According to new syllabus of all intermediate boards of Sindh.

Prof. Rawala's ....

PHYSICS

HELPING BOOK

For
Class XI
(All science students)

UNIT -11

Oscillations.

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Thoroughly revised and updated 2026 – 2027

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Unit.11**.....**

Oscillations

***Vibratory motion*:**

Vibratory or **oscillatory** motion is that type of motion which repeats itself after a definite interval of time. The vibrating body moves to and fro about a fixed point and completes each vibration in equal interval.

***Simple harmonic motion*:**

Q: What are the characteristics (or conditions) of simple harmonic motion?

***Ans*:** A body executing simple harmonic motion shows following characteristics:

1. Its motion is **vibratory** or **oscillatory**.
2. Some **restoring force acts on the vibrating system**.
3. **Acceleration** of the body executing simple harmonic motion SHM is **directly proportional to its displacement** "x" and **always directed towards its mean position** i.e.

$$\boxed{a \propto -x}$$

The negative sign shows that acceleration "a" of the body execution SHM is always directed towards its mean position.

(Or we can say that direction of acceleration "**a**" is always opposite to the direction of displacement "***x***", displacement is measured from the mean position on either side. When the body is moving **away** from the mean position "***x***" is **increasing** or displacement "***x***" will be **positive** but velocity of the body will **decrease**, **in other words**, its acceleration "**a**" will be **negative** i.e. negative "**a**" is against the direction of motion, in this case it is towards the mean position. These observations show that acceleration "**a**" will always be directed towards the mean position, whether the body moves towards or away from the mean position).

4. Energy of the system oscillates between kinetic energy and potential energy but the total energy remains constant, provided there is no energy loss due to external or internal frictional forces (such as those due to roughness of surfaces in contact, cohesive or adhesive force between the surfaces, shape of bodies, air resistance etc.).

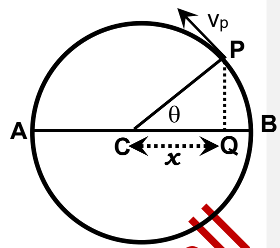
***S.H.M and circular motion*:**

Q.No.1: A point (or a particle) "P" is moving with a constant angular speed along the circumference of a circle of radius "x_o".

Derive an expression for instantaneous displacement "x" of its projection Q, vibrating along the diameter.

Ans. Consider a point "P" moving along a circular path of radius "x_o" with a constant angular speed "ω". It's projection along the diameter AB of the circular path is point "Q". The circular path may be considered as a part of our **reference circle**. As point "P" moves along the circular path its projection Q moves to and fro along the diameter of circle about point C. Hence motion of point P is **circular** or **angular** and that of Q is **vibratory**.

Point C is the center of the circle for P but since Q vibrates about this point, therefore, it is the **mean position** for Q.



Instantaneous displacement "x".

Instantaneous displacement of Q is its distance from the mean position C at that instant on either side.

Let x be the instantaneous displacement. Hence in triangle CPQ:

$$\cos\theta = \frac{CQ}{CP}$$

$$\cos\theta = \frac{x}{x_o} \qquad \boxed{x = x_o \cos\theta}$$

"x_o" is the maximum displacement of Q on either side of mean position, it is called amplitude of vibration.

This formula shows that when $x = \pm x_o$ (equal to radius of the circle) when the angle is 0° or 180° i.e. when Q is at B or at A. Under this condition its displacement is ***maximum*** and is equal to x_o.

- Hence the **amplitude** of the vibrating point Q is x_o (equal to the radius of the circle).

***Q.No.2*: For a body, executing S.H.M, at which point is its displacement:**
(1) *Minimum* **(2) *Maximum***

***Ans*:** The displacement of a vibrating body, on either side, at any instant, is measured from its mean or equilibrium position, hence it is:

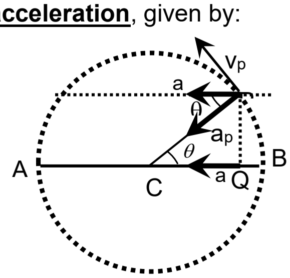
(1) Minimum at its mean position (= 0).

(2) Maximum at its extreme positions, at extreme positions its displacement is called ***amplitude of vibration***.

***Acceleration of Q at any instant*:**

Acceleration of Q at any instant is the component of acceleration of P at that instant parallel to diameter of the circle. But the acceleration of P is always directed towards the center and is known as **centripetal acceleration**, given by:

$$a_p = \frac{v_p^2}{x_o}$$



Where v_p is the magnitude of linear velocity of P (also called its tangential velocity, because it is along the tangent to the circle at a given instant). But $v_p = x_o\,\omega$.

$$\therefore \quad a_p = \frac{x_o^2\,\omega^2}{x_o} = x_o\,\omega^2$$

Resolving a_p into components we get:

Component of a_p parallel to the diameter $= a_p \cos\theta$

Component of a_p perpendicular to the diameter $= a_p \sin\theta$

But acceleration of Q = component of a_p parallel to diameter.

$$\therefore \quad a = -\,a_p \cos\theta$$

Negative sign shows that the acceleration of Q is always directed towards the mean position.

$\therefore \quad a = -x_o \omega^2 \cos\theta$ (since $a_p = x_o \omega^2$)

But $x_o \cos\theta = x$

$\therefore \quad \boxed{a = -\omega^2 x}$

Since ω represents the constant angular speed of P, therefore, ω^2 is constant, hence:

$\therefore \quad \boxed{a \propto -x}$

It shows that at any instant the acceleration of Q is directly proportional to displacement and always directed towards it's mean position, which proves that the motion of the projection point Q is simple harmonic *(*SHM*)*.

From the above formula we can see that the acceleration of Q is maximum when its displacement is maximum. Displacement of Q is maximum when it is at one of its extreme positions. Hence acceleration of Q is maximum at its extreme positions i.e. when it is at A or at B. At these points it's velocity changes rapidly. Maximum acceleration of Q is given by:

$$a_{max.} = -x_o \omega^2$$

Q.No.3*: *If body executes S.H.M, is its acceleration uniform?

Ans: Acceleration of a body executing S.H.M is ***not uniform***, because the magnitude of its acceleration continuously changes and at any instant depends upon its displacement 'x' . Acceleration of the body is maximum at extreme positions and zero at the mean position. During the vibratory motion of the body, at every point its acceleration is directed towards the mean position.

Q.No.4*: *For a body, executing S.H.M, at which point is its acceleration:
(1) Minimum (2) Maximum

Ans: The acceleration of a body, executing S.H.M is given by:

$$a \propto -x$$

Hence acceleration of the vibrating body will be **maximum** when "x" is maximum. The displacement "x" of the body will be maximum at extreme positions, although it's velocity will be minimum but it will have ***maximum acceleration*** (Its velocity will be changing rapidly). Hence:

(1) Minimum acceleration at mean position (= 0). Here its displacement is zero.
(2) Maximum acceleration at its extreme positions. Here its displacement is maximum.

Note that when a body executing S.H.M moves towards it's mean position it's velocity increases, ***it means*** that its acceleration is positive. We know that the direction of a positive acceleration is same as the direction of motion of a body. Since in this case the body is moving towards the mean position, therefore, the direction of acceleration will also be towards the mean position.

Similarly, when the body is moving away from the mean position, it's velocity decreases. ***In other words,*** now it has negative acceleration (called retardation or deceleration). Direction of negative acceleration is opposite to the direction of motion. Since in this case the body is moving away from the mean position, therefore, the direction of acceleration will be towards the mean position.

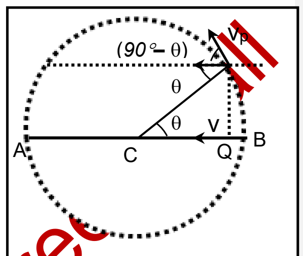
- We have seen that acceleration of a body executing S.H.M is always directed towards it's mean position.

<u>Q.No.5</u>: A point (or a particle) "P" is moving with a uniform speed along the circumference of a circle of radius "x_o".

Derive expressions for instantaneous and maximum speeds ('v and $v_{max.}$') of its projection Q vibrating along the diameter.

<u>Ans.</u> <u>Instantaneous speed of Q</u>:

Point P has two types of velocities, **<u>angular or circular velocity ω</u>** and **<u>linear velocity</u>**. Linear velocity of P is **<u>along the tangent</u>** to the circle at a given point and its magnitude is v_p. At any instant velocity of Q is the component of linear velocity of P parallel to the diameter.



Magnitude of component of v_p parallel to diameter

$$v = v_p \cos(90° - \theta)$$

But speed of Q = component of v_p parallel to diameter.

