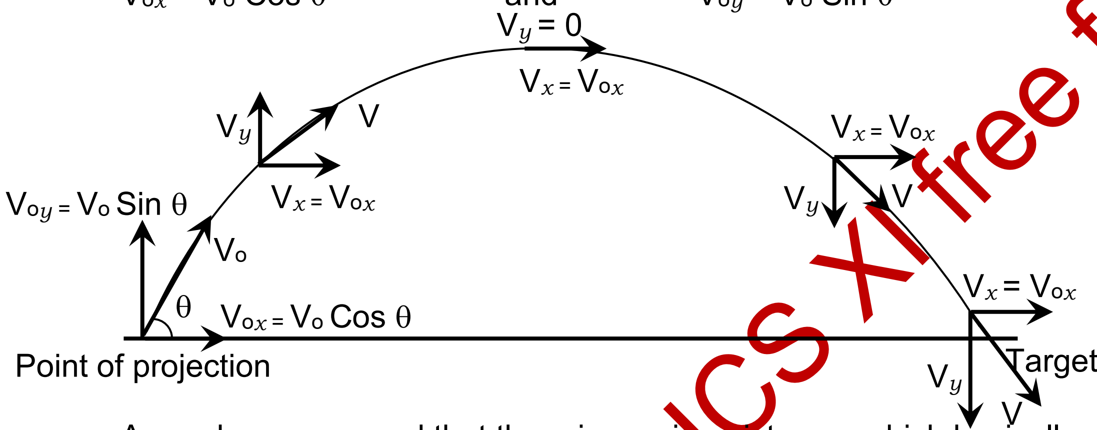
(iii) There is no effect of earth's rotation on projectile motion.

🗣(In reality rotation of the earth affects projectile motion when a projectile has to cover long distance to hit a target e.g. an intercontinental missile, which covers long distances, takes a long time during which the rotation of earth changes position of the target).

Explanation:

Let a projectile, such as a shell fired from a gun, be projected with initial velocity "V_o" making an angle "θ" with the horizontal. Assuming that the air resistance is negligible, its initial velocity "V_o" can be resolved into "x" and "y" components. Taking x-axis along the horizontal and y-axis along the vertical direction, V_{ox} and V_{oy} the x- and y-components of the initial velocity V_o are given by:

$V_{ox} = V_o \cos\theta$ and $V_{oy} = V_o \sin\theta$



As we have assumed that there is no air resistance, which basically affects the motion of a projectile in horizontal direction, therefore, no force is assumed to act on the projectile in horizontal direction ($F_x = 0$). Therefore, there will be no acceleration in x-direction, ***In other words***, component of its velocity V_x will remains constant throughout its motion.

🗣On the other hand, as the projectile (shell) has some weight, therefore, it will experience a constant downward force, in y-direction, (= its weight). Hence there will be a constant downward directed (in y-direction) acceleration "g". Since this acceleration is opposite to the direction of initial velocity "V_{oy}" in y –direction, at the point of projection (upward), hence we will take it with a minus sign "**–** g" throughout the projectile motion.

Velocity of the projectile at any point will have two components "V_x" and "V_y". Velocity "V" with which it hits the target (or its net velocity at any instant) can be calculated with the help of "V_x" and "V_y" at that instant:

$$V = \sqrt{V_x^2 + V_y^2} \qquad \text{Where } V_x = V_{ox}$$

All equations of motion are independently applicable to projectile motion in horizontal as well as to vertical directions.

Along x-axis.	Along y-axis.
$a_x = 0$	$a_y = -g$
$V_x = V_{ox} = V_o \cos\theta$	$V_y = V_{oy} - g\,t$
$x = V_{ox}\,t$	$y = V_{oy}\,t - \frac{1}{2} g\,t^2$

Time taken by projectile to reach the highest point "T".

In vertical direction force of gravity acts on the projectile as a result of which the projectile has a constant vertical acceleration 'g', this constant acceleration is opposite to the initial velocity in vertical direction at the point of projection. Hence $a_y = -g$. Similarly, due to this acceleration, as the projectile rises its velocity decreases and becomes zero at the highest point, projectile will take some time "T" to reach the highest point. Hence In the vertical direction:

$$V_{oy} = V_o \sin\theta$$
$$a_y = -g$$
$$V_y = 0 \text{ (at the highest point)}$$

Time to reach the highest point T =?

On substituting these values in the following equation of motion, we get:

$$V_y = V_{oy} - g\,t$$
$$0 = V_{oy} - g\,T$$
$$T = \frac{V_{oy}}{g} = \frac{V_o \sin\theta}{g}$$

Total time of flight "t".

Projectile takes time T to reach the highest point it will take same time T to return to the ground. Hence total time of flight will be "t = 2T".

The total time of flight or ***in other words*** time for which a projectile remains in the air will be: t = 2T

$$\boxed{t = \frac{2V_{oy}}{g}} \quad \text{or} \quad \boxed{t = \frac{2V_o \sin}{g}}$$

🗣Total time taken by a projectile to move from point of projection to target depends upon the angle of projection "θ" and upon its velocity "V_o" with which it is projected.

🗣For a given projectile projected with a certain velocity "V_o" since the value of Sin θ depends on the value of "θ" and <u>the value of Sin θ increases as "θ" is increased</u>, therefore, <u>larger</u> the angle of projection "θ" <u>longer</u> is the time of flight.

