**According to new syllabus of all intermediate boards of Sindh.**

***Prof. Rawala's ........***

# PHYSICS

## HELPING BOOK

***For***
***Class XI***
**(All science students)**

**2025 - 2026**

- ***Detailed notes.***
- ***Multiple Choice Questions (M.C.Q's).***
- ***Solution of text book numericals.***
- ***MCQs and solution of numericals from Karachi and Hyderabad board past papers.***



***Prof.***(r) ***Mohammed Hussain Rawala.***
**B.Sc. (Hons.), M.Sc.(Physics)**
Former head of physics department
Saint Patrick's college, Karachi.

# ***Contents***

## Unit. 9 ***Capacitors.***



## Unit. 10 ***D.C circuits.***



## Unit. 11 ***Oscillations.***



## Unit. 12 ***Acoustics.***



## Unit. 13 ***Physical optics.***



## Unit. 14 ***Communication.***

***To*** **HIM**

Who gives courage to struggle and chance to survive

**To** **those**

Who make its use in their life.

## Table of physical constants

| | |
|---|---|
| Specific heat of water | $C = 4200\ J/kg\text{-}°C$ |
| B.P of water under 1 atm. Pressure | $=100°C = 212°F = 373\ k$ |
| Melting point of ice under 1 atm. | $= 0°C = 32°F = 273\ k$ |
| Latent heat of vaporization of water | $= 2.26 \times 10^6\ J/kg.$ |
| Latent heat of fusion of ice | $= 3.36 \times 10^5\ J/kg.$ |
| Absolute zero | $= -273\ °C = 0\ k$ |
| Mass of the earth | $M_e = 5.98 \times 10^{24}\ kg.$ |
| Mass of the moon | $M_m = 7.36 \times 10^{22}\ kg.$ |
| Mass of the sun | $M_s = 1.97 \times 10^{30}\ kg.$ |
| Radius of the earth | $R_e = 6.38 \times 10^6\ m.$ |
| Radius of moon's orbit | $R_{m.} = 3.84 \times 10^8\ m$ |
| Universal gravitational constant | $G = 6.67 \times 10^{-11}\ N\text{-}m^2/kg^2.$ |
| Universal gas constant | $R = 8.313\ J/mole\text{-}k.$ |
| Boltzmann constant | $k = 1.38 \times 10^{-23}\ J/k.$ |
| Plank's constant | $h = 6.63 \times 10^{-34}\ J\text{-}s.$ |
| Rydberg's constant for hydrogen | $R_H = 1.097 \times 10^7\ m^{-1}.$ |
| Speed of light in vacuum | $c = 2.98 \times 10^8\ m/s.$ |
| Rest mass of electron | $m_e = 9.11 \times 10^{-31}\ kg.$ |
| Rest mass of proton | $m_p = 1.67 \times 10^{-27}\ kg.$ |
| Rest mass of neutron | $m_n = 1.68 \times 10^{-27}\ kg.$ |
| Charge on an electron | $e = -1.6021 \times 10^{-19}\ coul.$ |
| Charge on a proton | $e = +1.6021 \times 10^{-19}\ coul.$ |
| Permittivity of free space | $\varepsilon_o = 8.85 \times 10^{-12} C^2/N\text{-}m^2.$ |
| Permeability of free space | $\mu_o = 4\pi \times 10^{-7}\ web.\ /A\text{-}m.$ |
| Avogadro's number | $N_A = 6.02 \times 10^{23}$ |
| Radius of first orbit of hydrogen | $r = 0.53 \times 10^{-10}\ m = 0.53\ Å$ |

<u>Commonly used geometrical formulae</u>:

Area of a circle $= \pi r^2$ Circumference of a circle $= 2 \pi r$

Surface area of a sphere $= 4 \pi r^2$

Volume of a sphere $= \frac{4}{3} \pi r^3$

<u>Quadratic formula</u>: If $ax^2 + bx + c = 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

***Unit. 1*****…..**

# Physics and measurements

## ***Physics*:**

***Q.No.1*: What is physics?**

***Ans*:** Physics is a branch of science that deals with the study of matter and energy and the interaction between them; hence it is that branch of science that seeks to understand and describe the fundamental principles and laws governing the natural world.

Physics is basically an experimental science, it, therefore, depends upon the objective observations and accurate measurements of various natural phenomenon.

### ***Main branches of physics*:**

Physics is divided into many branches, each branch deals with different aspects of physical world.

- ***Classical mechanics*** **(*Newtonian physics*)** is based on old but still valid theories and supported by old experiments. Newtonian physics can be classified as **classical** in nature. Classical physics, broadly speaking, is divided into:

(*i*) Mechanics (*ii*) Electricity (*iii*) Magnetism (*iv*) Electromagnetism (*v*) Sound (*vi*) Heat. (*vii*) Optics. (*viii*) Thermodynamics. (*ix*) Spectroscopy etc.

**Electromagnetism** is basically a study of interaction between electric and magnetic fields, interaction of these fields with charged particles and current carrying conductors, production and propagation of electromagnetic waves etc.

**Thermodynamics** is basically a study of conversion of heat energy into mechanical energy (work) and vice versa. It also deals with the study of entropy, behavior of gases at different temperatures.

**Optics** is the study of properties of light such as refraction, reflection, interference and polarization etc.

- ***Modern physics*** deals with new ideas supported by phenomenon some of which could not be explained by classical physics.

Modern physics was developed successfully during the last century by many famous scientists such as Max. Plank, Einstein, Compton, De Broglie, Bohr, Rutherford etc.

Some of the branches into which modern physics is divided are:

(*i*) Quantum mechanics (*ii*) Relativity (*iii*) Nuclear physics (*iv*)Solid state physics (*v*) Electronics (*vi*) Plasma physics (*vii*) Particle physics (*viii*) Space physics etc.

**Quantum mechanics** is a branch of modern physics that deals with the behavior of particles at atomic and subatomic levels, deals with wave-particle duality, quantum superposition etc.

**<u>Einstein's theory of Relativity</u>** (special theory and general theory) deals with behavior of objects at speeds close to speed of light, such as a change in mass of a body, length contraction, time dilation etc. It also deals with space-time.

***<u>Q.No.2</u>:Give some applications of physics in various fields.***

- ***Engineering fields*:** Physics principles are applied in different branches of Engineering e.g. electrical engineer applies principles of electricity and magnetism, mechanical engineer applies laws of motion, momentum, torques etc. while designing a vehicle and so on.
- ***Energy and power*:** generation, transmission and utilization of electrical energy, production and conversion of nuclear energy for peaceful purposes, solar energy, energy storage in lithium battery etc. could not be achieved without the knowledge and application of physics.
- ***Electronics and communication*:** telecommunication, development of various electronic devices and information technology depend on the study of semiconductors, quantum mechanics and electromagnetism.
- ***Medical applications*:** Knowledge and applications of various branches of physics contributed in the development of X-rays, MRI, Ultrasound, laser surgery, radiation therapy etc.
- ***Material science*:** Research and understanding of properties of materials resulted in the development of new materials for various applications in electronics, transportation etc.
- ***Environmental studies*:** Physics plays a very important role in the study of climate change. It is used to monitor environmental changes and predict daily weather.
- ***Education and scientific literacy*:** Education in physics promotes critical thinking, problem solving skills and scientific mindset.

## ***<u>Physical quantities</u>:***

***<u>Q.No.1</u>: What is a base or a fundamental quantity?***

***<u>Ans</u>*:** Quantity which ***<u>cannot be</u>*** derived from another quantity or ***<u>cannot be</u>*** resolved into any other quantity more fundamental, is called a ***<u>base or a fundamental quantity</u>***.

***<u>Q.No.2</u>: What are derived quantities?***

***<u>Ans</u>*:** Quantities which ***<u>can be expressed</u>*** as some combination of base quantities are called ***<u>derived quantities</u>***.

- ***<u>Units</u>*** are standards of measurement used to express physical quantities.

Units of base quantities are called ***<u>base or fundamental units</u>*** and units of derived quantities are called ***<u>derived units</u>***.

Derived units are obtained by <u>multiplication</u>, <u>division</u> or <u>multiplication as well as division</u> of base units.

***<u>Q.No.3</u>: What is a system of units?***

***<u>Ans</u>*:** A set of base and derived units is called a system of units.
There are various systems of units such as C.G.S system, British engineering system and S.I or international system of units. Out of these S.I system is most modern and is widely used throughout the world in scientific, engineering and technological applications.

***<u>Q.No.4</u>: Name S.I base quantities?***

***<u>Ans</u>*:** There are seven base quantities in S.I system, they are:
(*i*) length (*ii*) mass (*iii*) time (*iv*) electric current (*v*) thermodynamic temperature
(*vi*) amount of substance, (*vii*) luminous intensity,

***<u>Q.No.5</u>:Name the S.I units of base or fundamental quantities?***

***<u>Ans</u>*:** S.I units of base quantities are:

| | *Quantity* | *S.I Unit* |
|---|---|---|
| 1. | Length. | meter m |
| 2. | Mass. | Kilogram kg. |
| 3. | Time. | second s |
| 4. | Electric current. | ampere A |
| 5. | Thermodynamic temperature. | Kelvin k |
| 6. | Amount of substance. | mole mol. |
| 7. | Luminous intensity. | candela cd |

***<u>Supplementary units</u>***

Supplementary units are units of quantities that are not covered by SI system of units. Supplementary units are dimensionless units and are used along with base unit in SI system.

Supplementary quantities are **<u>geometrical quantities</u>** related with angle at the center of a circle or of a sphere.

| *Physical Quantity* | *Supplementary Unit* | *Symbol* |
|---|---|---|
| Plane angle (angle at the center of a circle) | Radian | Rad |
| Solid angle (angle at the center of a sphere) | Steradian | Sr |

***<u>Q.No.6</u>*: *Define plane angle and its S.I unit.***

***<u>Ans</u>*:** It is the angle subtended at the center of a **<u>circle</u>** by an arc.

Angle subtended at the center of a circle by an arc is said to be I radian if length of the arc is equal to radius of the circle. 1 rad = 57.3°.

***<u>Q.No.7</u>*: *Define solid angle and its S.I unit.***

***<u>Ans</u>*:** It is the angle subtended at the center of a **<u>sphere</u>** by a certain area of its surface.

Angle subtended at the center of a sphere by the area of its surface is said to be I Steradian if the area of the surface is equal to square of radius of the sphere.

***<u>Q.No.6</u>*: *Name some derived quantities and give their S.I. units.***

***<u>Ans</u>*:** Some of the derived quantities and their S.I units are:

| | *Quantity*. | *S.I unit* |
|---|---|---|
| 1. | Area | square meter $m^2$ |
| 2. | Volume | cubic meter $m^3$ |
| 3. | Speed or velocity. | meter per second m/s |
| 4. | Acceleration. | meter per second square $m/s^2$ |
| 5. | Force. | kilogram-meter per second$^2$ or newton N |
| 6. | Torque or moment of force. | newton-meter N-m |
| 7. | Momentum. | kilogram-meter per second kg. m/s or N-s |
| 8. | Work, energy | Joule. J |
| 9. | Power | Joule/sec. or watt watt |

***<u>Note that</u>*:** While specifying units they should be written in <u>lowercase (small) letters</u> and in <u>singular form</u> except when magnitude of the quantity is greater than 1. Symbol of a unit of a quantity derived from name of a scientist must be written in capital letter (such as Newton N as a unit of force, Kelvin K as a unit of absolute or thermodynamic temperature and so on.)

***Q.No.7*: What is light year?**

***Ans*:** It is the unit of distance.

Long distances such as distance between stars, galaxies etc. are measured in light year.

One light year **is the distance travelled by light in vacuum in one year.**

Since S = v t $\therefore$ 1 light year (S) = $3 \times 10^8$ m/s $\times$ (365 $\times$ 24 $\times$ 60 $\times$ 60 s)

$\therefore$ 1 light year = $9.47 \times 10^{15}$ m

Our beautiful little galaxy, called ***Milky way galaxy***, is spiral shaped. Sun is a star of intermediate size and luminosity. Sun along with the whole solar system (including our beautiful little EARTH) revolves in Milky way at some distance from its center.

There are more than about **two billion stars** in our galaxy. Some **larger and some smaller than the sun**. There is a large number of galaxies in our universe.

Galaxy nearest to our galaxy is called **ANDROMEDA**, is at about 2 million light years away. (***It means that*** light coming from Andromeda galaxy will take about 2 million years to reach us !!!....)

***Q.No.8*: What is angstrom unit Å?**

***Ans*:** It is the unit of **length** (distance).

It is used when we come across short lengths. Wave length of light is very short, therefore, it is usually measured in angstrom unit.

1Å = $10^{-10}$ m

***Q.No.9*: What is electron volt (*ev*)?**

***Ans*:** Electron volt (ev.) is the unit of **energy**.

It is a small unit of energy. Energy in various nuclear reactions is measured in electron volt.

1 ev. = $1.6021 \times 10^{-19}$ J (Multiples: Mev. and Bev.)

# *Dimensions*:

***Q.No.1*: What are *dimensions* of a physical quantity?**

***Ans*:** Dimensions of a physical quantity represent the physical nature of the quantity. Dimensions of a physical quantity is some combination of dimensions of base quantities raised to some suitable power.

***Q.No.2*: What are dimensions of base quantities?**

***Ans*:** Dimensions of base quantities are "[L]" (for distance), "[M]" (for mass) "[T]" (for time), [A] for electrical current and [K] for thermodynamic temperature. Dimensions of all other quantities are derived from the dimensions of base quantities.