∴ speed of Q = $v = v_p \cos(90° - \theta)$

But $\cos(90° - \theta) = \sin\theta$

∴ $v = v_p \sin\theta$

Or $v = x_o\,\omega \sin\theta$ (Since $v_p = x_o\,\omega$)

But $\sin^2\theta + \cos^2\theta = 1$

∴ $\sin\theta = \sqrt{1 - \cos^2\theta}$

and $\cos\theta = \dfrac{x}{x_o}$

$$\sin\theta = \sqrt{1 - \frac{x^2}{x_o^2}}$$

$$\therefore \quad \boxed{v = x_o\,\omega\sqrt{1 - \frac{x^2}{x_o^2}}}$$

From trigonometry:
$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
Where α and β are any two angles, then
If $\alpha = 90°$ & $\beta = \theta$ Then
$\cos(90° - \theta) = \cos 90°\cos\theta + \sin 90°\sin\theta$
But $\cos 90° = 0$ & $\sin 90° = 1$
∴ $\cos(90° - \theta) = \sin\theta$

Or $v = x_o\,\omega\sqrt{\dfrac{x_o^2 - x^2}{x_o^2}}$ Or $v = \cancel{x_o}\,\omega\,\dfrac{\sqrt{x_o^2 - x^2}}{\cancel{x_o}}$

Hence the instantaneous speed of Q is given by:

or $\boxed{v = \omega\sqrt{x_o^2 - x^2}}$

<u>Maximum speed of Q</u>:

Speed of Q is <u>maximum</u> when it is passing through its <u>mean position</u> where it's displacement $x = 0$. Hence it's maximum speed $v_{max.}$ is given by:

$$v_{max.} = \omega\sqrt{x_o - 0}$$

$$\boxed{v_{max.} = x_o\,\omega}$$

$x_o\,\omega$ is the speed of P, hence maximum speed $v_{max.}$ of Q is equal to the speed v_p of P.

Minimum speed of Q:

Speed of Q is minimum (= 0) when it is at one of its extreme positions where $x = \pm x_o$.
Hence:

$$v = \omega\sqrt{x_o^2 - x^2} \qquad (\text{but } x = \pm x_o)$$

$$\therefore \quad v = \omega\sqrt{x_o^2 - (\pm x_o)^2}$$

$$\therefore \quad v = \omega\sqrt{x_o^2 - x_o^2} \qquad \therefore \quad v = 0$$

Relation between v and $v_{max.}$:

Instantaneous speed of Q is given by:

$$v = x_o\,\omega\sqrt{1 - \frac{x^2}{x_o^2}}$$

But $v_{max.} = x_o\,\omega$

Hence the instantaneous and maximum speeds of Q are related by:

$$\boxed{v = v_{max.}\sqrt{1 - \frac{x^2}{x_o^2}}}$$

Time period:

***Q.No.6**: A point (or a particle) "P" is moving with a uniform speed along the circumference of a circle of radius "x_o".*

Derive an expression for time period "T" of its projection Q, vibrating along the diameter.

Ans. Time taken by a vibrating body to complete one vibration is called it's time period T.

Angular speed "ω" of point P at the center of its circular path is given by:

$$\omega = \frac{\theta}{t}$$

Where "θ" is the angular displacement of point P in during time t.

Time in which P completes one revolution Q will complete one vibration.

But the angular displacement of P at the center of its circular path will be $\theta = 2\pi$ radian at the end of one revolution and t = T during this time vibrating point Q would have completed on vibration. Hence T is also the time period for Q:

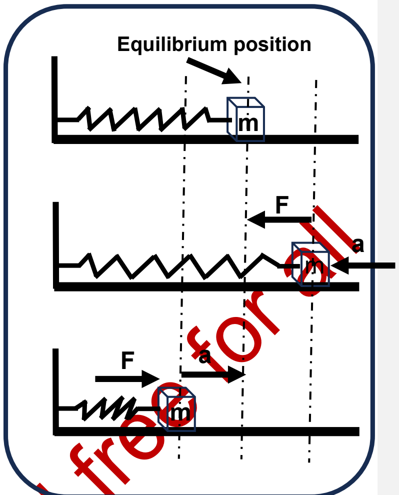
$$\omega = \frac{2\pi}{T} \qquad \text{Hence} \qquad \boxed{T = \frac{2\pi}{\omega}}$$

Horizontal motion under elastic restoring force:

***Q.No.7**: What is motion under elastic restoring force?*

Ans: When a body placed on a frictionless horizontal surface connected at the end of an elastic and light spring, whose other end is fixed to a rigid support, is pulled to one side and released then the body vibrates about its position of equilibrium.

Note that in this case the spring is **parallel** to the horizontal surface. **Weight** of the body is balanced by **normal reaction** of the surface. If **friction** between the vibrating body and the horizontal surface on which it vibrates is assumed to be **negligible** then the only **unbalanced force** acting on the body is the force exerted by the spring due to its elasticity. This force tends to restore the original conditions of the body i.e. it tends to bring the body back to its original position. It is called ***elastic restoring force***.



Vibratory motion of the body is **simple harmonic**, because the acceleration produced in the body, due to the elastic restoring force of the spring, is directly proportional to its displacement and is always directed towards its equilibrium (or mean) position

This type of vibratory motion of the body under position of equilibrium under the influence of elastic restoring force of the spring is called ***motion under elastic restoring force***.

***Q.No.2*: What is elastic restoring force?**

Ans*:** It is a force due to elasticity of the spring that tends to restore the original conditions, i.e. it tends bring the vibrating body back to its original position, hence it is called ***elastic restoring force.

***Q.No.3*: What is elastic constant or force constant of a spring?**

***Ans*:** It is the force required to produce unit elongation in the spring.

Spring constant $k = F/x$.

Its value depends upon the elastic properties of the spring. Its value is high for hard springs, as they require large force to elongate and vice versa.

***Q.No.4*: What is the unit of spring constant?**

***Ans*:** Since spring constant is F/x, therefore, its S.I unit is N/m or N m^{-1}.

***Q.No.5*: A body on a smooth horizontal surface performs vibratory motion under the influence of elastic restoring force.**

Prove that its motion is simple harmonic.

***Ans*:** Consider a body of mass 'm' attached to the end of a light and elastic spring, the other end of the spring is attached to a wall. To displace the body from its equilibrium position through some distance 'x' force required, according to Hook's law is given by:

$$F_{App.} \propto x$$
$$F_{App.} = k\,x$$

Where 'k' is the constant of proportionality and represents the ***spring constant*** (It is the force required to produce unit elongation in the spring, its value depends upon the ***elastic properties*** of the spring). As the body is pulled by applying the above force due to elasticity the spring will also exert an equal and opposite force. Force exerted by the spring tends to restore the original conditions (i.e. it tends to bring the body back to its original position). It is called ***elastic restoring force***, given by:

$$F_{elast.} = -\,k\,x$$

If now the applied force is removed, the elastic restoring force becomes unbalanced, body under the action of unbalanced force vibrates about its mean position, with some acceleration.
If 'a' is the acceleration produced in the vibrating body, then according second law of motion:

$$F_{elast.} = m\,a$$

Therefore,

$$m\,a = -k\,x$$

$$a = -\frac{k}{m}\,x$$

'k/m' is constant for a given spring and a body of given mass, hence:

$$a \propto -x$$

This relation shows that acceleration 'a' of the body vibrating under the influence of elastic restoring force is <u>directly proportional to the displacement and is always directed towards its mean position</u>. Hence motion of the vibrating body is simple harmonic.

Note that weight of the body is balanced by upward reaction of the surface on which it is kept.

***<u>Time period</u>*:**

<u>Q.No.1</u>: Derive a formula for time period of a body executing S.H.M under elastic restoring force?

***<u>Ans</u>*:** Time period of a vibrating body is the time in which it completes one vibration.
Acceleration of a projection point along the diameter of a reference circle is given by:

$$a = -\omega^2 x$$

But acceleration of a body executing simple harmonic motion under the influence of elastic restoring force is given by:

$$a = -\frac{k}{m}\,x$$

Comparing the two relations we get:

$$\omega^2 = \frac{k}{m} \quad \text{OR} \quad \omega = \sqrt{\frac{k}{m}}$$

Time period of a projection point executing S.H.M along the diameter of a reference circle is given by:

$$T = \frac{2\pi}{\omega}$$

On substituting $\omega = \sqrt{k/m}$ we get:

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Where "m" is mass of the body and "k" is the elastic constant of material of the spring.

<u>Q.No.2</u>: On what factors does the time period of a body executing S.H.M under elastic restoring force of a given spring depend?

<u>Ans</u>*:** Time period (time taken to complete one vibration) depends upon mass "m" of the body and upon the spring constant "k" of the spring. If mass of the body is <u>increased</u> its time period also <u>increases</u>, ***it means that on increasing mass (taking a heavier body) it slows down and takes a longer time to complete a vibration. But for large value of "k" the value of "T" is small.

Time period of a body executing S.H.M under the influence of elastic restoring force for a given spring is <u>directly proportional</u> to the <u>square root of its mass</u>.

***Energy*:**

***Q.No.1*: What happens to energy of a body executing S.H.M under the influence of elastic restoring force?**

Ans*:** Energy of the vibrating system continuously changes its form from K.E to P.E and vice versa. ***In other words, at any instant the total energy possessed by the vibrating system will be partially in the form of K.E and partially in the form of P.E and the total energy will be equal to the sum of K.E and P.E at that instant. The total energy remains constant if there are no energy losses due to frictional forces.