Maximum height reached "h":

When a projectile reaches the highest point on its path its velocity component in y-direction "V_y" is zero, hence for maximum height:

$$V_{oy} = V_o \sin\theta$$
$$V_y = 0 \quad \text{(at the highest point)}$$
$$a = -g$$
$$y = h$$
$$t = T = \frac{V_{oy}}{g}$$

On substituting the above data in the following equation, we get:

$$y = V_{oy}\,T - \tfrac{1}{2}\,g\,T^2$$
$$h = V_{oy} \times \frac{V_{oy}}{g} - \tfrac{1}{2}\,g\left\{\frac{V_{oy}}{g}\right\}^2$$
$$h = \frac{V_{oy}{}^2}{g} - \tfrac{1}{2}\,g\,\frac{V_{oy}{}^2}{g^2}$$
$$h = \frac{V_{oy}{}^2}{g} - \tfrac{1}{2}\,\frac{V_{oy}{}^2}{g}$$

$\therefore$ $\boxed{h = \frac{1}{2}\frac{V_{oy}^{\,2}}{g}}$ But $V_{oy} = V_o \sin\theta$ $\therefore$ $\boxed{h = \frac{1}{2}\frac{V_o^2 \sin^2\theta}{g}}$

From the above formula for maximum height reached by a projectile we can see that for a given projectile greater the angle of projection more is the height reached (also called vertical range).

Range of a projectile "R":

The total horizontal distance covered by a projectile from the point of projection to the point where it lands (target point) is called its range R.

Since in the horizontal direction there is no acceleration (because there is no force in this direction), therefore, distance covered by a projectile in x-direction during any time "t" after the takeoff is given by: $x = V_{ox}\, t$

Hence for total distance covered from the point of projection to target or ***in other words*** for range x = R.

$\therefore$ $R = V_{ox}\, t$

But $V_{ox} = V_o \cos\theta$ & $t = \frac{2 V_o \sin\theta}{g}$ (Total time of flight)

$\therefore$ $R = V_o \cos\theta \times \frac{2 V_o \sin\theta}{g} = \frac{2 V_o^2 \sin\theta \cos\theta}{g}$

But $2 \sin\theta \cos\theta = \sin 2\theta$ Hence range "R" of a projectile projected with a certain velocity "V_o", at an angle "θ" to the horizontal is given by:

$$\boxed{R = \frac{V_o^2 \sin 2\theta}{g}}$$

From the above formula we can see that the range of a projectile projected with a given initial velocity "V_o" depends upon the value of $\sin 2\theta$. Greater the value of $\sin 2\theta$ longer is the range of a projectile.

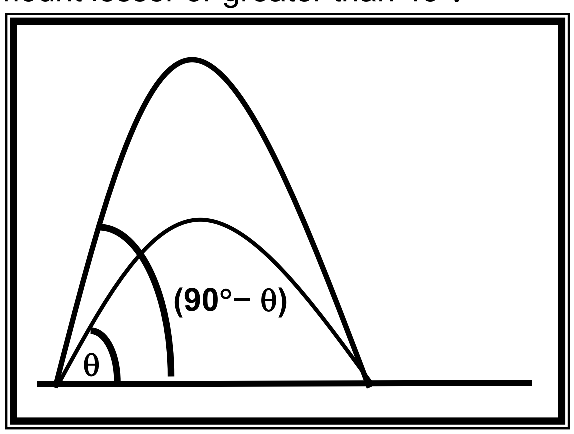
From the formula for range we can also see that range of a projectile will be equal for two angles of projection which are equal amount lesser or greater than 45°.

Maximum range "$R_{max.}$":

Range of a given projectile is maximum when $\sin 2\theta$ is maximum. The maximum value of $\sin 2\theta$ is 1". This is possible when:

$2\theta = 90°$ (Because $\sin 90° = 1$)

In other words: $\theta = 45°$



Range of a projectile will be maximum (i.e. it will cover maximum distance in horizontal direction) when it is projected at an angle of 45°.

Maximum range "$R_{max.}$" of a projectile is given by:

$$\boxed{R_{max.} = \frac{V_o^{\,2}}{g}} \qquad (\sin 2\theta = 1)$$

Section–A Multiple-Choice Questions MCQs:

Q.No.1: To get a resultant of 10 m, two displacement vectors of magnitude 6 m and 8 m should be connected:
(A) Parallel (B) antiparallel
(C) At angle of 45° (D) perpendicular to each other

Q.No.2: The velocity of a particle at an instant is 10 m/s and after 5 sec, velocity of the particle is 20 m/s. Velocity 3 sec before is:
(A) 8 (B) 4 (C) 6 (D) 7

Q.No.3:A ball is thrown upward with a velocity of 100 m/s. It will reach the ground after:
(A) 10 sec. (B) 20 sec. (C) 5 sec. (D) 40 sec

Q.No.4: Two projectiles are fired from the same point with the same speed at angles of 60° and 30 ° respectively. Which of the following is true?
(A) Their range will be the same.
(B) their maximum height will be same.
(C) Their landing velocity will be same.
(D) their time of flight will be same.

Q.No.5: The ratio of numerical values of average velocity and average speed of a body is always:
(A) Unity. (B) unity or less (C) unity or more.
(D) Less than unity.

Q.No.6: If the average velocity of a body becomes equal to the instantaneous velocity, body is said to be moving with:
(A) Uniform acceleration. (B) Uniform velocity.
(C) Variable velocity. (D) Variable acceleration.