On the basis of their dimensions, physical quantities are classified as:

***Dimensional quantities*:**

Quantities having dimensions are called dimensional quantities e.g. velocity, acceleration, force, momentum etc. are dimensional quantities.

***Dimensionless quantities*:**

Quantities having no dimension are called dimensionless quantities e.g. strain, angle etc. are dimensionless quantities.

***Dimensional constants*:**

Quantities having a constant value and at the same time having a dimension are called dimensional constants e.g. Universal gravitational constant "G", Plank's constant "$h$", speed of light in vacuum "c", permittivity of free space "$\varepsilon_o$", permeability of free space "$\mu_o$" etc. are dimensional constant.

***Dimensionless constants*:** Quantities having a constant value but do not have any dimension are called dimensionless constants, they are merely numbers e.g. 2, 3, $\pi$, $\sqrt{5}$, $(7)^2$ etc.

***Use of dimensional analysis*:** Dimensional analysis can be used for the following purposes:

1. For checking a result:

Any equation can be verified for correctness by finding out the dimensions of different terms.

2. Derivation of a result:

Various quantities can be derived with the help of dimensional analysis.

3. For changing units:

Units can be changed from one system to another.

***Limitations of dimensional analysis*:**

Some of the limitations of dimensional analysis are:

1. It doesn't give information about dimensional constant.
2. Relations involving trigonometric, exponential and logarithmic functions cannot be derived.
3. It does not provide any information about whether the quantity is scalar or a vector.

## ***Dimensions of various physical quantities*:**

***Find the dimensions of following quantities*:**

1. Area
2. Volume
3. Density
4. Speed or velocity
5. Acceleration
6. Force
7. Momentum
8. Work or energy (K.E, P.E or any form)
9. Power
10. Pressure
11. Torque
12. Frequency "$\nu$"
13. Wave length "$\lambda$"
14. Universal gravitational constant "G"
15. Plank's constant "$h$"
16. Angular velocity "$\omega$"
17. Angular acceleration "$\alpha$".
18. Angular momentum "L".
19. Electric charge "q".

**1-Area:**

Area = length × breadth

(Dimensions of area = dimensions of length × dimensions of breadth)

$= [L] \times [L] = [L^2]$.

- Dimensions of area are $[L^2]$.

**2-Volume:**

Volume = length × breadth × height

$= [L] \times [L] \times [L] = [L^3]$.

- Dimensions of volume are $[L^3]$.

**3- Density:**

$\text{Density} = \frac{\text{mass}}{\text{Volume}} = \frac{[M]}{[L^3]} = [M\,L^{-3}]$.

- Dimensions of density are $[ML^{-3}]$

**4- Speed or velocity:**

$\text{Speed} = \frac{\text{distance covered}}{\text{Time taken}} = \frac{[L]}{[T]} = [LT^{-1}]$

Dimensions of speed or velocity are $[LT^{-1}]$

**5- Acceleration:**

$\text{Acceleration} = \frac{\text{change in velocity } (\Delta v)}{\text{Time taken } (t)}$

$= \frac{[LT^{-1}]}{[T]} = [LT^{-1}] \times [T^{-1}] = [LT^{-2}]$.

- Dimensions of acceleration are $[LT^{-2}]$.

**6- Force (any type):**

Force = mass × acceleration

$= [M] \times [LT^{-2}] = [MLT^{-2}]$.

- Dimensions of force are $[MLT^{-2}]$

**7- Momentum:**

Momentum = mass × velocity

$= [M] \times [LT^{-1}] = [MLT^{-1}]$.

- Dimensions of momentum are $[MLT^{-1}]$.

**8- Work or energy (K.E, P.E any form)**

Work = Force × distance covered in the direction of force.

$= [MLT^{-2}] \times [L] = [ML^2T^{-2}]$.

- Dimensions of work or energy are $[ML^2T^{-2}]$
- $K.E = \frac{1}{2} m v^2$.

"$\frac{1}{2}$" is a dimensionless constant, hence the dimensions of K.E will be:

$= [M] \times [(LT^{-1})^2] = [ML^2T^{-2}]$

- Dimensions of K.E are $[ML^2T^{-2}]$.
- P.E = m g h.

Hence the dimensions of P.E will be:

$= [M] \times [LT^{-2}] \times [L] = [ML^2T^{-2}]$

- Dimensions of P.E are $[ML^2T^{-2}]$

This shows that dimensions of all forms of energy will be $[ML^2T^{-2}]$.

**9- Power:**

Power is rate of doing work or rate at which energy is converted from one form to another form.

$\text{Power} = \frac{\text{Work done or energy converted}}{\text{time}}$

Dimensions of work or energy= $[ML^2T^{-2}]$

$\therefore$ Dimensions of power $= \frac{[ML^2T^{-2}]}{[T]}$

Dimensions of power=$[ML^2T^{-2}] \times [T^{-1}] = [ML^2T^{-3}]$

- Dimensions of power are $[M\,L^2\,T^{-3}]$.

**10- Pressure:**

$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$

- Dimensions of pressure=$\frac{[MLT^{-2}]}{[L^2]}$

$= [MLT^{-2}] \times [L^{-2}] = [ML^{-1}T^{-2}]$

- Dimensions of pressure are $[ML^{-1}T^{-2}]$

**11- Torque:**

Torque = Force × moment arm.

Dimension of torque=$[MLT^{-2}] \times [L] = [ML^2T^{-2}]$

- Dimensions of torque are $[ML^2T^{-2}]$.

**12- Frequency:**

$\text{Frequency} = \frac{\text{Number of vibrations}}{\text{Time}}$

Dimensions of frequency = $\frac{\text{Dimensionless}}{\text{Dimension of time}}$

Dimensions of frequency = $\frac{\text{dimensionless}}{[T]}$

Dimensions of frequency = $\frac{1}{[T]} = [T^{-1}]$

- Dimensions of frequency are $[T^{-1}]$

**13- Wave length "λ".**

Wave length= distance between two consecutive crests or troughs.

- Dimensions of wave length are [T]

**14- Universal gravitational constant "G":**

According to law of gravitation:

$F = G\frac{m_1 m_2}{r^2} \qquad G = \frac{F r^2}{m_1 m_2}$

$\therefore$ Dimensions of G = $\frac{[MLT^{-2}]\times[L^2]}{[M][M]}$

$\therefore$ Dimensions of G= $[MLT^{-2}]\times[L^2]\times[M^{-2}]$

• Dimensions of Universal gravitational constant "G" are $[M^{-1}L^3T^{-2}]$.

**15- Plank's constant "$h$":**

According to quantum theory energy of each photon is: E = $h\nu$ $\therefore$ $h = \frac{E}{\nu}$

$\therefore$ Dimensions of $h = \frac{[ML^2T^{-2}]}{[T^{-1}]}$

$= [ML^2T^{-2}] \times [T]$

Dimensions of plank's constant are $[ML^2T^{-1}]$

**16- Angular velocity "ω":**

$\omega = \frac{\theta}{t}$

"θ" is a dimensionless quantity, hence

• Dimensions of ω are $= \frac{1}{[T]} = [T^{-1}]$.

**17- Angular acceleration:**

Angular acceleration $\alpha = \frac{\Delta\omega}{\Delta t}$

Dimensions of acceleration $= \frac{[T^{-1}]}{[T]}$

$= [T^{-1}] \times [T^{-1}] = [T^{-2}]$.

• Dimensions of angular acceleration are $[T^{-2}]$.

**18- Angular momentum "L":**

Angular momentum = m v r Sin θ

Sinθ is a dimensionless quantity, hence

Dimensions of angular momentum

$=[M]\times[LT^{-1}]\times[L]= [ML^2T^{-1}]$

• Dimensions of angular momentum are $[ML^2T^{-1}]$.

19- **Electric charge "q"**

Electric current is the rate of flow of electric charge through a cross section of a conductor:

Electric current = Electric charge / time

Electric charge = Electric current × time

Dimensions of charge is [A] and of time is [T]

Dimensions of electric charge = [A]×[T]

Dimensions of electric charge = [AT]

## ***Dimensional analysis*:**

## ***Basic rule for dimensional analysis*:**

While verifying the dimension vise correctness of an equation, the basic rule to be followed is that if an equation contains a number of terms which are to be added and/or subtracted, all terms must have the same dimensions.

### ***Prove that the following relations are dimensionally correct*:**

1. $v_f = v_i + a\,t$.
2. $S = v_i\,t + \frac{1}{2}\,a\,t^2$.
3. $2\,a\,S = v_f^2 - v_i^2$.

(1–3: Equations of uniformly accelerated rectilinear motion. (Equations of motion))

4. $E = m\,c^2$. 5. $E = h\,\nu$.
6. Total time of flight of a projectile (or total time taken by a projectile to move from point of projection to target) is given by: $t = 2\,V_o \sin\theta / g$
7. Maximum height reached by a projectile: $h = \frac{1}{2} V_o^2 \sin^2\theta / g$ or $h = V_o^2 \sin^2\theta / 2g$
8. $R = \frac{V_o^2 \sin 2\theta}{g}$ Horizontal range of a projectile.
9. $\nu\,\lambda = v$
10. Time period of a simple pendulum executing S.H.M (or when amplitude of vibrations is small). $T = 2\pi\sqrt{\frac{L}{g}}$
11. Frequency "$f$" with which a string must vibrate so that one loop is produced is given by:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

"$\mu$" is linear density of the string, "L" length of one loop and "T" is tension in the string.

12. Centripetal acceleration $a_c$: $a_c = \frac{v^2}{r}$
13. Centripetal acceleration $a_c$: $a_c = r\,\omega^2$
14. Centripetal acceleration $a_c$: $a_c = \frac{4\pi^2 r}{T^2}$
15. Centripetal force $F_c$: $F_c = \frac{m v^2}{r}$
16. Fringe spacing (Young's double slit experiment) $\Delta x = \lambda L/D$

## ***Solution:***

In order to prove that a physical relation is dimensionally correct we will have to show that both sides of an equation must have the same dimensions.

**1. $v_f = v_i + at$**

Substituting the dimensions of each quantity we get: $[LT^{-1}] = [LT^{-1}] + [LT^{-2}] \times [T]$

$[LT^{-1}] = [LT^{-1}] + [LT^{-1}]$

$\therefore \quad [LT^{-1}] = [LT^{-1}]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**2. $S = v_i t + \frac{1}{2} a t^2$**

"$\frac{1}{2}$" is a dimensionless quantity:

$[L] = [LT^{-1}] \times [T] + [LT^{-2}] \times [T^2]$

$[L] = [LT^0] + [LT^0]$ But $([T^0] = 1)$

Hence $[L] = [L] + [L] \quad \therefore \quad [L] = [L]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**3. $2aS = v_f^2 - v_i^2$**

"2" is a dimensionless quantity,

$[LT^{-2}] \times [L] = [(LT^{-1})^2] + [(LT^{-1})^2]$

$[L^2T^{-2}] = [L^2T^{-2}] + [L^2T^{-2}]$

$\therefore \quad [L^2T^{-2}] = [L^2T^{-2}]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**4. $E = mc^2$**

(Einstein's equation, "E" is energy, "c" is speed of light and "m" is mass converted into or produced from energy). Substituting the dimensions of each quantity we get: $[ML^2T^{-2}] = [M]\,[(LT^{-1})^2]$

$\therefore \quad [ML^2T^{-2}] = [ML^2T^{-2}]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**5. $E = h\nu$.**

According to quantum theory energy of each quantum (or photon) is given by this equation.