Potential energy of the system vibrating under elastic restoring force at any instant is given by: $P.E = \frac{1}{2} k x^2$

Similarly, its K.E at any instant is given by:

$$K.E = \frac{1}{2} k (x_o^2 - x^2)$$

Hence the total energy of the system executing S.H.M under the influence of elastic restoring force at any instant will be: $E = P.E + K.E$

$$E = \frac{1}{2} k x^2 + \frac{1}{2} k (x_o^2 - x^2)$$

$$\boxed{E = \frac{1}{2} k x_o^2}$$

'k' is the spring constant, 'x' is displacement of the vibrating body at any instant and 'x_o' is the **amplitude of vibration** (maximum displacement).

Hence energy of a system executing S.H.M under elastic restoring force for a given spring depends upon the amplitude of vibrations 'x_o'.

***Q.No.2*: Show that energy of a body executing simple harmonic motion under the influence of elastic restoring force is totally in the form of K.E at its mean position.**

***Ans*:** At any position potential energy of a body executing simple harmonic motion under elastic restoring force is given by: $P.E = \frac{1}{2} k x^2$

But at the mean position displacement of the body is zero i.e. $x = 0$

Therefore, at the mean position: $P.E = 0$ ………………… (i)

But at any position kinetic energy of a body executing simple harmonic motion under elastic restoring force is given by:

$$K.E = \frac{1}{2} k (x_o^2 - x^2)$$

Therefore, at the mean position ($x = 0$) K.E is given by:

$$\therefore \quad K.E = \frac{1}{2} k (x_o^2 - 0^2)$$

$$\therefore \quad K.E = \frac{1}{2} k x_o^2$$

But '$\frac{1}{2} k x_o^2$' is the total energy of the vibrating body. Therefore,

$$K.E = \text{Total energy} \ldots\ldots\ldots\ldots\ldots\ (ii)$$

From equations (i) and (ii) we conclude that the energy of the body executing S.H.M is totally in the form of K.E it does not have any P.E at the mean position.

Q.No.3*: *Show that energy of a body executing simple harmonic motion under the influence of elastic restoring force is totally in the form of P.E at extreme positions.

***Ans*:** At any position kinetic energy of a body executing simple harmonic motion under elastic restoring force is given by: $K.E = \frac{1}{2} k (x_o^2 - x^2)$

But at extreme positions ($x = \pm x_o$) K.E is given by:

$\therefore$ $K.E = \frac{1}{2} k \{x_o^2 - (\pm x_o)^2\}$

$\therefore$ $K.E = \frac{1}{2} k x_o^2 - \frac{1}{2} k x_o^2$

$\therefore$ $K.E = 0$ $(x = x_o)$

Potential energy of a body executing simple harmonic motion under elastic restoring force is given by: $P.E = \frac{1}{2} k x^2$

At extreme positions ($x = \pm x_o$)

Hence, $P.E = \frac{1}{2} K x_o^2$ $(x = x_o)$

But $\frac{1}{2} K x_o^2$ is the total energy, hence P.E of the vibrating system is maximum and is equal to the total energy of the body. Hence energy is totally in the form potential energy (P/E) at the extreme position whereas it's K.E is zero.

***Interconversion of K.E and P.E*:**

Energy of a body executing simple harmonic motion continuously changes its form from P.E to K.E and vice versa. Energy of a vibrating body at its mean position is totally kinetic (here its velocity is **maximum**) but at extreme position its energy is totally potential (here its velocity is **zero** for a fraction of a second) but displacement of the body from its mean position will be maximum (potential energy of the vibration body depends upon its displacement from the mean position). At any other position its energy is partially kinetic and partially potential (velocity is between **zero** and **maximum**)).

It the vibrating system is free of frictional forces then at any instant its total energy will be equal to the sum of its K.E and P.E at that instant such that its total energy remains constant.

Note that if the system is frictionless then it's energy will **remain constant** throughout its vibratory motion, although it constantly changes its form from K.E to P.E and vice versa such that it's total energy always **remains constant**.

If there are frictional forces present then a part of energy of the system will be used to do work against friction, as a result of which energy of the system will **decrease** in each vibration, due to which amplitude of vibration will decrease in each vibration. Ultimately the system will come to rest when its energy is totally used up in doing work against friction. This type of oscillations are called ***damped oscillations***.

***Motion under elastic restoring force*: (*Vertically*)**

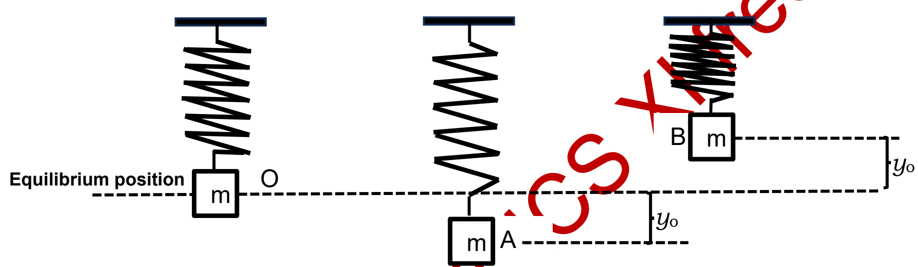
Consider a light body of mass "m" suspended at the end of a light elastic spring whose other end is attached to a rigid support. The suspended body comes to rest at its equilibrium position at point "O" after stretching the spring a little due to its weight.

If the body is now pulled down vertically through distance "y_o" to a point "A" then as soon as we start pulling the body downward, due to elasticity spring starts exerting reaction force upward. Force exerted by the spring is called **elastic restoring force**. This force tends to bring the body back to its equilibrium position i.e. it tends to restore original conditions. If the body is now released, it will move upward with some acceleration under the action of this force, its velocity **increases** as it moves up until it crosses its equilibrium position where its velocity becomes **maximum**. The body will continue moving upward but as soon as it crosses the equilibrium position, the spring will start compressing, hence the spring will now exert elastic restoring force against the direction of motion of the body, due to which velocity of the body starts **decreasing**, after covering same distance "y_o" above the equilibrium position body comes to rest at point "B". Compressed spring now pushes the body which moves back with increasing velocity. This process repeats itself again and again several times and we will see the body vibrating up and down about its position of equilibrium, between points "A" and "B". Distance "y_o" from point A to O or from O to B is **amplitude of vibration**. Point "O" is the **mean position** for the vibrating body. Motion of the body is **vibratory or oscillatory**.

It can be shown that acceleration "a" of the vibrating body at any instant is given by:

$$a \propto -y$$

Where "y" is the displacement of the body at any instant. From this equation we conclude that the acceleration "a" of the body at any instant is directly proportional to its vertical displacement "y" and is always directed towards its mean position, which shows that motion of the vibrating body is **simple harmonic motion (SHM)**.



Time period:

***Q.* Give the formula for time period of a body executing S.H.M in vertical direction under elastic restoring force?**

***Ans*:** Time period of a vibrating body is the time in which it completes one vibration.

Time period of a body executing S.H.M in vertical direction is given by:

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Where "m" is mass of the suspended body and "k" is the spring constant.

Q.No.1*: *On what factors does the time period of a body executing S.H.M In vertical under elastic restoring force depend?

Ans*:** Time period depends upon mass "m" of the body and upon the spring constant "k" of the spring. If mass of the body is **increased** its time period also **increases**, ***it means that on increasing mass (taking a heavier body) it slows down and takes a longer time to complete a vibration, actually time period "T" is directly proportional to $\sqrt{m}$. But for large value of "k" the value of "T" is small. Time period T is inversely proportional to $\sqrt{k}$.

***Simple pendulum*:**

***Q.No.1*: What is an ideal simple pendulum?**

***Ans*:** An ideal simple pendulum consists of a point mass suspended with the help of a weightless, inextensible and flexible string whose other end is attached to a rigid and non-yielding support.

🗣 ***It means that*** length of the string with which the point mass is suspended must not change as the pendulum vibrates, similarly the support must be rigid so that it should not vibrate along with the pendulum. In the laboratory the center of gravity of a small spherical metallic bob can be used to replace the heavy point mass.

***Q.No.2*: What is the type of motion executed by a simple pendulum?**

Ans*:** If bob of the pendulum is displaced from its equilibrium position and allowed to move freely then it performs to and fro motion. This vibratory motion of a simple pendulum for small ngular displacement is simple harmonic or ***in other words, motion of a simple pendulum is S.H.M when it's amplitude of vibrations is small.

***Q.No.3*: Prove that motion of a simple pendulum for small amplitude is simple harmonic S.H.M?**

***Ans*.** In order to prove that motion of simple pendulum is S.H.M we will find its acceleration. To find it's acceleration first of all we will have to find the unbalanced force acting on it.

At any displaced position, forces acting on the bob are:

(1) It's weight "W" acting vertically downward.

(2) Tension "T" in string acting along the string.

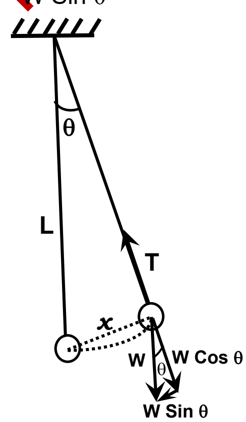
On resolving "W" into components parallel and perpendicular to the string we get:

Component of "W" parallel to the string $= W \cos\theta$

Component of "W" perpendicular to the string $= W \sin\theta$

Since the bob does not move parallel to the string, therefore, tension "T" in the string is balanced by "W Cos θ". Hence the unbalanced force acting on the bob is "W Sin θ", this force is always **directed towards the mean position** (unbalanced force always produces acceleration in its own direction). Under the action of this force the pendulum performs to and fro motion (This force is also the restoring force, as it tends to bring the bob back to its original mean position).