Q.No.7:At the top of a trajectory of a projectile, the acceleration is:
(A) Maximum. (B) minimum. (C) zero (D) g

Q.No.8: At what angle the range of a projectile becomes equal to the height of projectile.
(A) 65° (B) 45° (C) 76° (D) 30°

Q.No.9: The angle at which dot product becomes equal to the cross product is:
(A) 65° (B) 45° (C) 76° (D) 30°

Q.No.10: If the dot product of two non-zero vectors vanish, the vectors will be:
(A) in the same direction.
(B) opposite direction each other.
(C) perpendicular to each other. (D)zero

Q.No.11: $\hat{j} \times \hat{j}$ is equal to: ***(2003 Karachi Board)***
(A) j^2 (B) j (C) one (D) zero

Q.No.12: The dot product of unit vector $\hat{i}$ & $\hat{k}$ is:
(2003 Karachi Board)
(A) zero (B) 1 (C) −1 (D) $\hat{j}$

Q.No.13: If $\vec{F} = 4i - 2j$ and $\vec{d} = 3i + 4j$, the work done will be: ***(2003 Karachi Board)***
(A) 4 joule (B) 8 joule (C) 2 joule (D) 12 joules

Q.No.14: If $\vec{a} \cdot \vec{b} = 0$ when $\vec{a} \neq 0$, $\vec{b} \neq 0$ the two vectors are: ***(2004 Karachi Board)***
(A) parallel (B) opposite (C) perpendicular

Q.No.15: When $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, the angle between the vectors A and $\vec{B}$ is:
(2004 Karachi Board)
(A) Zero (B) 45° (C) 90°

Q.No.16: If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along x-,y- and z-axes respectively, then $\hat{k} \times \hat{j}$:
(2009 Karachi Board)
(A) $\hat{i}$ (B) $-\hat{i}$ (C) 1 (D) −1

Q.No.17: If a vector is divided by its own magnitude, the resulting vector is called:
(2009, 2003 Karachi Board)
(A) Position vector (B) unit vector
(C) null vector (D) free vector

Q.No.18: If $\vec{A} = a\hat{i}$ and $\vec{B} = b\hat{j}$, then $\vec{A} \times \vec{B}$ is equal to: ***(2012 Karachi Board)***
(A) 0. (B) $ab\hat{k}$ (C) $-ab\hat{k}$ (D) None of these.

Q.No.19: Two forces act together on an object; the magnitude of their resultant is minimum when the angle between them is: ***(2013 Karachi Board)***
(A) 0°. (B) 45° (C) 90° (D) 180°

Q.No.20: If $\vec{A} = 5\hat{i} + \hat{j}$ and $\vec{B} = 2\hat{k}$
Then $\vec{A} - \vec{B}$ is equal to: ***(2014 Karachi board)***
(A) $5\hat{i} + \hat{j} + 2\hat{k}$ (B) $5\hat{i} - \hat{j} - 2\hat{k}$
(C) $5\hat{i} + \hat{j} - 2\hat{k}$ (D) $5\hat{i} - \hat{j} + 2\hat{k}$

Q.No.21: If $\vec{A} \cdot \vec{B} = 0$, $\vec{A} \times \vec{B} = 0$ and $\vec{A} \neq 0$ vector $\vec{B}$ is equal to:***(2016,14,05 Karachi board)***
(A) Equal to $\vec{A}$. (B) Zero. (C) Perpendicular to $\vec{A}$.

(D) Parallel to $\vec{A}$.

Q.No.22: The magnitude of product $\hat{k}\cdot(\hat{j}\times\hat{i})$
(***2017, 2016 Karachi Board***)

(A) zero (B) 1 (C) –1 (D) $|\hat{k}|$

Q.No.23: Two perpendicular vectors having magnitudes of 4 units and 3 units are added; their resultant has the magnitude of:
(***2018, 08,09 Karachi Board***)

(A) 7 units (B) 12 units (C) 25 units (D) 5 units

Q.No.24: If $\vec{F} = 3\hat{i}$, and $\vec{d} = 6\hat{j}$, the work done will be
(***2019 Karachi Board***)

(A) zero. (B) 2 (C) 9 (D) 18

Q.No.25: $(\hat{i}\times\hat{j})\cdot(\hat{j}\times\hat{i})$ is: (***2019 Karachi Board***)

(A) –1 (B) $\hat{k}$ (C) 1 (D) zero

Q.No.26: If $\vec{A}\cdot\vec{B} = 0$, $\vec{A}\times\vec{B} = 0$ then $\vec{B}$ is:
(***2022 Karachi Board***)

(A) zero vector. (B) equal to $\vec{A}$.
(C) perpendicular to $\vec{A}$. (D) not parallel to $\vec{A}$.

Q.No.27: $\hat{i}\cdot(\hat{k}\times\hat{j})$ is equal to:
(***2022 Karachi Board***)

(A) 1 (B) –1 (C) 0 (D) $\hat{i}$

Q.No.28: If a vector has three rectangular components each equal to "a", magnitude of the vector will be: (***2022 Karachi Board***)

(A) $\sqrt{3}$ a (B) 3 a (C) a^3 (D) $\sqrt{3}$ a

Q.No.29: It is not a vector quantity:
(***2022 Karachi Board***)

(A) Force (B) Torque (C) Frequency (D)Weight

Q.No.30: When two bodies of unequal weights are dropped simultaneously from the same height, then:
(A) Heavier body will reach the ground earlier
(B) Lighter body will reach the ground earlier
(C) Both of them will reach the ground at the same time.

Q.No.31: Direction of retardation is:
(A) Same as the direction of motion.
(B) Opposite to the direction of motion.
(C) Perpendicular to the direction of motion.

Q.No.32: When a body moves with a constant speed in a circle: (***2005 Karachi Board***)
(A) its velocity is changing
(B) its acceleration is zero
(C) its acceleration is increasing
(D) its velocity is uniform.