$[ML^2T^{-2}] = [ML^2T^{-1}] \times [T^{-1}]$

$\therefore \quad [ML^2T^{-2}] = [ML^2T^{-2}]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**6.** Total time of flight of a projectile is given by: $t = \dfrac{2 V_o \sin\theta}{g}$

"2 and Sin θ" are dimensionless quantities,

$[T] = \dfrac{[LT^{-1}]}{[LT^{-2}]}$

$[T] = [LT^{-1}] \times L^{-1}T^2$

$[T] = [L^0T^{-1}] \quad \therefore \quad [T] = [T]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**7.** Maximum height reached by a projectile:

$h = \frac{1}{2}\dfrac{V_o^2 \sin^2\theta}{g}$ **OR** $h = \dfrac{V_o^2 \sin^2\theta}{2g}$

"$\frac{1}{2}$ and Sin θ" are dimensionless quantities,

$[L] = \dfrac{[(LT^{-1})^2]}{[LT^{-2}]}$ OR $[L] = [L^2T^{-2}] \times [L^{-1}T^2]$

$[L] = [LT^0] \quad \therefore \quad [L] = [L]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**8. $R = \dfrac{V_o^2 \sin 2\theta}{g}$**

"Sin 2θ" is a dimensionless quantity,

$[L] = \dfrac{[(LT^{-1})^2]}{[LT^{-2}]}$ OR $[L] = [(LT^{-1})^2] \times [L^{-1}T^2]$

$[L] = [L^2T^{-2}] \times [L^{-1}T^2]$

$[L] = [LT^0] \quad \therefore \quad [L] = [L]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**9. $\nu\lambda = v$**

we get: $[T^{-1}] \times [L] = [LT^{-1}]$

$\therefore \quad [LT^{-1}] = [LT^{-1}]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**10.** Time period of a simple pendulum:

$T = 2\pi\sqrt{\dfrac{L}{g}}$ "2π" is a dimensionless quantity:

$[T] = \left\{\dfrac{[L]}{[LT^{-2}]}\right\}^{\frac{1}{2}}$ or $[T] = ([L] \times [L^{-1}T^2])^{\frac{1}{2}}$

$[T] = (T^2)^{\frac{1}{2}} \quad \therefore \quad [T] = [T]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**11. $f$ (or $\nu$) $= \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$**

This formula gives us the "fundamental frequency" (or frequency with which a string vibrates when one stationary wave is formed). "T" is tension in the string, "L" length of one loop and "μ" is the linear density (or mass per unit length) of the string. "2" is a dimensionless quantity:

$$[T^{-1}] = \frac{1}{[L]}\left\{\frac{[MLT^{-2}]}{[M]/[L]}\right\}^{\frac{1}{2}}$$

$$[T^{-1}] = [L^{-1}]\,([MLT^{-2}] \times [M^{-1}] \times [L])^{\frac{1}{2}}$$

$$[T^{-1}] = [L^{-1}]\,([M^0 L^2T^{-2}])^{\frac{1}{2}} \quad (M^0 = 1)$$

$$[T^{-1}] = [L^{-1}]\,([L^2T^{-2}])^{\frac{1}{2}} = [L^{-1}] \times L\,T^{-1}$$

$$[T^{-1}] = [L^{0}] \times [T^{-1}] \qquad [L^{0}] = 1$$

$\therefore \quad [T^{-1}] = [T^{-1}]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**12.** Centripetal acceleration $a_c$:

$$\mathbf{a_c = \frac{v^2}{r}}$$

$$[LT^{-2}] = \frac{([LT^{-1}])^2}{[L]} \quad \text{OR} \quad [LT^{-2}] = [L^2T^{-2}] \times [L^{-1}]$$

$\therefore \quad [LT^{-2}] = [LT^{-2}]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**13.** Centripetal acceleration $a_c$: $\mathbf{a_c = r\,\omega^2}$

$[LT^{-2}] = [L]([T]^{-1})^2 \quad \therefore \quad [LT^{-2}] = [LT^{-2}]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**14.** Centripetal acceleration $a_c$

$$\mathbf{a_c = \frac{4\,\pi^2 r}{T^2}}$$

"$4\pi^2$" is a dimensionless quantity:

$$[LT^{-2}] = \frac{[L]}{[T^2]}$$

$[LT^{-2}] = [L \times T^{-2}] \quad \therefore \quad [LT^{-2}] = [LT^{-2}]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**15.** Centripetal force $F_c$:

$$\mathbf{F_c = \frac{m\,v^2}{r}}$$

$$[MLT^{-2}] = \frac{[M]\,[(LT^{-1})^2]}{[L]}$$

$[MLT^{-2}] = [ML^2T^{-2}] \times [L^{-1}]$

$\therefore \quad [MLT^{-2}] = [MLT^{-2}]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

**16.** Fringe spacing (Young's double slit experiment)

$$\mathbf{\Delta x = \frac{\lambda\,L}{d}}$$

Fringe spacing "$\Delta x$" is the distance between two adjacent bright or dark fringes (its dimension will be L). Hence:

$$[L] = \frac{[L][L]}{[L]} \quad \text{OR} \quad [L] = [L^2] \times [L^{-1}]$$

$\therefore \quad [L] = [L]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

## ***Uncertainty***:

Uncertainty is the degree of doubt in the measurement or an estimate.
There is always a certain degree of uncertainty in measurement which may be caused by the equipment used or techniques employed. Uncertainty is the difference between actual value and the standard value.

Uncertainty in a measurement from a single instrument is half the least count of the instrument used.

## ***Fractional and percentage uncertainties***:

Fractional uncertainty in a measurement is given by:

$$\text{Fractional uncertainty} = \frac{\text{Uncertainty in the measurement}}{\text{Value of the measurement}}$$

Whereas percentage uncertainty is actually the fractional uncertainty which is expressed in percentage,

$$\text{Percentage uncertainty} = \frac{\text{Uncertainty in the measurement}}{\text{Value of the measurement}} \times 100\%$$

## ***Errors***:

Error in a measurement is the difference (discrepancy) between measured and true (or actual) value.

### ***Systemic errors***:

Errors that have a clear cause and can be eliminated are known as systemic errors.

## ***Types of systematic errors***:

***Instrumental errors, Environmental errors, Observational errors*** and ***Theoretical errors*** are four different types of systematic errors.

- ***Instrumental error*** is caused by improper or defective instrument.
- ***Environmental error*** is caused by the surrounding environment.
- ***Observational error*** is caused by observer taking inaccurate measurements.
- ***Theoretical error*** is caused by wrong theoretical assumptions.

***Random errors***: Errors that occur randomly and apparently may not have any cause. These errors are difficult to identify and therefore, are difficult to fix.

## ***Graph***:

Graph shows relationship between various quantities that is easy to understand and remember.

***Q.No. 1***: ***What is meant by proportional quantities?***

***Ans***: If changes in one quantity depend upon the changes in some other quantity then they are called proportional quantities.

The exact mathematical relation between these quantities may be different. One of the quantities is called ***independent variable***, Whereas the other quantity is called ***dependent variable***.

***Independent variable*** is that which we change or changes by itself. The independent variable is sometimes also called an **input variable**.

***Dependent variable*** is that which changes because of changes in the independent variable. Sometimes it is also called an **output variable**.

Different types of graphs can be used to represent the relationship between dependent and independent variables. Choice of graph depends upon the type of data and nature of relationship between the variables.

***Q.No. 2: What is meant by directly proportional quantities?***

***Ans***: If on changing one physical quantity the other ***one also changes in the same way and in the same proportion*** i.e. by ***increasing*** one quantity (independent variable) the other quantity (dependent variable) also ***increases*** or vice versa. (***In other words,*** if one quantity is doubled the other also doubles or if one quantity is halved the other also reduces to half.) then the quantities are said to be ***directly proportional*** to each other.

🗣 Graph between directly proportional quantities is a ***straight line***.

***Q.No. 3: What is meant by inversely proportional quantities?***

***Ans***: If on changing one physical quantity the other ***one changes in the opposite way in the same proportion*** i.e. by ***increasing*** one quantity the other quantity ***decreases*** or vice versa. ***In other words,*** if one quantity is doubled (increased two times) the other one decreases by two times (means it becomes half) and so on, then the quantities are said to be ***inversely proportional*** to each other.

## ***Significant figures***:

Significant figures in a measurement are defined as the figures in a number which can be realized (i.e. number of digits which can actually be measured accurately with a given instrument.).

Greater the number of significant figures in a measurement greater is the accuracy and smaller is the percentage error in that measurement.

The limit of accuracy in a measurement is equal to the least count of the measuring instrument used.

***Q.No.1: Give general rules to find the significant figures in a given number.***

***Ans***: Rules which are generally followed to find significant figure are:

1. All non-zero digits are significant.
2. Zeroes lying between non-zero digits are significant.
3. Zeros to the right of decimal point and to the right of a non-zero digit are significant.
4. Zeros to the left of first non-zero digit in a number are not significant.
5. Zeros at the end of a number after the decimal point but the last non-zero digit are significant.
6. When two or more numbers are used in a calculation, the number of significant figures in the answer will be equal to the smallest number of significant figures in any one of the original factors.

## ***Section–A Multiple-Choice Questions MCQs:***

***Q.No.1***: The respective number of significant figures for the numbers 23.023, 0.0003, and $2.1 \times 10^{-3}$ are:

(a) 5, 1, 2 (b) 5, 1, 5 (c) 5, 4, 2 (d)4, 4, 2

***Q.No.2***: Which among the following is the supplementary quantity:

(a) Mass (b) Time (c) Solid angle (d) Luminosity

***Q.No.3***: The unit of solid angle is:

(a) Second (b) Steradian (c) Kilogram (d) Candela

***Q.No.4***: The quantity having the same unit in all systems of unit is:

(a) Mass (b) Time (c) Length (d) Temperature

***Q.No.5***: Random errors can be eliminated by:

(a) Number of observations and their mean.
(b)Measuring the quantity with more than one instrument
(c) Eliminating the cause (d) careful observations

***Q.No.6***: Systemic error can be:

(a) Either positive or negative (b) Positive only
(c) Negative only (d) Zero error

***Q.No.7***: $[MLT^{-2}]$ is the dimensional formula of:

(a) Strain (b) Displacement. (c) Force (d) Pressure

***Q.No.8***: Which of the following pair has the same dimension?

(a) Moment of inertia and Torque
(b) Impulse and Momentum
(c) Surface tension and Force
(d) Specific heat and Latent heat

***Q.No.9***: Dependent variable is:

(a) Cause (b) Cause and effect
(c) Effect (d) Reason

***Q.No.10***: The dimension of force is: **(*2013 Karachi Board*)**

• $[MLT]$ • $[MLT^{-1}]$ • $[MLT^{2}]$ • $[MLT^{-2}]$

***Q.No.11***: The dimensions of torque are:

• $[ML^{2}T]$. • $[ML^{2}T^{-2}]$. • $[ML^{2}T^{2}]$. • $[MLT^{-2}]$.

**(*2014, 2008 Karachi board*)**

***Q.No.12***: The dimension of "G" is:

**(*2016,2015,2011,2003 Karachi board*)**

• $[ML^{-2}T^{3}]$ • $[M^{-1}L^{-2}T^{3}]$ • $[M^{-1}L^{3}T^{-2}]$ • $[M^{-1}L^{2}T^{2}]$

***Q.No.13***: The unit of luminous intensity is:

**(*2018,2014,2007 Karachi board*)**

• Decibel. • Candela. • Diopter. • $W/m^{2}$.

***Q.No.14***: The dimensions of angular velocity are:

(***2018, 2002 Karachi Board***)

• $[ML^{0}T^{-1}]$ • $[ML^{0}T^{-2}]$ • $[M^{0}L^{0}T^{-1}]$ • $[M^{0}L^{0}T^{-3}]$

***Q.No.15***: The dimensions of physical quantity "force" are: (***2019 Hyderabad Board***)

(a) $M^{0}L^{1}T^{1}$ (b) $M^{1}L^{1}T^{-1}$
(c) $M^{0}L^{1}T^{-1}$ (d) $M^{1}L^{1}T^{-2}$

***Q.No.16***: The quantity having the same unit in different systems of unit is:

(***2025 Karachi Board***)

• Mass • Time • Length • Temperature

## ***Answers*:**

(1) (c) 5, 4, 2
(2) Solid angle.
(3) Steradian.
(4) Time.
(5) Number of observations and their mean
(6) Either positive or negative.
(7) Force.
(8) Impulse and Momentum.
(9) Effect.
(10) $[MLT^{-2}]$
(11) $[ML^{2}T^{-2}]$.
(12) $[M^{-1}L^{3}T^{-2}]$
(13) Candela.
(14) $[M^{0}L^{0}T^{-1}]$
(15) $M^{1}L^{1}T^{-2}$
(16) Time.

## ***Numericals*:**

***Q.No.1***: **What is the percent uncertainty in the measurement 3.67± 0.25 m?**

***Solution***:

Value of measurement = 3.67 m

Uncertainty in measurement = ± 0.25 m

$$\% \text{ uncertainty} = \frac{\text{Uncertainty in measurement}}{\text{Value of measurement}} \times 100\%$$

$$\% \text{ uncertainty} = \frac{0.25}{3.67} \times 100\%$$

**% uncertainty = 6.81 %**

Percentage uncertainty is **<u>6.81 %</u>**

***Q.No.2***: **What is the area and its approximate uncertainty of a circle with radius $3.7\times10^4$cm.**

***Solution***: $r = 3.7\times10^4$cm. A =?

$A = \pi r^2 = 22/7 \times (3.7\times10^4)^2 = 4.3\times10^9$ cm$^2$.

Area of the given circle is **<u>$4.3\times10^9$ cm$^2$</u>.**

***Q.No.3***: **An aero plane travels at 850 km/h. How long does it take to travel 1.00 km?**

***Data***:

Speed of aero plane v = 850 km/h.

Distance d = 1.00 km. Time t = ?

***Solution***:

$v = \frac{d}{t}$ $t = \frac{d}{v} = \frac{1.00}{850} = 0.00118$ h=**4.23 sec**

Aero plane will take **<u>4.23 sec.</u>** to travel 1.00 km.

***Q.No.4***: **A rectangular holding tank 25 m in length and 15.0 m in width is used to store water for a short period of time in an industrial plant. If 2980 m$^3$ water is pumped into the tank. What is the depth of water?**

***Data***:

L = 25 m W = 15.0 m D = ?

Volume V = 2980 m$^3$.

***Solution***:

Since volume of the tank = volume of water stored

∴ V = length(L) width(W) depth(D) ∴ D = V / L W

$D = \frac{2980}{25 \times 15}$ = **7.95 m.**

Depth D of the tank is **<u>7.95 m</u>.**

***Q.No.5***: **Find the volume of a rectangular underground water tank which has storage facility of 1.9 m by 1.2 m by 0.8 m?**

***Data***:

L= 1.9 m. W = 1.2 m. D = 0.8 m.

***Solution***:

V = L W D

V = 1.9 × 1.2 × 0.8 = **1.824 m$^3$.**

Volume of the tank is **<u>1.824 m$^3$.</u>**

***Q.No.6***: **Two students derive the following equations in which "x" refers to the distance traveled, "v" the speed, "a" the acceleration and "t" time and the subscript "o" means a quantity at time t = 0.**

**(a) $x = v\, t^2 + 2at$ and**

**(b) $x = v_o\, t + 2at^2$**

**Which of these could possibly be correct according to dimensional analysis.**

***Solution***:

Dimensionally an equation will be correct if dimensions on both sides of equation are same.