$\therefore$ Unbalanced force $= -W \sin\theta$



The negative sign shows that the unbalanced force is always directed towards the mean position. According to second law of motion:

Unbalanced force = m a

$\therefore \quad m\,a = -W \sin\theta$

$m\,a = -m\,g \sin\theta$

$\therefore \quad a = -g \sin\theta$

For small values of "θ" measured in radians, or ***in other words***, for small amplitude of vibration of the pendulum

$\sin\theta = \theta$

But $\quad \theta = \dfrac{x}{L}$

Where "x" is the instantaneous displacement of the bob from its mean position and "L" is length of the pendulum (measured from the point of suspension to center of the bob).

$\therefore \quad a = -g\,\dfrac{x}{L}$

For a given pendulum, at a given location g/L is constant,

$\therefore \quad a \propto -x$

The above relation shows that the acceleration of a simple pendulum at any instant is directly proportional to its displacement at that instant and is always directed towards it's mean position, hence motion of a simple pendulum for small amplitude is ***simple harmonic***.

***Q.No.4*: When is the tension in a string of a pendulum *maximum*?**

Ans: Tension in the string of a simple pendulum is maximum when the vibrating pendulum passes through its equilibrium or mean position. At this point the tension in the string will be equal to weight of bob of the pendulum.

The magnitude of tension in the string of a pendulum periodically changes as the pendulum vibrates and "θ" changes according to the following relation:

$T = W \cos\theta$

When the angular displacement of the bob "θ" is zero, that happens when the pendulum is passing through its mean position, the tension in the string will be maximum and is equal to weight of the suspended bob T = W (Cos 0°= 1).

***Q.No.5*: Derive a formula for *time period of a simple pendulum*, and show that it is *independent of mass and material* of its bob.**

Ans: Motion of a simple pendulum is S.H.M when its amplitude is small, under this condition its acceleration is given by:

$$a = -g\,\frac{x}{L}$$

But the equation for acceleration of a point executing S.H.M along the diameter of a reference circle is given by:

$$a = -\omega^2 x$$

Comparing the above two equations, we get:

$$\omega^2 = \frac{g}{L}$$

$$\therefore \quad \omega = \sqrt{\frac{g}{L}}$$

Time period of a body executing S.H.M is the time in which it completes one vibration. Time period of a point executing S.H.M along the diameter of a reference circle can be used to calculate time period of a pendulum by replacing "ω" by $\sqrt{g/L}$ in the following equation;

$$T = \frac{2\pi}{\omega}$$

$$\therefore \quad T = \frac{2\pi}{\sqrt{g/L}}$$

$$\therefore \quad \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

At a given location time period of a simple pendulum is directly proportional to $\sqrt{L}$, hence if length of the pendulum is increased its time period also increases according to the above relation. From the above formula we can also see that the time period of a simple pendulum is independent of ***mass*** and ***material*** of its bob.

Note that the motion of a simple pendulum is S.H.M as long as "θ" is **small** or the amplitude of its vibrations is **small**.

Also $\omega = \sqrt{g/L}$ for a simple pendulum executing S.H.M (or for a mass suspended with an inelastic string executing S.H.M).

$\omega = \sqrt{K/m}$ for a body attached at the end of an elastic spring executing S.H.M.

Second's pendulum:

Q.No.6: What is second's pendulum?

Ans: A pendulum whose time period is ***"2 seconds"*** is called second's pendulum. (This pendulum will take 1 sec. to move from one extreme to the other).

Q.No.7: What is the frequency of second's pendulum?

Ans: Frequency of a vibrating body is the number of vibrations completed in one second. Relation between time period "T" and frequency "f" is:

$$f = \frac{1}{T}$$

Since time period of a second's pendulum is 2 seconds hence its frequency is 0.5 hertz.

$$\therefore \quad f = \frac{1}{T} = \frac{1}{2} = 0.5 \text{ hertz}$$

and its length "L" where "$g = 9.8\ m/s^2$" is 0.99 m.

Q.No.8: What will be difference in the length of a second's pendulum on the surface of the earth and that on the surface of moon?

Ans: Time period 'T' of a simple pendulum executing S.H.M on the surface of any planet is given by:

$$\therefore \quad \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

Where 'g' is the acceleration due to gravity on the surface of a planet. For second's pendulum T = 2 second on any planet.

The value of "g" on the surface of moon is less (about $1.63\ m/s^2$) than the average value of "g" on the surface of the earth (about $9.8\ m/s^2$), The length of second's pendulum on earth is 0.99 m and on moon it is about 0.166 m.

In other words, length of second's pendulum will be lesser on moon than on the earth.

Q.No.9*: *What will be the length of a second's pendulum on the surface of Jupiter where the value of 'g' is about 2.63 times the value of 'g' on the earth (i.e. $g_J = 2.63\ g_e$)?

***Ans*:** Time period 'T' of a simple pendulum executing S.H.M on the surface of any planet is given by:

$$\therefore \quad \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

Where 'g' is the acceleration due to gravity on the surface of a planet. For second's pendulum T = 2 second on any planet.

The value of "g" on the surface of Jupiter is about 2.63 times the value of 'g' on the surface of the earth i.e. $g_J = 2.63\ g_e$.

Hence, $$2 = 2\pi\sqrt{\frac{L}{2.63\ g_e}}$$

Hence, $$2 = 2\pi\sqrt{\frac{L}{2.63 \times 9.8}}$$

Therefore, $$\boxed{L = 2.61\ m}$$

Length of second's pendulum on the surface of Jupiter will be ***2.61 m***.

Q.No.10*: *For simple harmonic motion will the time period change or not, by doubling the mass of the bob attached to:

(*a*) *Elastic spring.* *b*) *Inelastic string.* *Explain.* (***2011 Karachi Board***)

***Ans*:** (a) The time period of a bob attached to an elastic spring executing S.H.M (motion under elastic restoring force) is given by:

$$\therefore \quad \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

This formula shows that the time of the bob attached to an **elastic spring** and executing S.H.M for a given spring ***increases if mass of the bob is increased*** and vice versa.

Hence if mass of a bob is doubled its time period will ***increase by $\sqrt{2}$ times***.

(b) The time period of a bob attached to an inelastic string (a simple pendulum) executing S.H.M is given by:

$$\therefore \quad \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

This formula shows that the time period of the bob attached to an **inelastic string** (a simple pendulum) and executing S.H.M at a given location is ***independent of mass of the bob***. Hence if mass of the bob is ***increased time period of the pendulum will not change***.

Natural time period and natural frequency:

Q.No.1: What is natural time period of a vibrating body?

Ans*: The time period with which a body vibrates in the absence of external forces is known as its ***natural time period.

Hence a body which is capable to vibrate always vibrates with certain fixed time period. This time period is called ***natural time period*** of the body.

Simple pendulum of a given length (constant L) at a given location (constant g) always vibrates with a fixed time period which is known as its natural time period.

Q.No.2: What is natural frequency of a body?

***Ans*:** Number of vibrations completed by a vibrating body in one second is known as its frequency. (For waves it is the number of waves passing through a point in one second).

Since frequency is equal to the reciprocal of its time period, hence a body which can vibrate will always vibrate with a fixed frequency and a fixed time period. ***In other words,*** it will always complete the same number of vibrations in one second. This frequency is called ***natural frequency*** of the body.

Resonance:

Q.No.3: What is resonance?

Ans*:** When natural frequency of a body capable of vibrating exactly matches with the frequency of an external periodic force then the body starts vibrating with ***increasing amplitude. This phenomenon is known as ***resonance***.

All receivers, such as T.V, radio set, mobile phones, wireless etc. work on the principle of resonance. These receivers have a tuning circuit. In these tuning circuits the frequency of electrical oscillations is matched with the frequency of incoming signal. When the two frequencies match the receiver starts responding. As a result of which we hear sound and see picture.

***Free vibrator/oscillator*:(*OR ideal or vibrator*)**

An ideal vibrator/oscillator is that in which frictional forces are not present. If frictional forces are present then a part of energy of the vibrating system will be wasted in doing work against the frictional forces.

For example, (1) an ideal simple pendulum is that in which a **point mass** is suspended with the help of an **inextensible**, **weightless** string whose other end is fixed to a rigid support (a support which does vibrate along with the vibrating mass). A real pendulum has a metallic bob suspended by a string attached to a support. Centre of gravity of the bob can be considered as point mass and surface of the bob must be smooth and well-polished to minimize air resistance but viscous air drag remains, which is one of the factors to dissipate energy. Energy dissipated can, therefore be minimized but cannot be made exactly **zero**.

(2) A body executing S.H.M under the action of elastic restoring force must have no friction between the vibrating body and the surface on which it vibrates and also there must be no friction between adjacent turns of the spring. In reality friction between the lower surface of the vibrating body and the surface on which it vibrates can be minimized but cannot be eliminated completely.