Q.No.33: When a constant force is applied on a body, it moves with: (***2007 Karachi Board***)
(A) Constant speed.
(B) Constant velocity.
(C) Constant acceleration. (D) None

Q.No.34: If the average and instantaneous velocities of a body are the same, the body will move with: (***2013 Karachi Board***)
(A) Variable velocity. (B) Uniform velocity.
(C) Variable acceleration
(D) Uniform acceleration.

Q.No.35: A bus of weight 30000 N is moving with uniform velocity of 14 m/s. Its acceleration is
(***2017 Karachi Board***)

(A) 14 m/s. (B) zero.
(C) 7 m/s. (D) 9.8 m/s.

Q.No.36: If velocity of a body is decreasing, The direction of acceleration is:
(***2016 Karachi Board***)
(A) In the direction of velocity.
(B) Opposite to the direction of velocity.
(C) Perpendicular to the direction of velocity.
(D) 60° to the direction of velocity.

Q.No.37: If the time interval is very small ($\Delta t \to 0$), the rate of change of velocity of a bogy is called:
(***2019 Karachi Board***)
(A) average acceleration (B) Acceleration.
(C) Instantaneous acceleration
(D) constant acceleration

Q.No.38: A projectile is fired at an angle 'θ' with the horizontal. Its velocity will be minimum at:
(***2017 Karachi Board***)
(A) the point of projection. (B) the highest point.
(C) point of landing on the ground.
(D) all points of its path.

Q.No.39: A bullet is fired horizontally with 20 m/s, in the absence of air friction its horizontal velocity component after 2 s will be:
(***2018 Karachi Board***)

(A) 40 m/s. (B)20 m/s. (C) 10 m/s. (D) 5 m/s

Q.No.40: The horizontal range of a projectile depends upon: (***2010 Karachi Board***)
(A) The angle of projection.
(B) The velocity of projectile.
(C) 'g' at the place. (D) All of them.

Q.No.41: A projectile is thrown at an angle of 30° with the horizontal having a certain initial velocity. It will have the same range if thrown with the same velocity at an angle of: (***2009 Karachi Board***)

(A) 45° (B) 60° (C) 75° (D) 15°

Q.No.42: Motion on a curved path when one component of velocity is constant and the other is variable is called: (***2004 Karachi Board***)
(A)Circular motion (B)Projectile motion (C)Vibratory motion

Q.No.43: When the dot product of two vectors is negative then the angle between them is:
(***2015 Hyderabad Board***)

(A) 0° (B) 90° (C) 180° (D) 360°

Q.No.44: A …….. vector can be displaced parallel to itself. (***2015 Hyderabad Board***)

(A) Position. (B) Unit (C)Null. (D) Free.

Q.No.45: ……… component of velocity remains same during the motion of a projectile.
(***2015 Hyderabad Board***)

(A) Horizontal. (B) Vertical.
(C) Both (A) & (B) (D) None of these.

Q.No.46: The path followed by a projectile is
(***2016 Hyderabad Board***)

(A) Linear. (B)circular. (C)Parabola (D)Hyperbola.

Q.No.47: The dot product of i and j is:
(***2018 Hyderabad Board***)

(A) k (B) one (C) Zero (D)None of these

Q.No.48: Scalar product of two vectors obeys:
(***2018 Hyderabad Board***)

(A) Snell's law (B) commutative law.
(C) Associative law. (D) Both B and C.

Q.No.49: If $|\vec{A}\,.\,\vec{B}| = |\vec{A} \times \vec{B}|$ then the angle between $\vec{A}$ and $\vec{B}$ is: (***2018 Hyderabad Board***)

(A) 0° (B) 45° (C) 60° (D) 90°.

Q.No.50:If $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors , then $(\hat{i} \times \hat{j})$
(***2025 Karachi Board***)

(A) Zero (B) 1 (C) $\hat{j}$ (D) $\hat{k}$

Q.No.51:The dot product of unit vectors $\hat{i}$ and $\hat{k}$ is: (***2026 Karachi Board***)

(A) 1 (B) –1 (C) zero (D) $\hat{j}$

Answers:

(1) (D) perpendicular to each other
(2) (C) 6 m/s
(3) (B) 20 sec.
(4) (A) Their range will be the same.
(5) (A) Unity.
(6) (B) Uniform velocity.
(7) (D) g
(8) (C) 76°.
(9) (C) 45°
(10) (C) Perpendicular to each other.
(11) (D) zero.
(12) (A) zero.
(13) (A) 4 joules.
(14) (C) Perpendicular
(15) (C) 90°
(16) (B) $-\hat{i}$
(17) (B) unit vector.
(18) (B) $ab\hat{k}$.
(19) (D) 180°.
(20) (C) $5\hat{i}+\hat{j}-2\hat{k}$.
(21) (B) Zero.
(22) (C) –1
(23) (D) 5 units.
(24) (A) zero.
(25) (A) –1
(26) (A) zero vector.
(27) (B) –1
(28) (C) $\sqrt{3}$ a
(29) (C) Frequency
(30) (C) Both of them will reach the ground at the same time
(31) (B) Opposite to the direction of motion.
(32) (A) its velocity is changing
(33) (C) Constant acceleration.
(34) (B) Uniform velocity.
(35) (B) zero.
(36) (B) Opposite to the direction of velocity
(37) (C) Instantaneous acceleration
(38) (B) the highest point.
(39) (B) 20 m/s.
(40) (D) All of them.
(41) (B) 60°
(42) (B) Projectile motion.
(43) (C) 180°
(44) (A) Position.
(45) (A) Horizontal.
(46) (D) Hyperbola.
(47) (C) Zero
(48) (D) Both b and c.
(49) (B) 45°
(50) (D) $\hat{k}$
(51) (C) zero

Numericals:

Q.No.1: A helicopter is ascending at a rate of 12 m/s. At a height of 80 m above the ground a package is dropped.