Hence substituting dimensions of various quantities on both sides of given equations we get:

(a) $\mathbf{x = v\, t^2 + 2at}$ (2 is dimensionless)

$[L] = [LT^{-1}] \times [T^2] + [LT^{-2}]\times[T]$

$[L] = [LT^1] + [LT^{-1}]$

Since dimensions of both sides of this equation are **<u>not the same</u>** therefore, this equation is dimensionally **incorrect**.

(b) $\mathbf{x = v_o\, t + 2at^2}$ (2 is dimensionless)

$[L] = [LT^{-1}] \times [T] + [LT^{-2}]\times[T^2]$

$[L] = [LT^0] + [LT^0]$ $(T^0 = 1)$

$[L] = [L] + [L]$

$[L] = [L]$

Since both sides of above equation has **<u>same dimensions</u>** hence this equation is dimensionally **correct.**

***Q.No.7***: **One hectare is defined as $10^4$ m$^2$. One acre is $4\times10^4$ ft$^2$. How many acres are in one hectare? (1 m = 3.28 ft)**

***Data***:

1 hectare = $10^4$ m$^2$ 1 acre = $4\times10^4$ ft$^2$.

1 m =3.28 ft. No. of acres in 1 hectare=?

***Solution***:

Since 1 m = 3.28 ft. ∴ 1m$^2$ = $(3.28)^2$= 10.7584 ft$^2$

1 hectare = $10^4$ m$^2$ =10.7584 $\times10^4$ ft$^2$

No. of acres in 1 hectare = $\frac{10.7584 \times10^4}{4\times10^4}$ = **2.69**

Hence there are **2.69** acres in 1 hectare.

***Q.No.8*: A watch factory claims that its watches gain or lose not more than 10 seconds in a year. How accurate is this watch, express in percentage.**

***Solution*:**

Uncertainty = ± 10 sec. per year.

No. of seconds in one year = 365 × 24 × 60 × 60 = 31,536,000 seconds.

$$\% \text{ uncertainty} = \frac{10}{31{,}536{,}000} \times 100\%$$

% uncertainty = **0.0000 317 %** OR **$3.17 \times 10^{-5}$ %**

***Q.No.9*: Diameter of the moon is 3480 km. What is volume of the moon? How many moons would be needed to create a volume equal to that of the earth?**

**(Radius of earth = 6380 km)**

***Data*:**

Diameter of moon D = 3480 km

Radius of moon $R_m$ = 3480/2 = 1740 km.

Volume of moon V =?

Radius of the earth $R_e$ = 6380 km.

No. of moons equal to volume of the earth =?

***Solution*:**

Volume of moon $V_m = 4/3 \times \pi R_m^3$

$V_m = 4/3 \times 22/7 \times (1740)^3 = \mathbf{2.2075 \times 10^{10}\ km^3}$

Volume of earth $V_e = 4/3 \times \pi R_e^3$

$V_m = 4/3 \times 22/7 \times (6380)^3 = \mathbf{1.08824 \times 10^{12}\ km^3}$

No. of moons required to have volume equal to the volume of earth $= \frac{1.08824 \times 10^{12}}{2.2075 \times 10^{10}} = \mathbf{49.296}$

**49.296** moons will be required to have volume equal to the volume of the earth.

***Q.No.10*: Show that the following formulae are dimensionally correct:**

**(a) $V = f\lambda$ (b) $T = 2\pi\sqrt{\frac{m}{k}}$**

**(*2016, 2013 Karachi Board*)**

***Solution*:**

(a) $V = f\lambda$

Dimensions of "V", "$f$" and "$\lambda$" are $[LT^{-1}]$, $[T^{-1}]$ and [L] respectively. ∴ $[LT^{-1}] = [T^{-1}] \times [L]$

∴ $[LT^{-1}] = [LT^{-1}]$

Both sides of the equation have same dimensions, hence, the given equation is dimensionally correct.

(b) $T = 2\pi\sqrt{\frac{m}{k}}$

"k" represents the force constant or spring constant; it is given by:

$k = \frac{F}{x}$ (Hook's law)

∴ Dimensions of $k = \frac{[MLT^{-2}]}{[L]}$

∴ Dimensions of $k = [MLT^{-2}] \times [L^{-1}] = [ML^0T^{-2}]$

"2π" is a dimensionless quantity,

$[T] = \left\{\frac{[M]}{[ML^0T^{-2}]}\right\}^{\frac{1}{2}}$ ($[L^0] = 1$)

$[T] = ([M] \times [M^{-1}T^2])^{\frac{1}{2}}$

$[T] = (M^0\, T^2)^{\frac{1}{2}}$ ($[M^0] = 1$)

$[T] = [T]$

Since both sides of the equation have the same dimensions, therefore, the given formula is dimensionally correct.

***Q.No.11*: Show that the equation $f = \frac{1}{2l}\sqrt{\frac{F \times l}{m}}$ is dimensionally correct and find the dimensions of kinetic energy.**

(***2017 Karachi Board***)

***Solution*:**

"2" is a dimensionless quantity, hence:

$[T^{-1}] = \frac{1}{[L]}\left\{\frac{[MLT^{-2}][L]}{[M]}\right\}^{\frac{1}{2}}$

$[T^{-1}] = [L^{-1}]\,([ML^2T^{-2}] \times [M^{-1}])^{\frac{1}{2}}$

$[T^{-1}] = [L^{-1}]\,([M^0L^2T^{-2}])^{\frac{1}{2}}$ ($[M^0] = 1$)

$[T^{-1}] = [L^{-1}]\,([L^2T^{-2}])^{\frac{1}{2}}$

$[T^{-1}] = [L^{-1}] \times [L^1T^{-1}]$

$[T^{-1}] = [L^0T^{-1}]$ ($L^0 = 1$)

$[T^{-1}] = [T^{-1}]$

Since both sides of the equation have the same dimensions, therefore, the given equation is dimensionally correct.

Dimensions of K.E = $\frac{1}{2}$ m $v^2$.

"$\frac{1}{2}$" is a dimensionless constant, hence the dimensions of K.E will be:

$= [M] \times ([LT^{-1}])^2 = [ML^2T^{-2}]$

Dimensions of K.E are $[ML^2T^{-2}]$.

***Q.No.12*: Prove that the equation $T = 2\pi\sqrt{\frac{L}{g}}$ is dimensionally correct .**

(***2025 Karachi Board***)

***Solution*:**

Time period of a simple pendulum:

$$\mathbf{T = 2\pi\sqrt{\frac{L}{g}}}$$

"2π" is a dimensionless quantity:

$\therefore \quad [T] = \left\{\dfrac{[L]}{[LT^{-2}]}\right\}^{\frac{1}{2}}$

or $\quad [T] = ([L] \times [L^{-1}T^2])^{\frac{1}{2}}$

or $\quad [T] = [L]^{\frac{1}{2}} \times [L^{-1}T^2]^{\frac{1}{2}}$

or $\quad [T] = [L]^{\frac{1}{2}} \times [L]^{-\frac{1}{2}} \times [T^2]^{\frac{1}{2}}$

$[T] = [T^2]^{\frac{1}{2}} \quad \therefore \quad [T] = [T]$

Both sides of the equation have same dimensions, therefore, the given equation is dimensionally correct.

***Q.No.13*: Prove that the following equation is dimensionally correct .** (***2025 Karachi Board***)

$$F = \frac{m\,v^2}{r}$$

This is the formula for centripetal force.
Dimensions of any force = $[MLT^{-2}]$
Substituting the dimensions of F, m, v and r in the given equation we will get:

$\therefore \quad [MLT^{-2}] = \dfrac{[M]\ [LT^{-1}]^2}{[L]}$

$[MLT^{-2}] = [M]\,[L^2\,T^{-2}] \times [L^{-1}]$

$[MLT^{-2}] = [MLT^{-2}]$

Since both sides of the equation have the same dimensions, therefore, the given equation is dimensionally correct.

***Do you know that ..........***

Speed of light in vacuum is about $3 \times 10^8$ m/s.

***It means that*** it travels $3 \times 10^8$ m (300,000,000 meters or 300,000 kilometers) in one second.

$\therefore$ Distance traveled by light in one second = $3 \times 10^8$ m

Distance traveled by light in one minute = $3 \times 10^8 \times 60$ m

Distance traveled by light in one hour = $3 \times 10^8 \times 60 \times 60$ m

Distance traveled by light in one day (24 h)= $3 \times 10^8 \times 60 \times 60 \times 24$ m

Distance traveled in one year (365 days)=$3 \times 10^8 \times 60 \times 60 \times 24 \times 365$ m

Distance traveled in one year (365 days)= $9.46 \times 10^{15}$ m

- This distance traveled by light in one year is called "**Light year**".
- Hence: 1 light year = $9.46 \times 10^{15}$ m.

***Physics*** is the most ***logical, interesting*** and ***easy*** subject if you **Understand** it, if you don't then it becomes a ***difficult*** and ***boring*** subject.

Try to **understand** it you will never regret.

***This book is for those who want to study and* Understand *good quality physics.***

***Unit. 2*****…..** ✍

# Kinematics

## Scalars:

***Q.No.1*: What are scalars? Give some examples of scalars.**

***Ans*:** Scalars are physical quantities which ***do not have* direction**.

***Examples*:** Distance, speed, work, power, energy, time, temperature, electric charge, electric and magnetic flux, potential difference, density, volume, area, frequency, wave length, specific heat, etc.

Two or more scalars can be added, subtracted or multiplied like ordinary numbers.

## Vectors:

***Q.No.2*: What are vectors? Give some examples of vectors?**

***Ans*:** Vectors are those physical quantities which have ***magnitude as well as direction*** and ***obey vector algebra***.

- Hence vectors cannot be described completely without mentioning their direction.

***Examples:*** Velocity, displacement, acceleration, force, momentum, torque, angular displacement, angular Velocity, angular acceleration, angular momentum etc.

***Q.No.3*: How are vectors denoted?**

***Ans*:** Vectors are denoted by bold faced letters such as **A, B, C** etc., In hand written material it is a convention to put an arrow above the alphabet such as $\vec{A}$, $\vec{B}$, $\vec{C}$ etc.

***Q.No.4*: How is the magnitude of vector denoted?**

***Ans*:** The magnitude of a vector $\vec{F}$ is denoted by $|\vec{F}|$ it is read as "modulus of $\vec{F}$" or simply ***"mod. $\vec{F}$"***. It gives the absolute value of $\vec{F}$. The magnitude of a vector $\vec{F}$ is also denoted by F (without an arrow over the alphabet).

### *Representation of a vector*: (*Graphical representation*)

***Q.No.5*: How is a vector represented graphically?**

***Ans*:** Graphically a vector is represented by a directed line segment. The length of this straight line corresponds to the magnitude of the vector according to a suitable scale and it points in the direction of the vector.

## Addition of vectors:

***Q.No.1*: How are vectors added?**

***Ans*:** Because of their direction, vectors **cannot be** added as ordinary numbers. While adding two or more vectors their direction has to be considered. Hence vectors can be added by head – tail rule (Geometrical method) or by trigonometric method (Rectangular component method).

***Q.No.2*: What is *resultant* vector?**

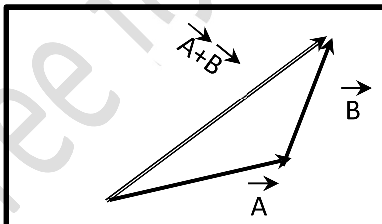
***Ans*:** Resultant of two or more vectors is a ***single vector*** whose ***magnitude*** and ***direction*** are such that it produces the ***same effect*** as the combined effect of given vectors.

## *Geometrical method* (*head to rule*):

***Q.No.3*: Describe head to tail rule method of vector addition?**

***Ans*:** In head to tail rule of vector addition, tail of second vector is joined with the head of first vector, tail of third is joined with head of second vector and so on, in this manner all the vectors to be added are joined to-gather. Their sum vector or their resultant vector is obtained by joining the head of last vector with the tail of first vector by a straight line. Length of this straight line corresponds to the ***magnitude*** of resultant an arrow is put at the end of this line joined with the head of last vector it shows its ***direction***.

Hence: $\vec{A} + \vec{B} = \vec{R}$




***Head to tail rule***

## *Properties of vector addition*:

Addition of vectors obeys:

(1) ***Commutative law***.
(2) ***Associative law***.

## *Commutative law of vector addition*:

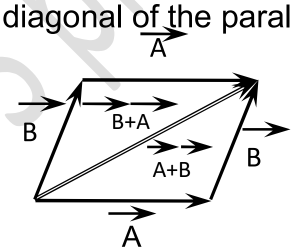
***Q.No.1*: Show that $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.**

**OR**

**Prove commutative law for vector addition?**

***Ans*:** To prove that $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. OR To prove commutative law for vector addition, both sides of the above equation must give the same resultant.

To find $\vec{A} + \vec{B}$, according to head to tail rule, head of $\vec{A}$ is joined with the tail of $\vec{B}$, but to find $\vec{B} + \vec{A}$ we will join the head of $\vec{B}$ with the tail of $\vec{A}$. We can see from the following diagram that in both the cases same diagonal of the parallelogram is obtained as their resultant.