In case of an ideal vibrator energy of the vibrating system will remain **constant**. Since energy of a vibrating system depends upon its amplitude of vibrations, therefore for an ideal vibrator its **amplitude** of vibrations will remain **constant**.

***Damped vibrations/oscillations*:**

In a real pendulum, or in a body vibrating under the action of an elastic restoring force or **in general,** in any real vibrating system, **dissipating forces** are always present. These dissipating forces (frictional forces) can be **minimized** but cannot be completely eliminated. As a result of which part of energy of the vibrating system **decreases**. This lost energy is used to do work against frictional forces. Due to decrease in energy of the system amplitude of the vibrating system **decreases**. Hence vibrations gradually die out and the vibrating system ultimately stops. Vibrations of this type are called ***damped vibrations or damped oscillations***.

Damping of a vibrating system can be eliminated by supplying the energy lost during each vibration.

For example, in case of a child taking swings, parent give a little push to keep the amplitude of the swing constant. During each push actually they supply energy lost in doing work to overcome dissipating forces.

Q- factor:

Q-factor may be defined as the ratio of energy stored to energy lost per oscillation.

$$Q = \frac{\text{Energy stored}}{\text{Energy lost}}$$

OR

$$Q = \frac{E_{stored}}{E_{lost}}$$

Q-factor is a **dimensionless quantity**. (since it a ratio of two similar quantities)

Section-A Multiple-Choice Questions MCQs:

Q.No.1**:** Two simple pendulums A and B with same lengths, and equal amplitude of vibrations but the mass of A is twice the mass of B, their periods are T_A and T_B and energies are E_A and E_B respectively. Choose the correct statement:

(A) $T_A = T_B$ and $E_A > E_B$ (B) $T_A < T_B$ and $E_A > E_B$
(C) $T_A > T_B$ and $E_A < E_B$ (D) $T_A = T_B$ and $E_A < E_B$

Q.No.2**:** In order to double the period of a simple pendulum:

(A) Its length should be doubled.
(B) Its length should be quadrupled.
(C) Its mass should be doubled.
(D) Its mass should be quadrupled.

Q.No.3**:** A simple harmonic oscillator has amplitude A and time period "t". Its maximum speed is:

(A) $\frac{4A}{t}$ (B) $\frac{2A}{t}$ (C) $\frac{4\pi A}{t}$ (D) $\frac{2\pi A}{t}$

Q.No.4**:** A spring attached by a load of weight W is vibrating with a period T. If the spring is divided in four equal parts and the same load is suspended from one of these parts, the new time period is:

(A) $\frac{T}{4}$ (B) 2 T (C) $\frac{T}{2}$ (D) 4 T

Q.No.5**:** The total energy of a particle executing simple harmonic motion is proportional to:

(A) Frequency of oscillation.
(B) Maximum velocity of motion.
(C) Amplitude of motion.
(D) Square of amplitude of motion.

Q.No.6**:** A child swinging on a swing in sitting position, stands up, then the time period of the swing will:

(A) Increase. (B) decrease.
(C) remains the same.
(D) increases if the child is tall and decreases if the child is short.

Q.No.7**:** If a boy oscillates at the angular frequency "ω_d" of the driving force, then the oscillations are called:

(A) Forced oscillations. (B) Coupled oscillations,
(C) Free oscillations. (D) Maintained oscillations

Q.No.8**:** A simple harmonic oscillator with a natural frequency "ω_N" is forced to oscillate with a driving frequency "ω_d". The resonance occurred when:

(A) "$\omega_N < \omega_d$" (B) "$\omega_N > \omega_d$"
(C) "$\omega_N = \omega_d$" (D) "$\omega_N \approx \omega_d$"

Q.No.9**:** In vehicles, shock absorber reduces jerks

(A) The shock absorber is an application of damped oscillations.
(B) Damping effect is due to the fractional loss of energy.
(C) Shock absorbers in vehicles reduce jerk. (D) All of these.

Q.No.10**:** A heavily damped system has a fairly flat resonance curve in:

(A) An acceleration - time graph.
(B) An amplitude - frequency graph.
(C) Velocity - time graph
(D) Distance – time graph.

Q.No.11**:** If the bob of a vibrating simple pendulum is suddenly detached from the string at its mean position, it's path will be: (***2005 Karachi Board***)

(A) a straight line (B) a circle.
(C) A parabola. (D) a hyperbola.

Q.No.12**:** which of the following exhibit simple harmonic motion. (***2010 Karachi Board***)

(A) A hanging spring supporting a weight.
(B) The balance of a wheel.
(C) The wheel of an automobile.
(D) The spring of a violin.

Q.No.13**:** Sin $\theta = \theta$ if θ is specifically less than: (***2012 Karachi Board***)

(A) 15°. (B) 10°. (C) 5°. (D) 1 radian.

Q.No.14**:** Time period of a simple pendulum depends upon: (***2013 Karachi Board***)

(A) mass (B) length (C) Acceleration due to gravity
(D) Both length and acceleration due to gravity

Q.No.15**:** If mass of bob of a simple pendulum is doubled, its time period will: (***2014 Karachi board***)

(A) be doubled. (B) becomes triple
(C) remains the same. (D) halved.

Q.No.16: For oscillatory motion of a simple pendulum, restoring force is:

(***2022 Karachi Board***)

(A)mg $\sin\theta$ (B)mg $\cos\theta$ (C) mg $\tan\theta$ (D) mg

Q.No.17: A particle is executing simple harmonic motion, particle has maximum velocity when it is:

(***2025 Karachi Board***)

(A) At maximum displacement.
(B) At equilibrium position.
(C) At half amplitude.
(D) At one quarter amplitude.

Q.No.18: In mass spring oscillator, if the attached mass is doubled, the time period will be: (***2026 Karachi Board***)

(A) T. (B) 2T.(C) $\sqrt{2}\,T$. (D) $T / \sqrt{2}$.

Answers:

(1) (A) $T_A = T_B$ and $E_A > E_B$
(2) (B) Its length should be quadrupled.
(3) (D) $2\pi A / t$
(4) (C) T/2
(5) (D) Square of amplitude of motion.
(6) (B) decrease.
(7) (A) Forced oscillations.
(8) (C) "$\omega_N = \omega_d$"
(9) (D) All of these.
(10) (B)An amplitude-frequency graph.
(11) (C) A parabola.
(12) (C) The wheel of an automobile.
(13) (C) 5°.
(14) (D) Both length and acceleration due to gravity.
(15) (C) remains the same.
(16) (A) mg $\sin\theta$.
(17) (B) At equilibrium position.
(18) (C) $\sqrt{2}\,T$.

Numericals:

***Q.No.1*: The period of oscillation of an object in an ideal spring and mass system is 0.50s and the amplitude is 5.0 cm.**

What is the speed at the equilibrium point? and the acceleration at the point of maximum extension of the spring?

***Data*:**

Time period $T = 0.50$ s
Amplitude $x_o = 5.0$ cm.$= 0.05$m
Speed at equilibrium $v = ?$
Acceleration at maximum extension $a = ?$

***Solution*:**

Speed of a body executing SHM is maximum while passing through its equilibrium position and is given by:

$$v = \frac{2\pi x_o}{T} = \frac{2 \times 3.142 \times 0.05}{0.5}$$

$\boxed{v = 0.6283 \text{ m/s}}$ OR $\boxed{v = 62.83 \text{ cm./s}}$

Acceleration of a body executing SHM is max. at its max. displacement position and is given by:

$$a_{max.} = -\,\omega^2 x_o \qquad \text{where } \omega = \frac{2\pi}{T}$$

Negative sign shows that acceleration of the body is always directed towards its equilibrium position. Magnitude of a_{max} is:

$\therefore\ a_{max} = (2\pi / T)^2 \times 0.05$ $\boxed{a_{max} = 7.9 \text{ m/s}^2}$

***Q.No.2*: A sewing machine needle moves with a rapid vibratory motion, like SHM. As it sews a seam. Suppose the needle moves 8.4 mm from its highest to its lowest position and it makes 24 stitches in 9.0 s. What is the maximum needle speed?**

***Data*:**

Distance between highest & lowest points $= 8.4$ mm.
Number of stitches $N = 24$
Time taken for 24 stitches $t = 9.0$ s.
Maximum needle speed $V_{max.} = ?$

***Solution*:**

Amplitude "x_o" of vibrating needle 'A' is the distance between the highest & lowest points, $x_o = \frac{8.4}{2}$ mm. $= 4.2$ mm $= 0.0042$ m

Time period (in this case time to complete one stitch)

$$T = \frac{\text{time to complete N stitches}}{\text{Total number of stitches N}}$$

$$T = \frac{9}{24} = 0.375 \text{ s.}$$

Speed of any body executing simple harmonic motion is maximum when it is passing through its mean position while coming from extreme position (in this case highest point), it is given by:

$$V_{max.} = \frac{2\pi x_o}{T} = \frac{2\pi \times 0.0042}{0.375}$$

$\boxed{V_{max.} = 0.070 \text{ m/s} = 7.0 \text{ cm./s}}$

🗣 Maximum speed of the needle is **<u>0.07 m/s</u> or <u>7.0 cm./s</u>.**

***Q.No.3*: An ideal spring with spring constant of 15 N/m is suspended vertically. A body of mass 0.60 kg. is attached to unstretched spring and released.**

(a) What is the extension of the spring when the speed is maximum?