How long does the package take to reach the ground?

***Data*:**

Initial velocity of the package = velocity with which helicopter is ascending v_i=12m/s

Net displacement of the package S =80 m.

Time taken to reach the ground t =?

***Solution*:**

Initial velocity of the package is equal to the velocity with which helicopter is ascending, hence after being dropped the package will move upward for a short time and then fall downward. ***In other words***, initial velocity of the package is upward and the acceleration "g" and the net displacement "S" are in the downward direction (opposite to the direction of v_i), Hence:

$v_i = +12$ m/s $a = -g$ & $S = -80$ m

Distance covered by the package is given by:

$$S = v_i\, t + \tfrac{1}{2}\, a\, t^2$$

$$-80 = 12 \times t + \tfrac{1}{2} \times -9.8 \times t^2$$

$$-80 = 12\,t - 4.9\,t^2$$

$$4.9\,t^2 - 12\,t - 80 = 0$$

This is a quadratic equation its solution by quadratic formula is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{(quadratic formula)}$$

$$t = \frac{-(-12) \pm \sqrt{(-12)^2 - 4\times4.9\times-80}}{2\times4.9}$$

$$t = \frac{12 \pm \sqrt{144 + 1568}}{9.8} = \frac{12 \pm \sqrt{1712}}{9.8}$$

$$t = \frac{12 \pm 41.98}{9.8}$$ Hence,

$$t = \frac{12 + 41.98}{9.8} \quad \text{OR} \quad t = \frac{12 - 41.98}{9.8}$$

$$t = \frac{53.38}{9.8} \quad \text{OR} \quad t = -\frac{29.38}{9.8}$$

$$t = 5.45 \text{ sec.} \quad \text{OR} \quad t = -2.99 \text{ sec.}$$

Since time cannot be negative, therefore, the package takes ***5.45 sec.*** to reach the ground.

Q.No.2: Two tug boats are towing a ship, each exerts a force of 6000 N, and the angle between the two ropes is 60°.

Calculate the resultant force on the ship.

***Data*:**

Magnitude of force F_1= 6000 N = 6×10^6 N

Magnitude of force F_2 = 6000 N = 6×10^6 N

Angle $\theta = 60°$

Magnitude of the resultant force F= ?

***Solution*:**

Let $\vec{F}_1$ and $\vec{F}_2$ be the force exerted by the two ropes making an angle "θ" with each other, then the magnitude of resultant "F", according to ***law of cosine*** is given by:

$$F = \sqrt{F_1^2 + F_2^2 + 2\,F_1 F_2 \cos\theta}$$

$$F = \sqrt{(6\times10^3)^2 + (6\times10^3)^2 + 2\times6\times10^3\times6\times10^3 \cos 60}$$

$$F = \sqrt{36\times10^6 + 36\times10^6 + 72\times10^6 \times 0.5}$$

$$F = \sqrt{108\times10^6} \qquad \boxed{F = 10392\text{ N}}$$

Magnitude of the resultant force acting on the ship is ***10392 N***.

Q.No.3: A car starts from rest and moves with a constant acceleration. During the 5th second of its motion, it covers a distance of 36 m, calculate:

(*i*) The acceleration of the car.

(*ii*) The total distance covered by the car during this time.

***Data*:**

Initial velocity $v_i = 0$

Distance covered during 5th second S=36m

Acceleration a =?

Total distance covered in 5 s, S_5 =?

***Solution*:**

(*i*) Let the total distance covered by the car in 5 seconds be S_5, given by:

$$S_5 = v_i\, t + \tfrac{1}{2}\, a\, t^2 = 0 + \tfrac{1}{2}\, a\, (5)^2$$

$S_5 = 12.5\,a$ …………… (*i*)

Distance S_4 covered in 4 sec. is given by:

$$S_4 = v_i\, t + \tfrac{1}{2}\, a\, t^2 = 0 + \tfrac{1}{2}\, a\, (4)^2$$

$S_4 = 8\,a$ ………………. (*ii*)

Distance covered during 5Th second will be:

$$S_5 - S_4 = S$$

$$12.5\,a - 8\,a = 36$$

$4.5\,a = 36 \qquad a = \dfrac{36}{4.5}$

$\boxed{a = 8\ m/s^2}$

Acceleration of the car is ***8 m/s²***.

(*ii*) Total distance covered by the car in 5 sec. is given by:

$S_5 = v_i\,t + \frac{1}{2}\,a\,t^2$

$S_5 = 0 + \frac{1}{2} \times 8 \times (5)^2 \qquad \boxed{S_5 = 100\ m}$

Total distance covered by the car is ***100m***

***Q.No.4*: Show that range of a projectile at complimentary angles is same.**

Ans: Range of a projectile is given by:

$$R = \frac{V_o^2 \sin 2\theta}{g}$$

From the formula for range of a projectile we can see that for a given value of "V_o" range of a projectile depends upon the value of Sin 2θ. This formula shows that range of a projectile will be <u>equal for two angles of projection</u> which are equal amount lesser or greater than 45°, since: (45°+ α) and (45°− α) are any two angles of projection which are equal amount "α" greater or lesser than 45° for which range of the projectile will be equal. These angles are ***complimentary angles*** i.e. sum of these angles is 90°.