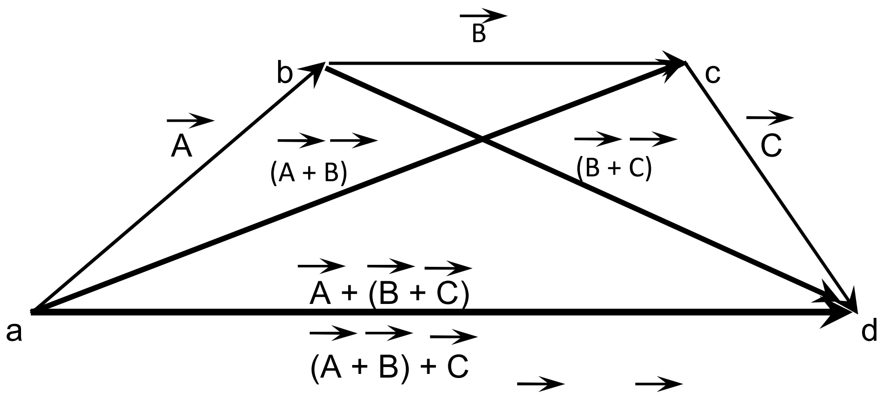

$\therefore$ $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. Proved.

Hence vectors can be added in ***any order***. In other words, we can say that vector addition obeys ***commutative law***.

## *Associative law*:

***Q.No.2*: Show that $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$**

***Ans*:** To prove that $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$. OR To prove associative law for vector addition, first of all we will add vectors "$\vec{A}$" and "$\vec{B}$" their resultant "$\vec{ac}$" will then be added to "$\vec{C}$", their resultant "$\vec{ad}$" will be obtained i.e. $(\vec{A} + \vec{B}) + \vec{C}$




Similarly for getting R.H.S, we add vector "$\vec{B}$" and "$\vec{C}$" first and their resultant represented by "$\vec{bd}$" is then added to vector "$\vec{A}$" we will get their resultant represented by "$\vec{ad}$".
This will give us the R.H.S of the above equation i.e. $\vec{A} + (\vec{B} + \vec{C})$.

Since the same resultant "$\vec{ad}$" is obtained, therefore, addition of vectors obeys associative law. It means that vectors can be added making any group.

Hence: $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$. Proved.

- Vectors can be added in any ***grouping***.

***Q.No.3*: What is the resultant of vectors having the same direction?**

***Ans*:** If two or more vectors to be added are in the same direction then the magnitude of their resultant will be equal to the sum of magnitudes of the individual vectors, whereas it's direction is same as the direction of original vectors.

***Q.No.4*: What is the resultant of two opposite vectors?**

***Ans*:** Magnitude of resultant of two opposite vectors will be equal to the difference of magnitudes of individual vectors, whereas the direction of resultant is same as the direction of the larger vector.

- The resultant of two vectors equal in magnitude but opposite in direction is a ***null*** vector (vector whose magnitude is zero).

***Q.No.5*: What is the resultant of a vector and its negative vector?**

***Ans*:** Resultant of a vector and its negative is a null vector.

***In other words***, resultant of two vectors equal in magnitude but opposite in direction is always a null vector.

## ***Magnitude of resultant vector*: (*Law of Cosine*)**

If $\vec{A} + \vec{B} = \vec{R}$, where $\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$, and If "$\theta$" is the angle between them, then the magnitude of the resultant vector "$\vec{R}$" or magnitude of sum of two vectors $\vec{A}$ and $\vec{B}$ can be determined by ***Law of Cosine***, according to which:

$$\therefore \quad R^2 = A^2 + B^2 + 2\,A\,B\,\text{Cos}\,\theta$$

$$\therefore \quad R = \sqrt{A^2 + B^2 + 2\,A\,B\,\text{Cos}\,\theta}$$

OR

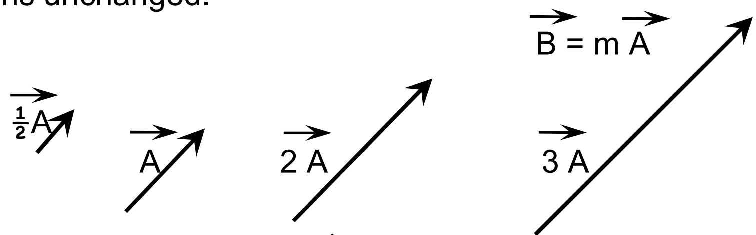
$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2\,A\,B\,\text{Cos}\,\theta}$$

***Note that*** "$\theta$" is the angle between $\vec{A}$ and $\vec{B}$ when their tails are at the same point.

Above equations give us the magnitude of resultant of two vectors $\vec{A}$ and $\vec{B}$ or they give us the magnitude of sum of two vectors $\vec{A}$ and $\vec{B}$.

## Multiplication of a vector by a positive number:

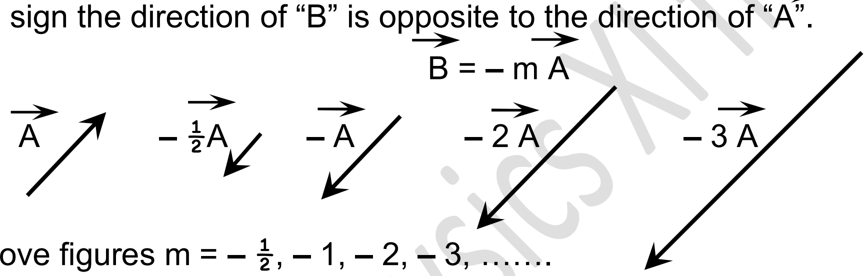
When a vector "$\vec{A}$" is multiplied by a positive number "m" a new vector "$\vec{B}$" is obtained, whose magnitude is equal to "m" times the magnitude of the original vector "$\vec{A}$" it's direction remains unchanged.




In the above figures m = $\frac{1}{2}$, 1, 2, 3, …….

## Multiplication of a vector by a negative number:

When a vector "$\vec{A}$" is multiplied by a negative number "– m" the magnitude of new vector "$\vec{B}$" thus obtained is equal to "m" times the magnitude of the original vector "$\vec{A}$" but because of the negative sign the direction of "$\vec{B}$" is opposite to the direction of "$\vec{A}$".




In the above figures m = $-\frac{1}{2}$, – 1, – 2, – 3, …….

- Hence if a negative sign is placed before a vector it's direction ***reverses***, it's direction changes by an angle of ***180*** °.

***Note that*** in the above multiplications vectors $\vec{A}$ and $\vec{B}$ are of the same type, having the same unit. For example, if a force '$\vec{F}$' is multiplied by "2" we get "2 $\vec{F}$" which is also a force whose magnitude is twice the magnitude of "$\vec{F}$" its unit is same as the unit of $\vec{F}$.

## Division of a vector by a number:

***Q.No.1: Describe division of a vector by a number.***

***Ans:*** Division a vector "$\vec{A}$" by a non-zero number "n" is equivalent to the multiplication of "$\vec{A}$" by "m" where "m" is the reciprocal of "n" (m = 1/n).

$$\frac{\vec{A}}{n} = m\vec{A}$$

The direction of vector "m$\vec{A}$" depends upon the sign of "m".

## Unit vectors:

***Q.No.1: What is a unit vector?***

***Ans:*** A Unit vector is that whose magnitude is **"1" (*one*)**.

- A unit vector is used to specify direction.
- Unit vectors are represented by small alphabets with a cap "^" on it.

***Q.No.2*: How is a unit vector calculated?**

***Ans*:** Unit vector "$\hat{a}$" parallel to a given vector $\vec{A}$ can be calculated by dividing the given vector "$\vec{A}$" by its magnitude A. Hence

$$\text{Unit vector} = \frac{\text{vector}}{\text{Magnitude of the vector}}$$

$$\hat{a} = \frac{\vec{A}}{A}$$

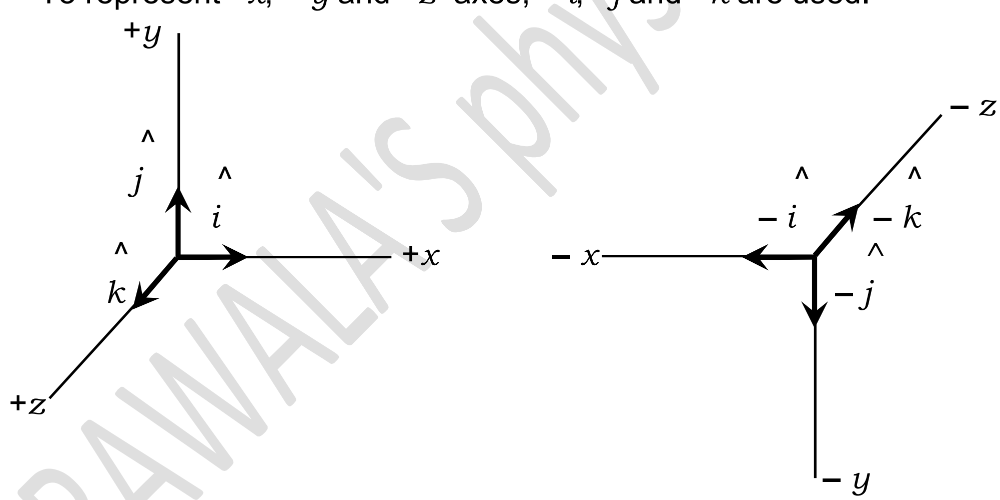
***In other words***, any vector "$\vec{A}$" can be split into two parts, one part A represents it's magnitude and the other part "$\hat{a}$" represents it's direction.

$$\vec{A} = A\,\hat{a}$$

***Q.No.3*: What are rectangular unit vectors or $\hat{i}, \hat{j}$ and $\hat{k}$?**

***Ans*:** In order to represent the three-dimensional coordinate system three mutually perpendicular vectors are used. These three mutually perpendicular vectors, called ***rectangular unit vectors*** are $\hat{i}, \hat{j}$ and $\hat{k}$. It means that their magnitude is "1" but $\hat{i}$ is along $+x$-axis, $\hat{j}$ is along $+y$-axis and $\hat{k}$ is along +z-axis. Hence they are used to represent $x$, $y$ and $z$-axes.

To represent $-x$, $-y$ and $-z$-axes, $-\hat{i}$, $-\hat{j}$ and $-\hat{k}$ are used.




***Q.No.4*: What is a *free vector*?**

***Ans*:** A free vector is that vector which can be moved parallel to itself without changing its magnitude and/or direction and applied at any point.

***Q.No.5*: What is a *position vector*?**

***Ans*:** A position vector is that vector which is used to give position of a point or anything with respect to a fixed point, such as the origin of a coordinate system (***It means that*** it tells you how far and in what direction is a point from the origin of a coordinate system).

Position vector is usually represented by $\vec{r}$. Components of a position vector are called its ***coordinates***. In terms of its coordinates a position vector is given by:

$$\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

In terms of magnitude of its coordinates, the magnitude of position vector is given by:

$$|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$$

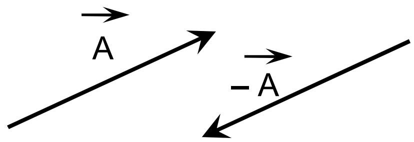
$|\vec{r}| = r$ is the magnitude of position vector.

- Position vector is actually the distance of a given point from the origin, in a particular direction (its M.K.S unit is meter).

***Q.No.6: What is negative of a vector?***

***Ans:*** Negative of a vector is another vector whose magnitude is same but the direction is opposite.

***Negative of a vector:***




***Q.No.7: What is null vector?***

***Ans:*** A null vector is that vector whose magnitude is ***zero***.

# Resolution of vectors:

***Q.No.1: What is resolution of vectors?***

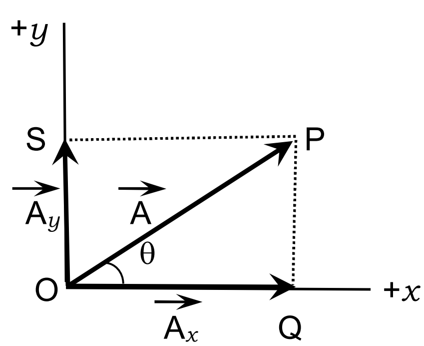
***Ans:*** The process of splitting a vector into two or more components (parts) is called resolution of vectors. Once the process of resolution is complete, the vector is said to have been resolved.

- Two dimensional vectors are resolved into two mutually perpendicular components, one along $x$- and the other along $y$-axis. Similarly, a three-dimensional vector can be resolved into three mutually perpendicular components, along $x$-, $y$- and $z$-axes.
- Since these components are mutually perpendicular and along sides of a rectangle, hence these components are also known as the ***rectangular components***.
- ***Note that*** a component along a given axis represents the effective value of the vector in that particular direction.

***Q.No.2: Derive formulae for rectangular components of a two dimensional vector $\vec{A}$ making an angle "θ" with +x-axis?***

***Ans:*** Let a vector "$\vec{A}$" be represented by a directed line segment "$\overrightarrow{OP}$" making an angle "θ" with $+x$-axis as shown.

Angle "θ" is taken in anticlockwise sense starting from +x-axis.



If perpendiculars are drawn from the head of $\vec{A}$ on the two axes then "$\vec{OQ}$" represents "$\vec{A_x}$" $x$-component of vector $\vec{A}$ and "$\vec{OS}$" represents "$\vec{A_y}$" $y$- component of vector $\vec{A}$.

Since $\vec{QP} = \vec{OS}$, Therefore, $\vec{A_y}$ can also be represented by QP.