(b) What is the maximum speed?

***Data*:**

Spring constant $k = 15$ N/m.
Mass of suspended body $m = 0.6$ kg.
(a) Extension of spring $x_o = ?$
(b) Maximum speed $V_{max.} = ?$

***Solution*:**

(a) Extension force $F = k\,x_o$

$mg = k\,x_o$ ($F = mg$ weight of body)

$\therefore\ x_o = \frac{m\,g}{K} = \frac{0.6 \times 9.8}{15} = \mathbf{0.392\ m.}$

(b) Maximum speed is given by:

$$V_{max.} = \sqrt{\frac{k}{m}}\, x_o$$

$$\therefore\ V_{max.} = \sqrt{\frac{15}{0.6}} \times 0.392$$

$$V_{max.} = \sqrt{25} \times 0.392 = 5 \times 0.392$$

$V_{max.} =$ **<u>1.96 m/s</u>** or **<u>2 m/s.</u>**

🗣 Extension is **<u>0.392 m</u>** and maximum speed is **<u>1.96 m/s</u>** or **<u>2 m/s.</u>**

***Q.No.4*: A body is suspended vertically from an ideal spring of spring constant 2.5 N/m. The spring is initially in its relaxed position. The body is then released and oscillates about its equilibrium position. The motion is described by**

y = (4.0 cm) sin [(0.70 rad/s) t].

What is the maximum kinetic energy of the body.

***Data*:**

Spring constant $k = 2.5$ N/m.

Maximum kinetic energy of body K.E= ?

Motion of body described by

y = (4.0 cm) sin [(0.70 rad/s) t].

Amplitude of oscillations $x_o = 4.0$ cm. = 0.04 m

***Solution*:**

Maximum kinetic energy of the vibrating body is given by:

$$K.E_{max.} = \tfrac{1}{2}\, k\, x_o^2$$

$$\therefore \quad K.E_{max.} = \tfrac{1}{2} \times 2.5 \times (0.04)^2 = \underline{\mathbf{0.002\ J}}.$$

🗣Maximum kinetic energy of the vibrating body is **0.002 J** or **2 mJ**. (milli joules)

***Q.No.5*: The period of oscillation of a simple pendulum does not depend on mass of the bob. By contrast the period of a mass-spring system does depend on mass. Explain the apparent contradiction.**

***Solution*:**

Time period of oscillations of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Time period of oscillations of a mass attached to an elastic spring is given by:

$$T = 2\pi\sqrt{\frac{m}{K}}$$

These equations show that:

- ❖ At a given location (for given "g") time period "T" of a simple pendulum depends upon its length "L" but it is independent of its mass "m".

But

- ❖ Time period "T" of a mass-spring system depends upon mass "m" of the vibrating body and upon spring constant "K" of the spring.

***Q.No.6*: What is the period of a simple pendulum of a 6.0 kg mass oscillating on a 4.0 m long string?**

***Data*:**

Mass of pendulum $m = 6.0$ kg.

Length of string $L = 4.0$ m.

Time period $T = ?$

***Solution*:**

Time period of oscillations of a pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T = 2\pi\sqrt{\frac{4}{9.8}} \qquad \text{squaring both sides:}$$

$$T^2 = 4\pi^2\,\frac{4}{9.8} = 16.1136$$

$$\therefore \quad T = \sqrt{16.1136} = \underline{\mathbf{4.01\ s}}.$$

🗣Time period of the pendulum is **4.01 s**.

***Q.No.7*: A pendulum of length 75 cm and mass 2.5 kg swings with a mechanical energy of 0.015 J. What is the amplitude?**

***Data*:**

Length of pendulum $L = 75$ cm.= 0.75 m

Mass of pendulum $m = 2.5$ kg.

Energy of pendulum $E = 0.015$ J

Amplitude of pendulum $x_o = ?$

***Solution*:**

Amplitude "x_o" of vibrations of the pendulum in terms of its energy can be found using formula for energy of mass-spring system (as both execute SHM).

Energy "E" of a vibrating system is given by:

$$E = \tfrac{1}{2}\, K\, x_o^2 \ ,,,,,,,,,,,,,,,,,,, \ (i)$$

But comparing formula for "T" of a simple pendulum with a formula for "T" for mass-spring system we find that:

$$\sqrt{\frac{m}{K}} = \sqrt{\frac{L}{g}}$$

Squaring both sides and re-arranging we get:

$$K = \frac{m\,g}{L} = \frac{2.5 \times 9.8}{0.75} = \mathbf{32.67\ N/m.}$$

Substituting the values of known quantities in equation (i) we get:

$$0.015 = \tfrac{1}{2} \times 32.67 \times x_o^2$$

$$x_o^2 = \frac{2 \times 0.015}{32.67} = 9.1827\times10^{-4}$$

$$x_o = \sqrt{9.1827\times10^{-4}} = \mathbf{0.030\ m}.\ \text{or}\ \mathbf{3.0\ cm}$$

Amplitude "x_o" of vibrating pendulum is **0.030 m**. or **3.0 cm**.

***Q.No.8*: A pendulum of length "L_1" has a period of T_1 = 0.950 s. length of the pendulum is adjusted to a new value "L_2" such that T_2 = 1.00 s.**

What is the ratio L_2/L_1

***Data*:**

Time period of pendulum T_1 = 0.95 s (of length L_1 before adjustment)

Time period of pendulum T_2 = 1.00 s (of length L_2 after adjustment)

Ratio of lengths L_2/L_1 =?

***Solution*:**

Time period of pendulum before adjustment:

$$T_1 = 2\pi\sqrt{\frac{L_1}{g}}$$

Time period of pendulum after adjustment:

$$T_2 = 2\pi\sqrt{\frac{L_2}{g}}$$

Dividing T_2 by T_1,

$$\frac{T_2 = \cancel{2\pi}\sqrt{\frac{L_2}{\cancel{g}}}}{T_1 = \cancel{2\pi}\sqrt{\frac{L_1}{\cancel{g}}}}$$

* $$\frac{T_2}{T_1} = \sqrt{\frac{L_2}{L_1}}$$

Squaring both sides we get:

$$\frac{L_2}{L_1} = \frac{T_2^2}{T_1^2}$$

$$\frac{L_2}{L_1} = \frac{(1.0)^2}{(0.95)^2} = \frac{1}{0.9025} = \mathbf{1.108}$$

The ratio of lengths (L_2/L_1) is **1.108**.

***Q.No.9*: Calculate the length of second's pendulum on the surface of moon where acceleration due to gravity is 0.167 times that on the earth's surface.**

(*1995 Karachi Board*)

***Data*:**

Acceleration due to gravity on the surface of moon g_m =0.167 g

Time period of second's pendulum T=2 sec.

Length of second's pendulum on Moon L=?

***Solution*:**

Time period on the surface of moon is given by

$$T = 2\pi\sqrt{\frac{L}{g_m}}$$

OR $$2 = 2\pi\sqrt{\frac{L}{0.167\times g}}$$

Squaring both sides we get:

$$4 = 4\pi^2 \frac{L}{0.167\times 9.8}$$

$$4 = 4\pi^2 \frac{L}{1.6366}$$

$$L = \frac{4\times1.6366}{4\pi^2} \quad \boxed{L = 0.166\ m = 16.6\ cm}$$

Length of second's pendulum on the surface of moon is ***0.166 m*** OR ***16.6 cm***.

Q.No.10*: Calculate the length of second's pendulum at a place where g= 10 m/s^2.** (1997 Karachi Board***)

***Data*:**

Time period of second's pendulum T = 2 sec.

Acceleration due to gravity g =10m/s^2

Length of second's pendulum L=?

***Solution*:**

Time period of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$2 = 2\pi\sqrt{\frac{L}{10}}$$

Squaring both sides we get:

$$4 = 4\pi^2\frac{L}{10} \quad \text{OR} \quad L = \frac{4\times10}{4\pi^2}$$

$$\boxed{L = 1.01\ m.}$$

Length of second's pendulum is ***1.01 m.*** or ***101 cm***. at a place where g =10 m/s^2.

***Q.No.11*: A body of 0.5 kg. attached to a spring is displaced from its equilibrium position and released. If the spring constant is 50 N/m, find:**

(*i*) Time period. (*ii*) The frequency.

(*1998 Karachi Board*)

***Data*:**

Mass of the body m = 0.5 kg

Force constant K = 50 N/m

(*i*) Time period T =?

(*ii*) Frequency f =?

Solution:

(*i*) Time period of oscillations of a mass attached to an elastic spring is given by:

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$T = 2\pi\sqrt{\frac{0.5}{50}}$$

$$T = 2\pi\sqrt{0.01} = 2\pi \times 0.1$$

$$\boxed{T = 0.63 \text{ sec.}}$$

(*ii*) $f = \frac{1}{T} = \frac{1}{0.63}$

$$\boxed{f = 1.59 \text{ Hz.}}$$

Q.No.12: **Find the length of second's pendulum on planet Jupiter where the value of 'g' is 2.63 times the value of 'g' on the surface of the earth.**

(***2000 Karachi Board***)

Data:

Length of the pendulum L =?