For example, range of a given projectile will be equal at 60° and 30° because 30°<45°< 60° (30° is 15° less than 45° and 60° is 15° greater than 45° and 30°+ 60° = 90°).

Similarly range of the projectile will be equal at 15° and 75° angles of projection, because 15° < 45° < 75° (15° is 30° less than 45° and 75° is 30° greater than 45° and 15°+ 75° = 90°).

***Q.No.5*: At what angle the range of a projectile becomes equal to the height of projectile?**

Ans: Range of projectile is $R = \dfrac{V_o^2 \sin 2\theta}{g}$

Maximum height reached $h = \dfrac{V_o^2 \sin^2\theta}{2g}$

$R = h \quad \text{if} \quad \sin 2\theta = \dfrac{\sin^2\theta}{2}$

This happens when θ = 76°. Hence at θ = 76°

$\sin 2\times 76 = \sin 152 = 0.5$

$\dfrac{\sin^2\theta}{2} = \dfrac{\sin^2 76}{2} = \dfrac{(\sin 76)^2}{2} = \dfrac{(0.9703)^2}{2} = \dfrac{0.9415}{2} = 0.5$

***Q.No.5*: A mortar shell is fired at a ground level target 500 meter distance with an initial velocity of 90 m/s. What is its launch angle?**

Data:

Horizontal range	$R_H = 500$ m
Initial velocity	$V_o = 90$ m/s
Launch angle	θ = ?

Solution:

Horizontal range of a projectile is given by:

$R_H = \dfrac{V_o^2 \sin 2\theta}{g}$

$500 = \dfrac{(90)^2 \sin 2\theta}{9.8}$

$\sin 2\theta = \dfrac{500 \times 9.8}{90 \times 90}$

$\sin 2\theta = \dfrac{4900}{8100} = 0.6049$

$2\theta = \sin^{-1} 0.6049$

$2\theta = 37.2° \qquad \boxed{\theta = 18.6°}$

But range of a projectile is same for two launch angles which are equal amount greater or lesser than 45°. Hence the second possible launch angle will be: 90° −18.6° = 71.4°.

$\boxed{\textit{Launch angle} = 71.4°}$

***Q.No.6*: Find the angle between**

$\vec{A} = 2\,\hat{i} + 2\,\hat{j} - \hat{k}$

$\vec{B} = 6\hat{i} - 3\,\hat{j} + 2\,\hat{k}$

Solution:

Angle between vectors $\vec{A}$ and $\vec{B}$ is given by: $\theta = \cos^{-1} \dfrac{\vec{A}\cdot\vec{B}}{AB}$

$$\theta = \cos^{-1}\frac{(2\hat{i} + 2\hat{j} - \hat{k})\cdot(6\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{(2)^2+(2)^2+(-1)^2}\,\sqrt{(6)^2+(-3)^2+(2)^2}}$$

Since $\hat{i}\cdot\hat{j} = 0,\ \ \hat{j}\cdot\hat{k} = 0\ $ and $\ \hat{k}\cdot\hat{i} = 0$

$$\therefore \quad \theta = \cos^{-1}\frac{12\,\hat{i}\cdot\hat{i} - 6\hat{j}\cdot\hat{j} - 2\hat{k}\cdot\hat{k}}{\sqrt{4+4+1}\,\sqrt{36+9+4}}$$

Since $\hat{i}\cdot\hat{i} = 1,\ \ \hat{j}\cdot\hat{j} = 1\ $ and $\ \hat{k}\cdot\hat{k} = 1$

$\therefore \quad \theta = \cos^{-1}\dfrac{12-6-2}{\sqrt{9}\sqrt{49}}$

$\therefore \quad \theta = \cos^{-1}\dfrac{4}{3\times 7}$

$\therefore \quad \theta = \cos^{-1}\dfrac{4}{21} = \cos^{-1} 0.1905$

$\boxed{\theta = 79.01^\circ}$

The angle between vectors is ***79°***.

_Q.No.7_: Find the work done in moving an object along a vector: $\vec{r} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ if applied force is: $\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$

Solution:

Work done is the dot product of applied force $\vec{F}$ and displacement $\vec{r}$. Therefore,

$\text{Work} = \vec{F}\cdot\vec{r}$

$\text{Work} = (2\hat{i} - \hat{j} - \hat{k})\cdot(3\hat{i} + 2\hat{j} - 5\hat{k})$

Since $\hat{i}\cdot\hat{j} = 0,\quad \hat{j}\cdot\hat{k} = 0$ and $\hat{k}\cdot\hat{i} = 0$

$\text{Work} = 6\,\hat{i}\cdot\hat{i} - 2\,\hat{j}\cdot\hat{j} + 5\,\hat{k}\cdot\hat{k}$

But $\hat{i}\cdot\hat{i} = 1,\quad \hat{j}\cdot\hat{j} = 1$ and $\hat{k}\cdot\hat{k} = 1$

$\text{Work} = 6 - 2 + 5 \quad \boxed{\textit{Work} = 9}$

Q.No.8:

If vectors $\vec{A} = a\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{B} = \hat{i} + a\hat{j} + \hat{k}$ are perpendicular to each other, then find the value of "a".