In $\Delta$ OQP: $\cos\theta = \dfrac{OQ}{OP}$ But $OQ = |\vec{A_x}| = A_x$, $OP = |\vec{A}| = A$

$\therefore$ $\cos\theta = \dfrac{A_x}{A}$

$$\boxed{A_x = A\cos\theta}$$

In terms of its magnitude and direction (using a rectangular unit vector "$\hat{i}$") $x$-component of any vector "$\vec{A}$" is given by:

$$\boxed{\vec{A_x} = A\cos\theta\ \hat{i}}$$

Similarly, in $\Delta$ OQP:

$\sin\theta = \dfrac{QP}{OP}$ But $QP = |\vec{A_y}| = A_y$, $OP = |\vec{A}| = A$

$\therefore$ $\sin\theta = \dfrac{A_y}{A}$

$$\boxed{A_y = A\sin\theta}$$

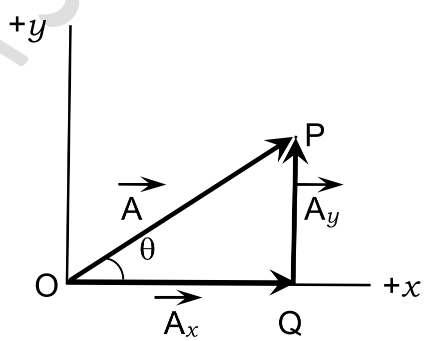
In terms of its magnitude and direction (using a rectangular unit vector "$\hat{j}$") $y$-component of any vector "$\vec{A}$" is given by:

$$\boxed{\vec{A_y} = A\sin\theta\ \hat{j}}$$

## ***Composition of a vector with the help of its components***:

If components of a vector are known then the vector itself can be determined by taking the vector sum of its components.

Hence if $\vec{A_x}$ and $\vec{A_y}$ are the of components of a vector "$\vec{A}$", then by head to tail rule, if head of $\vec{A_x}$ is joined with tail of $\vec{A_y}$ their resultant will be vector "$\vec{A}$", as shown:




## ***Magnitude of $\vec{A}$***:

To find the magnitude of vector $\vec{A}$, consider $\Delta$OPQ: In this triangle, according to Pythagoras theorem: $(OP)^2 = (OQ)^2 + (QP)^2$

But $OP = A$ (magnitude of vector $\vec{A}$)

$OQ = A_x$ (magnitude of $x$-component $\vec{A_x}$)

$QP = A_y$ (magnitude of $y$-component $\vec{A_y}$)

$\therefore \quad A^2 = A_x^2 + A_y^2 \quad \therefore \quad \boxed{A = \sqrt{A_x^2 + A_y^2}}$

This formula gives us the magnitude of a two-dimensional vector "A" in terms of magnitudes of its components $A_x$ and $A_y$.

## Direction of $\vec{A}$:

To find the direction of the vector $\vec{A}$, or ***in other words*** the angle "θ" which it makes with $+x$-axis, consider ΔOPQ, In this triangle,

$\tan\theta = \dfrac{QP}{OQ}$ But $OQ = A_x$ (magnitude of $x$-component $\vec{A_x}$)

$QP = A_y$ (magnitude of $y$-component $\vec{A_y}$)

$\therefore \quad \tan\theta = \dfrac{A_y}{A_x} \quad \therefore \quad \boxed{\theta = \tan^{-1}\dfrac{A_y}{A_x}}$

***Q.No.3: How is magnitude of a three-dimensional vector calculated using its rectangular components?***

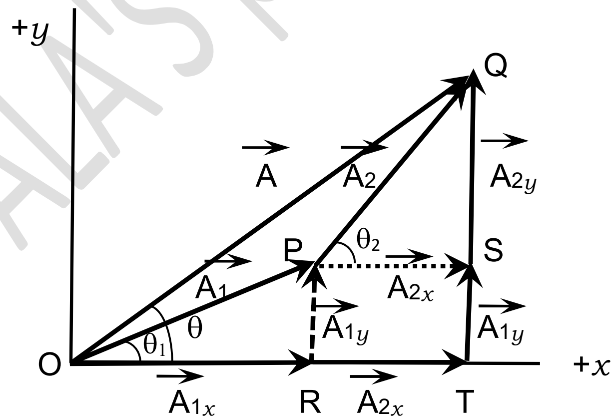
***Ans:*** If $A_x$, $A_y$ and $A_z$ are magnitude of components of a vector, then magnitude of the vector $\vec{A}$ can be calculated by: $\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}}$

## Addition of vectors by rectangular components:

In order to understand the addition of vectors by rectangular component method, consider two vectors "$\vec{A_1}$" making an angle "$\theta_1$" and "$\vec{A_2}$" making an angle "$\theta_2$" with +x-axis.

First of all, we will follow head to tail rule to add them and then we will add them by their rectangular components. We must get the same resultant (answer vector) by both the methods.

To add these vectors by head to tail rule method, we will join the head of $\vec{A_1}$ with the tail of $\vec{A_2}$. Their resultant "$\vec{A}$" will be obtained by joining the tail of $\vec{A_1}$ with the head of $\vec{A_2}$. Hence line $\vec{OQ}$ represents their resultant $\vec{A}$ making an angle "θ" with x-axis.




Now if we resolve $\vec{A_1}$ and $\vec{A_2}$ into $x$- and $y$-components, we get:

$\vec{OR} = A_{1x}\hat{i} = A_1 \cos\theta_1 \hat{i} \qquad \vec{PS} = A_{2x}\hat{i} = A_2 \cos\theta_2 \hat{i}$

$\vec{RP} = A_{1y}\hat{j} = A_1 \sin\theta_1 \hat{j} \qquad \vec{SQ} = A_{2y}\hat{j} = A_2 \sin\theta_2 \hat{j}$

∴ OT = OR + RT

But: RT = PS = $A_{2x}$ and OT = $A_x$

∴ OT = OR + RT

∴ $A_x = A_{1x} + A_{2x}$

OR $A_x = A_1 \cos\theta_1 + A_2 \cos\theta_2$.

Similarly: TQ = TS + SQ

But TS = RP = $A_{1y}$ and SQ = $A_{2y}$

∴ $A_y = A_{1y} + A_{2y}$

OR $A_y = A_1 \sin\theta_1 + A_2 \sin\theta_2$

$A_x$ and $A_y$ represent the magnitudes of rectangular components of the resultant vector "$\vec{A}$". Hence the ***magnitude*** of the resultant vector "$\vec{A}$" is determined by:

$$\boxed{A = \sqrt{A_x^2 + A_y^2}}$$

Similarly, the ***direction*** of the resultant vector $\vec{A}$, or ***in other words*** the angle which it makes with +x-axis, is given by:

∴ $$\boxed{\theta = \tan^{-1}\frac{A_y}{A_x}}$$

The resultant vector obtained by this method is represented by $\vec{OQ}$, making an angle "θ" with +$x$-axis.

- Hence whether vectors are added by head to tail rule or with the help of their rectangular components, we get the same resultant.
- Addition of vectors by rectangular component method is more accurate, it is especially use full in adding three dimensional vectors.
- By this method ***any number*** of vectors can be added at a time, in this method of addition of vectors diagrams are not needed, values of trigonometric ratios at various angles to calculate values of components are needed.

## ***Steps to be taken for addition of vectors by rectangular component method:***

1. Resolve each given vector into its $x$- and $y$-components.
2. Add magnitude of all $x$-components, with their + or – signs, to get the magnitude of $x$-component $A_x$ of the resultant:

∴ $A_x = A_{1x} + A_{2x} + A_{3x} + \ldots\ldots\ldots\ldots\ldots$

OR $A_x = A_1 \cos\theta_1 + A_2 \cos\theta_2 + A_3 \cos\theta_3 + \ldots\ldots\ldots$

3. Add magnitude of all the $y$-components, with their + or – signs, to get the magnitude of $y$-component $A_y$ of the resultant:

∴ $A_y = A_{1y} + A_{2y} + A_{3y} + \ldots\ldots\ldots\ldots\ldots$

OR $A_y = A_1 \cos\theta_1 + A_2 \cos\theta_2 + A_3 \cos\theta_3 + \ldots\ldots\ldots$

## ***Magnitude of the resultant:***

Now calculate the magnitude of resultant A by:

$$A = \sqrt{A_x^2 + A_y^2}$$

OR $$\boxed{A = \sqrt{A_x^2 + A_y^2}}$$

## ***Direction of the resultant:***

To find the direction of resultant $\vec{A}$, calculate the angle "θ" which the resultant makes with +$x$-axis by:

$$\theta = \tan^{-1} \frac{A_y}{A_x}$$

🗣 ***Note that*** the above formula will give us the angle which the resultant makes with +$x$-axis in the first quadrant in anticlockwise sense.

🗣 ***Also note that*** the positive or negative sign of a component of the resultant depends upon the location of the resultant vector in first, second, third or fourth quadrants. If the resultant vector is in any quadrant, other than the first quadrant, then it's direction must be corrected.

# ***Product of vectors:***

## ***Scalar or dot product:***

***Q.No.1: What is scalar or dot product of vectors? Give some examples.***

***Ans:*** If multiplication of two vectors results in a ***scalar quantity*** then such type of multiplication of vectors is known as ***scalar product***.

OR

When two parallel vectors are multiplied then the quantity obtained will be a ***scalar***, this type of multiplication is known as ***scalar product***.

Since this type of product of vectors is represented by a dot placed between vectors to be multiplied, hence it is also called ***dot product***.

***Examples.1:***

Work is the scalar or dot product of force "$\vec{F}$" and the resulting displacement "$\vec{d}$". It is a scalar quantity, although it is the product of two vectors.

$$\text{Work} = \vec{F} \cdot \vec{d}$$

***Examples.2:***

Electric flux "ϕ" is the number of lines of electric force passing normally through a surface, it is a scalar quantity. It is equal to the scalar or dot product of electric intensity (or strength of electric field) "$\vec{E}$" and vector area "$\Delta\vec{A}$" of the surface.

$$\text{Electric flux } \phi = \vec{E} \cdot \Delta\vec{A}$$

***Examples.3:***

Power is the scalar product of force "$\vec{F}$" and velocity "$\vec{v}$". Power is a scalar quantity.

$$\text{Power} = \vec{F} \cdot \vec{v}$$

***Q.No.2: What is the formula for finding the magnitude of scalar quantity obtained in scalar or dot product.***

***Ans:*** Formula for finding the magnitude of scalar quantity obtained in dot product of two vectors is:

$$\vec{A} \cdot \vec{B} = A\,B \cos\theta$$

Where "θ" is the angle between vectors $\vec{A}$ and $\vec{B}$.

***Q.No.3**: Show that dot product of two non-zero vectors is zero when vectors are perpendicular to each other.*

***Ans**:* Magnitude of scalar or dot product of any two vectors is given by:

$$\vec{A} \cdot \vec{B} = A\,B\,\cos\theta$$

If given vectors are perpendicular to each other, angle "θ" between them will be 90°:

$$\vec{A} \cdot \vec{B} = A\,B\,\cos 90° \qquad (\cos 90° = 0)$$

$$\vec{A} \cdot \vec{B} = 0$$

Hence if scalar product of two non-zero vectors is zero then the vectors are perpendicular to each other (θ = 90°).

***Q.No.4**: Show that $\hat{i} \cdot \hat{j} = 0$.*

***Ans**:* According to the definition of dot product:

$$\hat{i} \cdot \hat{j} = i\,j\,\cos\theta$$

Since "$\hat{i}$" is along $x$-axis and "$\hat{j}$" is along $y$-axis, hence the vectors to be multiplied are ***perpendicular*** to each other and angle "θ" between them is 90°. Therefore,

$$\hat{i} \cdot \hat{j} = i\,j\,\cos 90° \quad (\cos 90° = 0) \qquad \text{OR} \qquad \hat{i} \cdot \hat{j} = 1 \times 1 \times 0$$

$$\therefore \quad \boxed{\hat{i} \cdot \hat{j} = 0}$$

Similarly: $\hat{j} \cdot \hat{k} = 0$ and $\hat{k} \cdot \hat{i} = 0$.

***Q.No.5**: At what angle is value of scalar or dot product of two vectors of given magnitude maximum.*

***Ans**:* Magnitude of scalar or dot product of any two vectors is given by:

$$\vec{A} \cdot \vec{B} = A\,B\,\cos\theta$$

Maximum value of Cos θ is "1" when "θ" is 0° or when vectors are ***parallel*** to each other. Hence:

$$\vec{A} \cdot \vec{B} = A\,B\,\cos 0° \qquad (\cos 0° = 1)$$

$$\vec{A} \cdot \vec{B} = A\,B$$

***Hence*** scalar product of two vectors is maximum when vectors are ***parallel*** to each other or the angle between them is "0°".

***Q.No.6**: Show that $\vec{A} \cdot \vec{A} = |\vec{A}|^2$ OR $\vec{A} \cdot \vec{A} = A^2$.*

***Ans**:* Since the angle between vectors to be multiplied $\vec{A}$ and $\vec{A}$ is zero, therefore,

$$\vec{A} \cdot \vec{A} = A\,A\,\cos 0° \qquad (\text{Sine } \vec{A} \parallel \vec{A})$$

$$\vec{A} \cdot \vec{A} = A^2 \qquad (\cos 0° = 1)$$

- Hence dot product of a vector by itself is equal to the square of magnitude of the vector.