Time period of second's pendulum T=2s

Acceleration due to gravity $g_J = 2.63g_e$

Solution:

Time period of a pendulum on the surface of Jupiter is given by:

$$T = 2\pi\sqrt{\frac{L}{g_J}}$$

$$2 = 2\pi\sqrt{\frac{L}{2.63\times g_e}} \quad \text{squaring both sides}$$

$$4 = 4\pi^2 \frac{L}{2.63 \times 9.8}$$

$$L = \frac{4 \times 2.63 \times 9.8}{4\pi^2} \qquad \boxed{L = 2.61 \text{ m.}}$$

Length of second's pendulum on the surface of Jupiter is ***2.61 m.***

Q.No.13:**A body of mass 0.5 kg. is attached to the end of a spring placed on a smooth horizontal table and is performing SHM.**

Find the acceleration of the body when it has displacement of 0.6 m. The spring constant of the spring is 150 N/m.

(***2001 Karachi Board***)

Data:

Mass of the body m = 0.5 kg.

Acceleration of the body a = ?

Displacement x = 0.6 m

Spring constant K= 150 N/m.

Solution:

Acceleration of a body attached to a spring executing SHM is given:

$a = -\frac{K}{m}x$ where negative sign shows that 'a' is always towards the mean position. Hence the magnitude of acceleration is given by:

$$a = \frac{K}{m}x = \frac{150\times 0.6}{0.5} \qquad \boxed{a = 180 \text{ m/s}^2.}$$

Q.No.14: **Compute the acceleration due to gravity on the surface of the moon where a simple pendulum 1.5 m long makes 100 vibrations in 605 seconds.**

(***2002 Karachi Board***)

Data:

Length of the pendulum L = 1.5m

Number of vibrations = 100

Time taken = 605 sec.

Time period $T = \frac{605}{100} = 6.05$ s

Acceleration due to gravity on the surface of moon g_m = ?

Solution:

Acceleration due to gravity on the surface of moon is given by:

$$T = 2\pi\sqrt{\frac{L}{g_m}}$$

Squaring both sides we get:

$$T^2 = 4\pi^2 \frac{L}{g_m}$$

$$(6.05)^2 = 4\pi^2 \times \frac{1.5}{g_m}$$

$$g_m = \frac{4\pi^2 \times 1.5}{(6.05)^2} = \frac{59.218}{36.6025}$$

$$\boxed{g_m = 1.618 \text{ m/s}^2}$$

Q.No.15: **A mass at the end of a spring oscillates with a simple harmonic motion with a period of 0.40 sec. Find the acceleration when the displacement is 4.0 cm.** (***2003 Karachi Board***)

Data:

Time period T = 0.40 sec.

Displacement x = 4.0 cm= 0.04 m

Acceleration a =?

Solution:

$$T = 2\pi\sqrt{\frac{m}{K}}$$

Squaring both sides we get:

$$(0.40)^2 = 4\pi^2 \frac{m}{K}$$

$$0.16 = 4\pi^2 \frac{m}{K}$$

$$\frac{m}{K} = \frac{0.16}{4\pi^2}$$

But $F = m\,a = K\,x$

$$\frac{m}{K} = \frac{x}{a}$$

$\therefore\quad 0.16 = 4\pi^2 \dfrac{x}{a}$

$$a = \frac{4\pi^2 \times x}{0.16} = \frac{4\pi^2 \times 0.04}{0.16}$$

$\therefore\quad \boxed{a = 9.87\ m/s^2.}$

***Q.No.16*: The period of vibration of a body of mass 25 gm. attached to a spring, vibrating on a smooth horizontal surface, when it is displaced 10 cm to the right of its extreme position, is 1.57 sec. and the velocity at the end of the displacement is 0.4 m/s. Determine the (a) spring constant (b)Total energy and (c) amplitude.**
(***2004 Karachi Board***)

Data:

Mass of body $m = 25gm = 0.025$ kg
Displacement $x = 10$ cm $= 0.10$ m
Time period $T = 1.57$ sec
Velocity $v = 0.4$ m/s
Spring constant $K = ?$
Total energy $E = ?$
Amplitude $x_o = ?$

Solution:

Time period of a mass attached to an elastic spring executing S.H.M is given by:

$T = 2\pi\sqrt{\dfrac{m}{K}}$ squaring both sides

$$T^2 = 4\pi^2 \frac{m}{K}$$

$$(1.57)^2 = 4\pi^2 \frac{0.025}{K}$$

$$K = \frac{4\pi^2 \times 0.025}{(1.57)^2} \qquad \boxed{K = 0.4\ N/m}$$

Spring constant is ***0.4 N/m.***

Energy of the system at any instant is partially K.E and partially P.E, hence:

Total energy E = K.E + P.E

$$E = \tfrac{1}{2} m v^2 + \tfrac{1}{2} K x^2$$

$$E = \tfrac{1}{2}\times 0.025\times(0.4)^2 + \tfrac{1}{2}\times 0.4\times(0.1)^2$$

$$E = 2\times10^{-3} + 2\times10^{-3}$$

$$\boxed{E = 4\times 10^{-3}\ J.}$$

Energy of the system is ***4×10^{-3}J.***

But $\quad E = \tfrac{1}{2} k\, x_o^2$

$\therefore\quad 4\times10^{-3} = \tfrac{1}{2}\times 0.4\ x_o^2$

$$x_o^2 = \frac{4\times10^{-3}\times 2}{0.4} = 0.02$$

$\therefore\quad \boxed{x_o = 0.1414\ m.}$

Amplitude of oscillation is ***0.1414 m***

***Q.No.17*: A body of mass 32 gm. attached to an elastic spring is performing SHM. Its velocity is 0.4 m/s when the displacement is 8 cm. towards right. If the spring constant is 0.4 N m^{-1}, Calculate: (*i*) Total energy. (*ii*) The amplitude of its motion**
(***2008,2000,1995,93 Karachi Board***)

Data:

Mass $m = 32$ gm. $= 0.032$ kg
Velocity $v = 0.4$ m/s
Displacement $x = 8$ cm. $= 0.08$m
Spring constant $K = 0.4$ N m^{-1}
(*i*) Total energy $E = ?$
(*ii*) The amplitude of motion $x_o = ?$

Solution:

Total energy of mass attached to an elastic spring and executing SHM is given by:

$$E = K.E + P.E = \tfrac{1}{2} mv^2 + \tfrac{1}{2} K x^2$$

$$= \tfrac{1}{2}\times 0.032\times(0.4)^2 + \tfrac{1}{2}\times 0.4\times(0.08)^2$$

$$= 2.56\times10^{-3} + 1.28\times10^{-3}$$

$\therefore\quad \boxed{E = 3.84\times 10^{-3}\ J}$

But Total energy $E = \tfrac{1}{2} k x_o^2$

$\therefore\quad 3.84\times10^{-3} = \tfrac{1}{2}\times 0.4\times x_o^2$

$$x_o^2 = \frac{2\times 3.84\times10^{-3}}{0.4} = 0.0192$$

$\therefore\quad x_o = \sqrt{0.0192} \qquad \boxed{x_o = 0.1386\ m}$

The amplitude of vibrations is ***0.1386m.***

***Q.No.17*: A sonometer wire of length 1m, when plucked at the center, vibrates with a frequency of 256 Hz.**

Calculate the wavelength and the speed of the waves in the wire.

(***2009, 2005 Karachi Board***)

***Data*:**

Length of the wire	$L = 1m$
Frequency	$f = 256$ Hz.
Wave length of wave	$\lambda = ?$
Speed of waves	$v = ?$

***Solution*:**

When a wire is plucked from its center it vibrates with the fundamental frequency, forming one loop. Hence wave length of the wave formed in it:

$\lambda = 2L = 2\times1$ $\boxed{\lambda = 2\ m}$

Similarly speed of wave:

$v = f\lambda = 256 \times 2$ $\boxed{v = 512\ m/s}$

***Q.No.18*: A string 2 m long and a mass of 0.004 kg. is stretched horizontally by passing one of its ends over a pulley and the string is attached with one kg. mass to it vertically.**

Find the speed of the transverse wave on the string and the frequency of the fundamental and fifth harmonic at which the string vibrates.

(***2010,02,03 Karachi Board***)

***Data*:**

Length of the string	$L = 2$ m
Mass of the string	$m = 0.004$ kg
Mass suspended	$M = 1$ kg
Speed of the wave	$V = ?$
Fundamental frequency	$f_1 = ?$
Frequency of the fifth harmonic	$f_5 = ?$

***Solution*:**

Speed of the transverse wave on a string is given by:

$V = \sqrt{\dfrac{T\,L}{m}}$ Since $T = Mg$

$\therefore\ V = \sqrt{\dfrac{M\,g\,L}{m}} = \sqrt{\dfrac{1\times 9.8\times2}{0.004}}$

$V = \sqrt{\dfrac{19.6}{0.004}} = \sqrt{4900}$

$\boxed{V = 70\ m/s.}$

Speed of the transverse waves on the string is ***70 m/s***.