(*2018 Karachi Board*)

Solution:

$\vec{A}\cdot\vec{B} = 0 \qquad (\text{When } \vec{A}\perp\vec{B})$

$(a\hat{i} + \hat{j} - 2\hat{k})\cdot(\hat{i} + a\hat{j} + \hat{k}) = 0$

But $\hat{i}\cdot\hat{j} = 0,\quad \hat{j}\cdot\hat{k} = 0$ and $\hat{k}\cdot\hat{i} = 0$

$\therefore \quad a\,\hat{i}\cdot\hat{i} + a\,\hat{j}\cdot\hat{j} - 2\,\hat{k}\cdot\hat{k} = 0$

$a + a - 2 = 0$

OR $\quad 2a - 2 = 0 \qquad \boxed{a = 1}$

Q.No.9:

If $\quad \vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ $\quad \vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$

Find a unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$.

(*2017,16,14,11,2,1999 Karachi Board*)

Data:

$\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$

$\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$

Unit vector $\perp$ to $\vec{A}$ and $\vec{B}$ $\quad \hat{U} = ?$

Solution:

By definition: $\quad \vec{A}\times\vec{B} = |\vec{A}\times\vec{B}|\,\hat{U}$

$\hat{U}$ is a unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$. But

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$\vec{A}\times\vec{B} = \hat{i}(-6\times-1-3\times-3) + \hat{j}(-3\times4 - 2\times-1)$
$\quad + \hat{k}(2\times3 - 4\times-6)$

$\vec{A}\times\vec{B} = \hat{i}(6+9) + \hat{j}(-12+2) + \hat{k}(6+24)$

$\vec{A}\times\vec{B} = 15\hat{i} - 10\hat{j} + 30\hat{k}$

$|\vec{A}\times\vec{B}| = \sqrt{(15)^2 + (-10)^2 + (30)^2}$

$|\vec{A}\times\vec{B}| = \sqrt{225+100+900} = \sqrt{1225} = 35$

$\therefore \quad \hat{U} = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

$\hat{U} = \dfrac{15\hat{i} - 10\hat{j} + 30\hat{k}}{35} = \dfrac{15}{35}\hat{i} - \dfrac{10}{35}\hat{j} + \dfrac{30}{35}\hat{k}$

Hence a unit vector perpendicular to $\vec{A}$ as well as $\vec{B}$ will be:

$$\boxed{\hat{U} = \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}}$$

_Q.No.10_: If one of the rectangular components of force 50 N is 25 N. Find the value of the other.(*2010 Karachi Board*)

Data:

Let $\quad F = 50$ N. and $F_x = 25$ N

Then $\quad F_y = ?$

Solution:

Since $F_x = F\cos\theta$ OR $25 = 50\cos\theta$

$\cos\theta = \dfrac{25}{50} = 0.5$ OR $\theta = \cos^{-1} 0.5$

$\underline{\boldsymbol{\theta = 60^\circ}}$

The other component is:

$F_y = F \sin\theta = 50 \sin 60°$

$F_y = 50 \times 0.866$ $\boxed{F_y = 43.3\ N}$

***Q.No.11*: Two forces of magnitude 10 N and 15 N are acting at a point. The magnitude of their resultant is 20 N. Find the angle between them (*2004, 1993 Karachi Board*)**

***Data*:**

Magnitude of $F_1 = 10\ N$
Magnitude of $F_2 = 15\ N$
Magnitude of resultant $F = 20N$
Angle between F_1 & F_2 $\theta = ?$

***Solution*:**

According to law of Cosine, the magnitude of the resultant is given by:

$$F^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos\theta$$

Where θ is the angle between F_1 & F_2.

$$(20)^2 = (10)^2 + (15)^2 + 2 \times 10 \times 15 \times \cos\theta$$

$$400 = 100 + 225 + 300 \cos\theta$$

$$400 - 325 = 300 \cos\theta$$

$$\cos\theta = \frac{75}{300} = 0.25 \quad \therefore \quad \theta = \cos^{-1} 0.25$$

$$\boxed{\theta = 75.53°}$$

***Q.No.12*: Two forces of equal magnitude are acting at a point, find the angle between the two forces when the magnitude of resultant is also equal to the magnitude of either of these forces**

***Data*:** (*2003 Karachi Board*)

Force F_1 = Force F_2
Magnitude of their resultant F = magnitude of F_1 or F_2
Angle between F_1 and F_2, $\theta = ?$

***Solution*:**

Magnitude of resultant of any two forces when angle ***between*** them is θ, according to law of Cosine, is given by:

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

Since $F_1 = F_2 = F$, therefore,

$$\therefore \quad F = \sqrt{F^2 + F^2 + 2FF\cos\theta}$$

Squaring both sides we get:

$$F^2 = 2F^2 + 2F^2\cos\theta$$

$$F^2 - 2F^2 = 2F^2\cos\theta$$

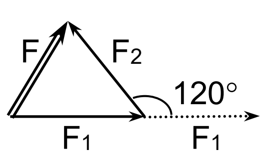
$$\therefore \quad -F^2 = 2F^2\cos\theta$$

$$\cos\theta = \frac{-F^2}{2F^2}$$

$$\cos\theta = -\tfrac{1}{2}$$

$$\theta = \cos^{-1} -\tfrac{1}{2}$$

$$\therefore \quad \boxed{\theta = 120°}$$



$\therefore$ Angle <u>between</u> the two forces is ***<u>120</u>°***.

***Q.No.13*: Two possible angles to hit a target by a mortar shell fired with initial velocity of 98 m/s are 15° and 75°. Calculate the range of projectile and the minimum time required to hit the target? (*2004,1995 Karachi Board*)**

***Data*:**

Initial velocity $V_o = 98$ m/s
Angles for same range $\theta = 15°$&$75°$
Range of projectile R = ?
Minimum time required t = ?