***Q.No.7**: Show that $\hat{i} \cdot \hat{i} = 1$.*

***Ans**:* According to the definition of dot product:

$$\hat{i}\cdot\hat{i} = i\,i\,\text{Cos}\,\theta \qquad (\theta = 0°,\ \hat{i} \parallel \hat{i},\ i = 1)$$

$\hat{i}\cdot\hat{i} = 1\times1\times \text{Cos}\,0°$ OR $\hat{i}\cdot\hat{i} = 1\times1\times1$ (Cos 0°= 1)

$\therefore$ $\boxed{\hat{i}\cdot\hat{i} = 1}$

Similarly, $\hat{j}\cdot\hat{j} = 1$ and $\hat{k}\cdot\hat{k} = 1$.

## ***Laws obeyed by scalar or dot product:***

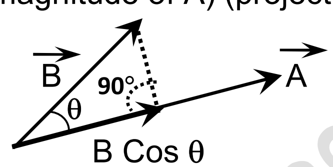
### ***Commutative law:***

***Q.No.1: Show that $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$. OR***

***Show that scalar product obeys commutative law.***

***Ans***: Scalar or dot product of two vectors is basically equal to the magnitude of first vector multiplied by the projection of second vector on the first vector.

i.e. $\vec{A}\cdot\vec{B}$ = (magnitude of $\vec{A}$) (projection of $\vec{B}$ on $\vec{A}$)




To find $\vec{A}\cdot\vec{B}$ we will multiply the magnitude of $\vec{A}$ by the projection of $\vec{B}$ on $\vec{A}$. But the projection of $\vec{B}$ on $\vec{A}$ is B Cos θ, where "θ" is the angle between vectors $\vec{A}$ and $\vec{B}$, hence:

$$\vec{A}\cdot\vec{B} = A\,B\,\text{Cos}\,\theta$$

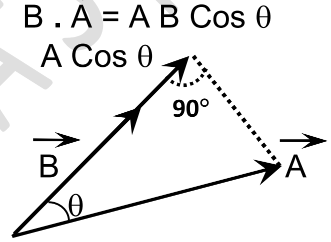
Similarly, $\vec{B}\cdot\vec{A}$ = (magnitude of $\vec{B}$)(projection of $\vec{A}$ on $\vec{B}$)

In the following diagram we can see that the projection of $\vec{A}$ on $\vec{B}$ is A Cos θ. Hence:

$$\vec{B}\cdot\vec{A} = (B)(A\,\text{Cos}\,\theta)$$

$$\vec{B}\cdot\vec{A} = A\,B\,\text{Cos}\,\theta$$

Projection of $\vec{A}$ on $\vec{B}$ is taken by dropping a perpendicular from the head of $\vec{A}$ in the direction of B




Right hand side of both equations is same, therefore, $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$.

- Scalar or dot product obeys commutative law i.e. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$.
- It means that the scalar product of vectors can be taken in any ***order***.

***Q.No.2: What is the scalar product of vectors opposite to each other?***

***Ans***: By definition magnitude of scalar product of two vectors is given by:

$$\vec{A}\cdot\vec{B} = A\,B\,\text{Cos}\,\theta.$$

In this case the angle between $\vec{A}$ and $\vec{B}$ is 180°

$$\vec{A}\cdot\vec{B} = A\,B\,\text{Cos}\,180° \qquad (\text{Cos}\,180° = -1)$$

$$\vec{A} \cdot \vec{B} = -AB$$

Scalar or dot product of vectors opposite to each other is ***negative***.

***Q.No.3: Give scalar or dot product of vectors in terms of their rectangular components.***

***Ans:*** In terms of their rectangular components, dot product of any two vectors is given by:

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_k \hat{k}) . (B_x \hat{i} + B_y \hat{j} + B_k \hat{k})$$

$$\vec{A} \cdot \vec{B} = (A_x B_x + A_y B_y + A_k B_k)$$

(This result is obtained by taking $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$ and $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$)

## ***Vector or cross product:***

***Q.No.1: What is vector or cross product?***

***Ans:*** If multiplication of two vectors results in a ***vector quantity*** then such type of multiplication of vectors is known as ***vector product.***

OR

When two perpendicular vectors are multiplied then the quantity obtained will be a ***vector***, this type of multiplication is known as ***vector product***.

Since this type of product of vectors is represented by a cross placed between vectors to be multiplied hence it is also called ***cross product***.

$$\therefore \quad \boxed{\vec{A} \times \vec{B} = \vec{C}}$$

- The new vector "$\vec{C}$" thus obtained is known as ***product vector***.

***Q.No.2: What is the magnitude of vector product?***

***Ans:*** The magnitude of vector which is obtained as a result of vector product of vectors $\vec{A}$ and $\vec{B}$ (It is also called product vector) is given by:

OR $$\boxed{\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}}$$

Where "$\hat{n}$" is a unit vector it shows the direction of product vector, "θ" is the angle between vectors $\vec{A}$ and $\vec{B}$.

***Q.No.3: How is the direction of product vector found?***

***Ans:*** Direction of the product vector is found with the help of right-hand rule.

***Q.No.4: State right hand rule.***

***Ans:*** According to right hand rule, first of all tails of representative lines of vectors to be multiplied is joined to gather. Then rotate the vector which appears first in the product towards the second through smaller of the two possible angles. Then curl the fingers of right hand in the direction of rotation, stretch the thumb it will point in the direction of the product vector.

The product vector is always ***perpendicular*** to the plane formed by the original vectors $\vec{A}$ and $\vec{B}$, it is always perpendicular to both $\vec{A}$ as well as $\vec{B}$.

***Q.No.5: Show that cross product of two vectors is zero when vectors are parallel to each other.***

***Ans:*** Vector or cross product of any two vectors is given by:

$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$$

If given vectors are parallel to each other, angle "θ" between them will be 0°:

$$\vec{A} \times \vec{B} = A\,B\,\text{Sin}\,0°\,\hat{n} \qquad (\text{Sin}\,0° = 0)$$

$$\vec{A} \times \vec{B} = 0$$

Hence if vector product of two non-zero vectors is zero then the vectors are ***parallel*** or ***opposite*** to each other (θ = 0°, θ = 180°).

***Q.No.6*: Show that $\vec{A} \times \vec{A} = 0$**

***Ans***: By definition: $\vec{A} \times \vec{A} = A\,A\,\text{Sin}\,\theta\,\hat{n}$

∴ $\vec{A} \times \vec{A} = A\,A\,\text{Sin}\,0°$ ($\vec{A}$ and $\vec{A}$ are parallel to each other)

∴ $\boxed{\vec{A} \times \vec{A} = 0}$ (Since Sin 0° = 0)

- Hence vector product of a any vector $\vec{A}$ by itself will be zero.

Similarly, $\hat{i} \times \hat{i} = 0$, $\hat{j} \times \hat{j} = 0$ and $\hat{k} \times \hat{k} = 0$.

***Q.No.7*: If vectors are perpendicular to each other? (Or the angle between them is 90°) their vector or cross product is maximum.**

***Ans***: Vector product of any two vectors is given by:

$$\vec{A} \times \vec{B} = A\,B\,\text{Sin}\,\theta\,\hat{n}$$

When θ = 90°or when vectors $\vec{A}$ and $\vec{B}$ are ***perpendicular*** to each other.

$$\vec{A} \times \vec{B} = A\,B\,\text{Sin}\,90°\,\hat{n} \qquad (\vec{A} \perp \vec{B},\ \theta = 90°)$$

$$\boxed{\vec{A} \times \vec{B} = A\,B\,\hat{n}} \qquad (\text{Sin}\,90° = 1)$$

- Vector product has a ***maximum*** magnitude if vectors to be multiplied are ***perpendicular*** to each other.

***Q.No.8*: Show that $\hat{i} \times \hat{j} = \hat{k}$.**

***Ans***: By definition:

$$\hat{i} \times \hat{j} = \hat{i}\ \hat{j}\ \text{Sin}\,90°\,\hat{n}$$

(θ = 90° because $\hat{i}$ is along $x$-axis and $\hat{j}$ is along $y$-axis, $\hat{i} \perp \hat{j}$.)

∴ $\hat{i} \times \hat{j} = 1 \times 1 \times 1\,\hat{n}$ ( $\hat{i} = \hat{j}$ = 1, Sin 90°=1)

∴ $\hat{i} \times \hat{j} = 1\,\hat{n}$

Direction of $\hat{i} \times \hat{j}$, on applying right hand rule, comes out to be along + $z$-axis. ***In other words***, the vector obtained is of magnitude "1" and its direction is along + $z$-axis (hence it is a unit vector along +$z$-axis). The unit vector along + $z$-axis is $\hat{k}$. Hence:

$$\boxed{\hat{i} \times \hat{j} = \hat{k}}$$

Similarly, $\hat{j} \times \hat{k} = \hat{i}$ and $\hat{k} \times \hat{i} = \hat{j}$

**Commutative law:**

***Q.No.9*: Show that $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ OR $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$**

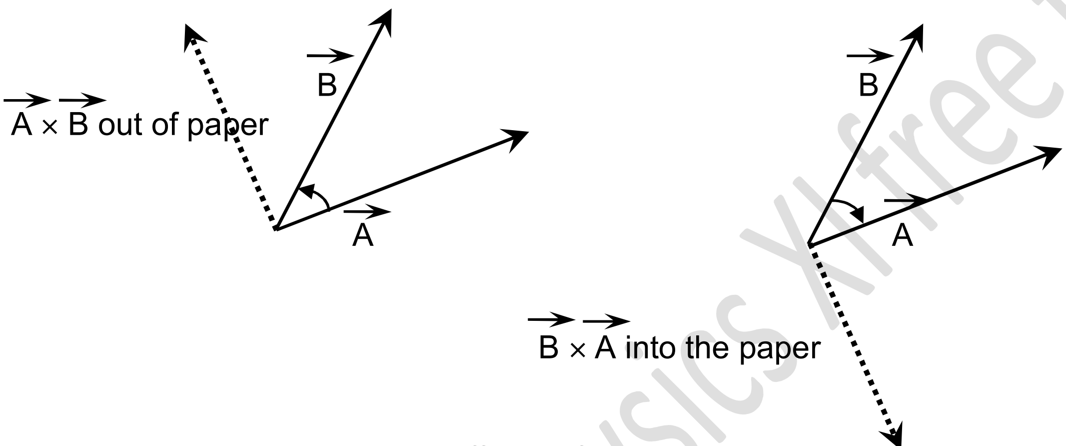
**OR**

***Show that vector product <u>does not obey commutative law</u>.***

***Ans*:** By definition:

$\vec{A} \times \vec{B} = A\,B\,\text{Sin}\,\theta\,\hat{n}$ and $\vec{B} \times \vec{A} = B\,A\,\text{Sin}\,\theta\,\hat{n} = A\,B\,\text{Sin}\,\theta\,\hat{n}$

These equations show that the magnitude of $\vec{A} \times \vec{B}$ is same as the magnitude of $\vec{B} \times \vec{A}$. But since both products give vector quantities, hence applying right hand rule to the following figure we see that $\vec{A} \times \vec{B}$ is directed ***out of paper*** whereas $\vec{B} \times \vec{A}$ is directed ***<u>into the paper</u>***.




Two vectors are said to be different if either their magnitude, direction or magnitude as well as their direction is different. Since the directions of $\vec{A} \times \vec{B}$ and that of $\vec{B} \times \vec{A}$ is different, therefore, they are ***<u>not equal</u>***.

- ***In other words***, vector or cross product ***<u>does not obey</u>*** commutative law, because on changing the order of vectors to be multiplied the direction of product vector comes out to be opposite. It means that:

$$\boxed{\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}}$$

If a negative sign is placed before a vector then it's direction reverses, Since $\vec{B} \times \vec{A}$ gives a vector quantity, therefore, the direction of $-\vec{B} \times \vec{A}$ after reversing becomes same as the direction of $\vec{A} \times \vec{B}$ and since their magnitude is also equal therefore,

$$\boxed{\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}}$$

- Hence $\vec{B} \times \vec{A}$ is <u>negative</u> of $\vec{A} \times \vec{B}$.

But $\vec{A} \times \vec{B} = A\,B\,\text{Sin}\,\theta\,\hat{n}$ $\therefore$ $\boxed{\vec{B} \times \vec{A} = -A\,B\,\text{Sin}\,\theta\,\hat{n}}$

***Q.No.10*: Show that $\hat{j} \times \hat{i} = -\hat{k}$.**

***Ans*:** Vector product ***<u>does not obey</u>*** commutative law because on changing the order of vectors to be multiplied the direction of product vector reverses. In the present case $\hat{j} \times \hat{i}$

comes out to be a unit vector along –z-axis, which is $-\hat{k}$. Hence:

$$\hat{j} \times \hat{i} = -\hat{k}$$

Similarly, $\hat{k} \times \hat{j} = -\hat{i}$ and $\hat{i} \times \hat{k} = -\hat{j}$

***Q.No.11*: How is a unit vector perpendicular to $\vec{A}$ as well as $\vec{B}$ and to the plane formed by $\vec{A}$ and $\vec{B}$ determined?**

***Ans*:** Since the direction of $\vec{A} \times \vec{B}$, on applying right hand rule, comes out to be perpendicular to $\vec{A}$ as well as $\vec{B}$ and to the plane formed by $\vec{A}$ and $\vec{B}$, therefore, if $\hat{n}$ represents the direction of $\vec{A} \times \vec{B}$ then:

$$\vec{A} \times \vec{B} = A\,B\,\sin\theta\,\hat{n} \quad \text{or} \quad \vec{A} \times \vec{B} = |\vec{A} \times \vec{B}|\,\hat{n}$$

Hence a unit vector <u>perpendicular</u> to any two vectors $\vec{A}$ , $\vec{B}$ and to the plane formed by them is given by:

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$\hat{n}$ is a unit vector <u>perpendicular</u> to $\vec{A}$, $\vec{B}$ and to the plane formed by $\vec{A}$ and $\vec{B}$.