Fundamental frequency is given by:

$f_1 = \dfrac{V}{2\,L}$

$f_1 = \dfrac{70}{2\times2}$ $\boxed{f_1 = 17.5\ Hertz}$

Fundamental frequency is ***17.5 Hertz***.

Frequency for n segments is given by:

$f_n = n\dfrac{V}{2\,L}$ or $f_5 = 5\times\dfrac{70}{2\times2}$

$\boxed{f_5 = 87.5\ Hertz}$

(Or $f_5 = 5\,f_1 = 5\times17.5 = 87.5$ Hertz)

Frequency for 5th harmonic is ***87.5 Hertz***.

***Q.No.19*: A simple pendulum completes 4 vibrations in 8 seconds on the surface of the earth. Find the time period on the surface of the moon where the acceleration due to gravity is one sixth of that on the earth.**

(***2010,09,07,1993Karachi Board***)

***Data*:**

Time taken for 4 vib. $t = 8$ sec.

Time period on earth $T_e = \dfrac{8}{4}$ s $= 2$ s.

Time period on moon $T_m = ?$

Acceleration due to gravity on moon

$g_m = 1/6\ g_e$

***Solution*:**

Time period of a pendulum on the surface of earth is given by:

$T_e = 2\pi\sqrt{\dfrac{L}{g_e}}$

OR $2 = 2\pi\sqrt{\dfrac{L}{9.8}}$

Squaring both sides we get:

$4 = 4\pi^2\dfrac{L}{9.8}$ $\therefore$ $L = \dfrac{4\times9.8}{4\pi^2} = 0.99$ m

Time period on the surface of the moon:

$T_m = 2\pi\sqrt{\dfrac{L}{g_m}}$

$T_m = 2\pi\sqrt{\dfrac{0.99}{1/6\ g_e}}$ OR $T_m = 2\pi\sqrt{\dfrac{0.99}{1.633}}$

$T_m = 2\pi\times0.7785$ $\boxed{T_m = 4.89\ sec}$

Time period of the given pendulum on the surface of moon is ***4.89 sec.***

***Q.No.20*: A string 1 m long and of mass 0.004 kg. is stretched with a force. Calculate the force if the speed of the wave in the string is 140 m/s.**

(***2012 Karachi Board***)

***Data*:**

Length of the string L = 1 m.
Mass of the string m = 0.004 kg
Speed of the wave V = 140 m/s
Stretching force = tension in the string T = ?

***Solution*:**

Speed of a wave in a string is given by:

$$V = \sqrt{\frac{T\,L}{m}}$$

$$140 = \sqrt{\frac{T\times 1}{0.004}}$$

Squaring both sides, we get:

$$(140)^2 = \frac{T}{0.004}$$

$$19600 = \frac{T}{0.004}$$

$$T = 19600 \times 0.004 \qquad \boxed{T = 78.4\ N}$$

***Q.No.21*: A body hanging from a spring is set into motion and the period of oscillation is found to be 0.5 sec. After the body has come to rest, it is removed. How much shorter will the spring be when it comes to rest.**

(***2015, 2013, 02 Karachi Board***)

***Data*:**

Time period T = 0.5 s.
Decrease In length x =?

***Solution*:**

Force constant:

$$k = \frac{F}{x} = \frac{m\,g}{x}$$

But the time period of the oscillating body attached to a spring is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Therefore, $$T = 2\pi\sqrt{\frac{m}{m\,g/x}}$$

$$T = 2\pi\sqrt{\frac{x}{g}}$$

Squaring both sides we get:

$$T^2 = 4\pi^2\,\frac{x}{g}$$

$$x = \frac{T^2\,g}{4\pi^2}$$

$$x = \frac{(0.5)^2\times 9.8}{4\pi^2} \qquad \boxed{x = 0.062\ m}$$

The spring will be shorter by ***0.062 m*** after the oscillating suspended body has been removed from it.

***Q.No.22*: A 100 cm long string vibrates in 4 loops at 50 Hz. The linear density of the string is 4×10^{-4} gram./cm. Calculate the tension in the string.**

(***2017 Karachi Board***)

***Data*:**

Length L = 100 cm = 1 m
Frequency f_4 = 50 Hz.
Number of loops = 4
Linear density $\mu = 4\times10^{-4}$ g/cm

$$= 4\times10^{-4}\times\frac{100}{1000}\ \text{kg/m} = 4\times10^{-5}\ \text{kg/m}$$

Tension T =?

***Solution*:**

Since $f_4 = 4\,f_1$

$\therefore$ $f_1 = f_4 / 4 = 50 / 4 = 12.5$ Hz.

But $$f_1 = \frac{1}{2\,L}\sqrt{\frac{T}{\mu}}$$

$$12.5 = \frac{1}{2\times1}\sqrt{\frac{T}{4\times10^{-5}}}$$

$$25 = \sqrt{\frac{T}{4\times10^{-5}}}$$ squaring both sides, we get

$$(25)^2 = \frac{T}{4\times10^{-5}}$$

$$T = 625\times4\times10^{-5} \qquad \boxed{T = 0.025\ N}$$

***Alternate method*:**

Speed v of waves is given by:

$$v = \lambda\,f = 0.5\times50 = 25\ \text{m/s}$$

But speed of stationary waves in a stretched string is given by:

$$v = \sqrt{\frac{T}{\mu}} \qquad v^2\times\mu = T$$

$$T = (25)^2\times4\times10^{-5}$$

$$T = 625\times4\times10^{-5} \qquad \boxed{T = 0.025\ N}$$

Tension in the string is ***0.025 N.***

Q.No.23: A 15 kg. block is suspended by a spring of spring constant 5×10^3 N/m. Calculate the frequency of vibrations of the block displaced from its equilibrium position and then released. (***2018 Karachi Board***)

Data:

m = 15 kg. k = 5×10^3 N/m. f = ?

Solution:

Time period of the vibrating block:

$T = 2\pi\sqrt{\dfrac{m}{k}}$ $\quad T = 2\pi\sqrt{\dfrac{15}{5\times10^3}}$

But $f = \dfrac{1}{T}$ $\quad \therefore\ f = \dfrac{1}{2\pi}\sqrt{\dfrac{5\times10^3}{15}}$

$\therefore\ f = \dfrac{1}{2\pi}\sqrt{333.3}$ $\quad f = \dfrac{18.26}{2\pi}$

$\boxed{f = 2.91\ \text{Hz.}}$

Q.No.24: A string 2 m long and of mass 0.004 kg. is stretched horizontally by passing one end over a pulley and attaching a 1 kg mass to it. Find the speed of transverse wave on the string and the frequency of the second harmonic. (***2019, 2016 Karachi Board***)

Data:

Length of the string L = 2 m
Mass of the string m = 0.004 kg
Mass suspended M = 1 kg
Speed of the wave V = ?
Frequency of the second harmonic f_2 = ?

Solution:

Speed of the transverse wave on a string is given by:

$V = \sqrt{\dfrac{T\,L}{m}}$ $\quad$ Since T = Mg

$\therefore\ V = \sqrt{\dfrac{M\,g\,L}{m}} = \sqrt{\dfrac{1\times9.8\times2}{0.004}}$

$V = \sqrt{\dfrac{19.6}{0.004}} = \sqrt{4900}$ $\quad \boxed{V = 70\ m/s.}$

Speed of the transverse waves on the string is ***70 m/s.***

Frequency for n segments is given by:

$f_n = n\,\dfrac{V}{2\,L}$ or $f_2 = 2\times\dfrac{70}{2\times2}$

$\boxed{f_2 = 35\ \text{Hertz}}$

Frequency for second harmonic is ***35 Hertz.***

Q.No.25: A mass at the end of spring oscillates with a period of 0.4 sec. Find the acceleration when displacement is 6 cm. (***2019, 2003 Karachi Board***)

Data:

Time period T = 0.4 sec.
Displacement x = 6 cm = 0.06 m
Acceleration a =?

Solution:

$T = 2\pi\sqrt{\dfrac{m}{K}}$

Squaring both sides we get:

$(0.4)^2 = 4\pi^2\dfrac{m}{K}$ $\quad 0.16 = 4\pi^2\dfrac{m}{K}$

$\dfrac{m}{K} = \dfrac{0.16}{4\pi^2}$

But $\quad F = m\,a = K\,x$

$\dfrac{m}{K} = \dfrac{x}{a}$

$\therefore\quad 0.16 = 4\pi^2\dfrac{x}{a}$

$a = \dfrac{4\pi^2\times x}{0.16}$ or $a = \dfrac{4\pi^2\times0.06}{0.16}$

$\therefore\ \boxed{a = 14.80\ m/s^2.}$

Q.No.26: A body of mass 5 gram oscillates as a simple pendulum. Calculate the time period of the pendulum if its length is 1.2 m. (***2025 Karachi Board***)

Data:

Mass of the body m = 5 gm.
Length of the pendulum L = 1.2 m
Time period T = ?

Solution:

Time period of a simple pendulum is independent of its mass, it is given by:

$T = 2\pi\sqrt{\dfrac{L}{g}} = 2\pi\sqrt{\dfrac{1.2}{9.8}}$

Squaring both sides we get:

$T^2 = 4\pi^2\,\dfrac{1.2}{9.8} = 4.834$

$T = \sqrt{4.834} = $ **2.198 sec.**