***Solution*:**

Range of a projectile is given by:

$$R = \frac{V_o^2}{g}\sin 2\theta$$

$$R = \frac{(98)^2}{9.8}\sin 2\times 15° \text{ (range at 15°)}$$

$$R = \frac{9604}{9.8}\sin 30°$$

$$\therefore \quad R = \frac{9604 \times 0.5}{9.8} \qquad \boxed{R = 490\ m}$$

Range of the projectile is ***<u>490 m</u>*** for both the angles. The projectile will take minimum time to hit the target when it is fired at **<u>smaller</u>** of the two possible angles, hence

$$T = \frac{2V_o \sin\theta}{g}$$

$$T = \frac{2 \times 98 \times \sin 15°}{9.8} \qquad \boxed{T = 5.176\ sec}$$

The projectile takes <u>minimum time</u> of ***<u>5.176 sec</u>***. at 15°to hit the target.

***Q.No.14*:**

$\vec{A} = 3\hat{i} + \hat{j} - 2\hat{k}$ & $\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$

Find the projection of A on B.

(*2013 Karachi Board*)

***Solution*:**

By definition:

$$\vec{B} \cdot \vec{A} = B \text{ (projection of } \vec{A} \text{ on } \vec{B})$$

Hence:

$$\text{Projection of } \vec{A} \text{ on } \vec{B} = \frac{\vec{B} \cdot \vec{A}}{B}$$

$$\vec{B} \cdot \vec{A} = (3\hat{i} + \hat{j} - 2\hat{k}) \cdot (-\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{B} \cdot \vec{A} = -3\,\hat{i}\cdot\hat{i} + 3\,\hat{j}\cdot\hat{j} - 8\,\hat{k}\cdot\hat{k}$$

$$\vec{B} \cdot \vec{A} = -3 + 3 - 8$$

$\vec{B} \cdot \vec{A} = -8$

Similarly: $B = \sqrt{(-1)^2 + (3)^2 + (4)^2}$

$B = \sqrt{1 + 9 + 16} = \sqrt{26}$

$\therefore$ $\boxed{\textbf{Projection of } \vec{A} \textbf{ onto } \vec{B} = \dfrac{-8}{\sqrt{26}}}$

Q.No.15: **A body starts from rest and moves with constant acceleration 10 m/sec², how much distance will it travel in the 4th sec of its motion.** (***2019 Karachi Board***)

Data:

Initial velocity $v_i = 0$

Acceleration $a = 10\ m/s^2$

Distance covered during 4th second $S = ?$

Solution:

Distance traveled in 4 seconds S_4:

$S_4 = v_i t + \frac{1}{2} a t^2$

$S_4 = 0 + \frac{1}{2}\, 10\, (4)^2$

$S_4 = 80\ m$ .... (*i*)

Distance traveled in 3 sec. S_3:

$S_3 = v_i t + \frac{1}{2} a t^2$

$S_3 = 0 + \frac{1}{2}\, 10\, (3)^2$

$S_3 = 45\ m$ .... (*ii*)

Distance traveled during 4th second will be:

$S = S_4 - S_3 = 80 - 45 = \mathbf{35\ m}$

Distance traveled during 4th sec is ***35 m.***

Q.No.16: **Two forces $\vec{F}_1 = 3\hat{i} - 2\hat{j} + 5\hat{k}$ and $\vec{F}_2 = \hat{i} + 6\hat{j} + 2\hat{k}$ act on a body which is displaced along $\vec{r} = 4\hat{i} - \hat{j} + 3\hat{k}$. Calculate work done on the body.** (***2022 Karachi Board***)

Solution:

Net force acting on the body $\vec{F} = \vec{F}_1 + \vec{F}_2$

$\vec{F} = (3\hat{i} - 2\hat{j} + 5\hat{k}) + (\hat{i} + 6\hat{j} + 2\hat{k})$

$\vec{F} = 4\hat{i} + 4\hat{j} + 7\hat{k}$

$\text{Work} = \vec{F} \cdot \vec{r} = (4\hat{i} + 4\hat{j} + 7\hat{k}) \cdot (4\hat{i} - \hat{j} + 3\hat{k})$

$\text{Work} = 16\, \hat{i} \cdot \hat{i} - 4\, \hat{j} \cdot \hat{j} + 21\, \hat{k} \cdot \hat{k}$

$\text{Work} = 16 - 4 + 21$ $\boxed{\textbf{Work = 33 Joules.}}$

Q.No.17: **At what angle the horizontal range of projectile becomes equal to its vertical range? Prove mathematically.** (***2025 Karachi Board***)

Data:

Angle $\theta = ?$

Horizontal range R_H = Vertical range = Maximum height "h" reached

Solution:

Range of projectile is $R_H = \dfrac{V_o^2 \sin 2\theta}{g}$

Maximum height reached or vertical range:

$h = \dfrac{V_o^2 \sin^2 \theta}{2g}$

But $R_H = h$

$\dfrac{\cancel{V_o^2} \sin 2\theta}{\cancel{g}} = \dfrac{\cancel{V_o^2} \sin^2 \theta}{2\cancel{g}}$

OR $\sin 2\theta = \dfrac{\sin^2 \theta}{2}$

This happens when $\theta = 76°$. Hence at $\theta = 76°$

$\sin 2\times 76 = \sin 152 = 0.5$

$\dfrac{\sin^2 \theta}{2} = \dfrac{\sin^2 76}{2} = \dfrac{(\sin 76)^2}{2} = \dfrac{(0.9703)^2}{2} = \dfrac{0.9415}{2} = 0.5$

At an angle of **<u>76°</u>** horizontal range of projectile becomes equal to its vertical range.