***Q.No.12*: Give the determinant form in which cross product may be written?**

***Ans*:** If vectors $\vec{A}$ and $\vec{B}$ are expressed in terms of their rectangular components then their vector product can be found by solving their determinant i.e.

if $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ Then:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- This is the determinant form of vector or cross product.

***Q.No.13*: Give some examples of vector product.**

***Ans*: *Example.1*:** Torque "τ" is the turning effect of a force produced in a body about a certain axis. It is a vector quantity. It is equal to the vector or cross product of position vector "$\vec{r}$" and force "$\vec{F}$".

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- Direction of torque is determined by right hand rule. Its direction is same as the direction of $\vec{r} \times \vec{F}$ .

***Example.2*:** When a charged particle carrying charge "q" moving with some velocity "$\vec{v}$" enters a magnetic field of strength (called magnetic flux density)"$\vec{B}$" it experiences a force "$\vec{F}$", given by:

$$\vec{F} = q\,(\vec{v} \times \vec{B})$$

Magnitude of this force is "q v B Sin θ", where "θ" is the angle at which the charged particle enters a magnetic field or it is the angle between the direction of magnetic field "$\vec{B}$" and the direction of "$\vec{v}$". The direction of this force is determined by right hand rule (It's direction is same as the direction of $\vec{v} \times \vec{B}$) it is always perpendicular to "$\vec{v}$" , "$\vec{B}$" and to the plane in which $\vec{v}$ and $\vec{B}$ are.

***Example.3*:** Angular momentum is a <u>vector quantity</u>, basically It is the momentum of a body whose motion is angular. Angular momentum of a body about a fixed point (such as origin of a coordinate system or axis of rotation) is the vector or cross product of its position vector "$\vec{r}$" and its linear momentum "$\vec{p}$".

$$\vec{l} = \vec{r} \times \vec{p}$$

Where "$\vec{r}$" gives position of the body (or a point) with respect to the fixed point (such as axis of rotation of the body or origin of a coordinate system).

***Q.No.14*: *Give properties of vector or cross product.***

***Ans*:** Vector or cross product shows following properties:

(*i*) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

It shows that cross product does not obey commutative law.

(*ii*) $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

(*iii*) $(\vec{A} + \vec{B}) \times \vec{C} = \vec{A} \times \vec{C} + \vec{B} \times \vec{C}$

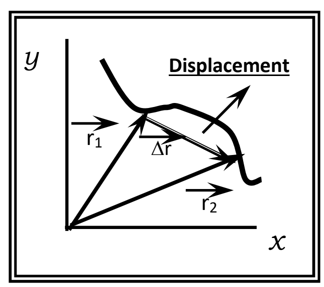
(*iv*) $\hat{i} \times \hat{i} = 0$, $\hat{j} \times \hat{j} = 0$ and $\hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, and $\hat{k} \times \hat{i} = \hat{j}$,

$\hat{j} \times \hat{i} = -\hat{k}$, $\hat{k} \times \hat{j} = -\hat{i}$ and $\hat{i} \times \hat{k} = -\hat{j}$

# ***Displacement*:**

***Q.No.1*: *What is displacement?***




***Ans*:** It is the shortest straight-line distance between initial and final positions of a body. Or it is the change in position of a body.

Position of a body with respect to a fixed point (origin of axes) is given by a position vector; hence its displacement will be the difference of final and initial position vectors i.e.

$$\vec{\Delta r} = \vec{r_2} - \vec{r_1}$$

• Displacement is a ***vector quantity***; its <u>magnitude</u> is equal to the distance between the two positions and it is <u>directed</u> towards the <u>final position</u> of the body.

• It's S.I unit is ***"m"***.

# ***Speed*:**

***Q.No.2*: *What is speed of a body?***

***Ans*: *It is the distance covered by a body in unit time.***

Or ***It is the rate of distance covered.***

$$\text{Speed} = \frac{\text{Distance covered.}}{\text{Time taken}}$$

$$v = \frac{s}{t}$$

- Speed is a ***scalar*** quantity.
- S.I unit of speed is ***m/s*** or ***ms*** $^{-1}$. Its dimensional formula is $[LT^{-1}]$

## ***Velocity***:

***Q.No.3: What is velocity?***

***Ans: It is the distance covered by a body in unit time in a particular direction.***

OR ***it is speed of a body in a particular direction.***

OR ***It is the rate of displacement of the body.***

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$\vec{v} = \frac{\vec{s}}{t}$$

- Velocity is a ***vector*** quantity. Its direction is same as the ***direction of displacement*** (or direction of motion).
- S.I unit of velocity is ***m/s*** or ***ms*** $^{-1}$ (same as the unit of speed).
- Its dimensional formula is $[LT^{-1}]$
- In terms of its rectangular components, it is given by:

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

***Q.No.4: What is average velocity?***

***Ans: It the total displacement of the body divided by total time taken.***

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

- It is determined by dividing the total displacement of the body by the time taken.

***Q.No.5: What is instantaneous velocity?***

***Ans***: Velocity of a body at a ***particular instant*** (or during a very short interval of time, or at a particular point) is called its instantaneous velocity.

- It is determined by first finding the displacement of the body during a very short interval of time and then dividing that displacement by the short time interval.

$$\vec{v}_{inst.} = \lim_{\Delta t\to 0} \frac{\Delta \vec{s}}{\Delta t}$$

" $\lim_{\Delta t\to 0}$ " is read as limit Δt approaches to zero, it means that the time interval is so short that it is close to zero (but not equal to zero).

***Q.No.6: When does velocity become variable?***

***Ans***: Velocity is a ***vector quantity***, hence like any vector quantity it becomes variable when either there is a change in its magnitude (or ***in other words***, there is a change in speed of the body), direction or there is a change in magnitude and direction both.

***Q.No.7*: What is uniform velocity?**

***Ans*:** Uniform velocity is that which ***does not change*** in magnitude and/or in direction.

Or it is the equal distance covered by a body in equal intervals of time in a particular direction (however short the time interval may be).

- When a body moves with a uniform velocity it's instantaneous and average velocities are equal.

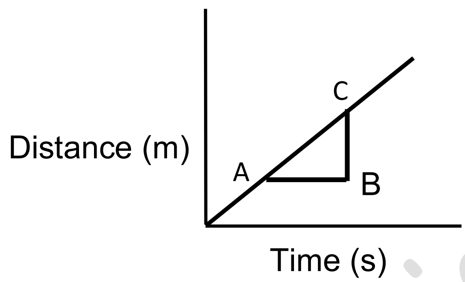
$$\vec{v}_{av.} = \vec{v}_{inst.} \qquad \vec{v} = \text{uniform.}$$

***Q.No.8*: How is velocity of a body calculated from its distance-time graph?**

***Ans*:** If the direction of motion is not changing then the velocity of a body can be calculated from its distance-time graph by finding slop of the graph.

***Q.No.9*: What is the shape of distance-time graph when a body moves with a uniform velocity?**

***Ans*:** When a body moves with a uniform velocity it's distance-time graph will be a straight line, passing through origin (as shown in diagram).




(Distance-time graph)

$$\text{Slope (velocity)} = \frac{BC}{AB}$$

$$\left\{ \begin{matrix} BC = S \\ AB = \Delta t \end{matrix} \right\}$$

- Slope of this type of graph is ***same*** at all points, which shows that velocity of the body is same everywhere, ***In other words*** this shape of distance-time graph is for a body moving with a ***uniform velocity***.

***Q.No.10*: What is the shape of distance-time graph of a body moving with a variable speed, in the same direction?**

***Ans*:** For a body moving with a variable speed the distance-time graph will be a curve. The shape of the curve depends upon the variation in the speed of body (velocity of a body may also be variable due to a change in its direction).

- In this case, slope of the curve at a given point on the curve is found, which gives instantaneous speed of the body at that instant. The slope of the curve will ***not be*** the same at all points (***in other words***, its speed will not be the same at different instants).

## ***Acceleration*:**

***Q.No.11*: What is *acceleration*?**

***Ans*: Rate of change of velocity of a body is called its acceleration.**

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \quad \text{Or} \quad \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

- Acceleration is a ***vector*** quantity; its direction is ***same*** as the direction of ***change in velocity "$\Delta \vec{v}$"*** of the body.
- If $v_f > v_i$ i.e. if velocity of a body ***increases*** then its acceleration will be ***positive***, the

direction of acceleration and the direction of motion will be ***same***.

- If $v_f < v_i$ i.e. if velocity of a body ***decreases*** then its acceleration will be ***negative***, It is also called ***retardation*** or ***deceleration***, in this case the direction of acceleration and the direction of motion will be ***opposite***.
- S.I unit of acceleration is ***"m/s²"*** or ***"m s⁻²"***
- Its dimensional formula is $[LT^{-2}]$ .

***Q.No.12: What is instantaneous acceleration?***

***Ans:*** It is the acceleration of a body at a particular instant or during a very short interval of time.

- It can be calculated by finding the change in velocity during a very short interval of time and then dividing this change in velocity by the short interval of time.

$$\vec{a}_{ints.} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t}$$

" limit " is read as "limit Δt approaches to zero", it means that the time
Δt→0 interval is so short that it is close to zero (but not equal to zero).

***Q.No.13: What is uniform acceleration?***

***Ans:*** When there is an equal change in velocity in equal intervals of time (whether the time interval is short or long) then the acceleration of the body is said to be uniform.

- When a body has uniform acceleration it's average and instantaneous accelerations will be equal (It means that the rate of change of velocity of the body will be equal whether it is measured during a short or a long interval of time).

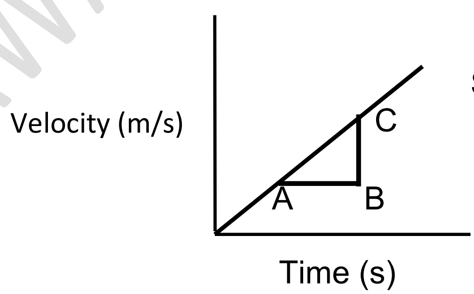
***Q.No.14: How is acceleration of a body calculated from its velocity-time graph?***

***Ans:*** Acceleration of a body is equal to the ***slop of its velocity-time graph***.

- The slope of velocity-time graph for uniform acceleration is same at all instants.
- The slope of velocity-time graph for non- uniform acceleration is different at different instants.
- The slope of velocity-time graph for retardation is negative.
- The slope of velocity-time graph for uniform velocity is zero (because motion of the body is non-accelerated).

***Q.No.15: What is the shape of velocity-time graph if acceleration of the body is uniform?***

***Ans:*** If a body moves with a uniform acceleration the shape of its velocity-time graph will be a straight line, passing through origin.




(Velocity-time graph)

$$\text{Slope (acceleration)} = \frac{BC}{AB}$$

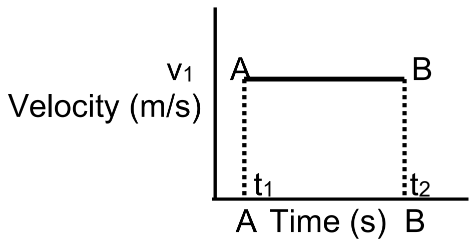
$$\left\{\begin{matrix} BC = \Delta v \\ AB = \Delta t \end{matrix}\right\}$$

***Q.No.16: What is the acceleration of a body moving with uniform velocity?***

***Ans:*** When a body moves with a uniform velocity i.e. its velocity does not change due to any reason then it's acceleration will be ***zero***.

***Q.No.17*: What is the shape of velocity-time graph if *velocity* of the body is *uniform*?**

***Ans*:** If a body moves with a uniform velocity the shape of its velocity-time graph will be a straight line, parallel to time axis.




Slope (acceleration) = 0

$\Delta v = v_1 - v_1 = 0$

$\Delta t = AB = t_2 - t_1$

***Q.No.18*: When does motion of a body become accelerated?**

***Ans*:** Motion of a body becomes accelerated when it moves with a ***variable velocity***.

***It means*** that motion of a body will be accelerated only when either there is a change in magnitude, change in direction or there is a change in magnitude as well as direction of its velocity.

***Q.No.19*: What type of acceleration is produced when only the direction of velocity changes?**

***Ans*:** When there is a change only in the direction of velocity of a body the resulting acceleration is called ***centripetal acceleration***.

- ***In this case*** the path followed by the body will be circular in shape, and the direction of acceleration will be towards the center of the circular path.

***Important note*:** *The angle between velocity $\vec{v}$ and centripetal acceleration $\vec{a_c}$ will be 90°, Similarly the angle between velocity $\vec{v}$ and the force (centripetal force) will be 90°.

***Q.No.20*: What is non-uniform acceleration?**

***Ans*:** Acceleration of a body will be non-uniform if there is an unequal change in its velocity in equal intervals of time.

***Q.No.21*: Is it possible for a body to have acceleration when moving with a:**
**(*i*) Constant velocity. (*ii*) Constant speed.**

***Ans*:** A body can have acceleration only when it's velocity changes. Velocity of a body is said to have changed when there is either a change in its magnitude, direction or a change in magnitude as well as direction.

(*i*) A body cannot have acceleration when it moves with a constant velocity.

(*ii*) A body can have acceleration when it moves with a constant speed, because speed represents only the magnitude of velocity of the body. Direction of its velocity may be changing. Hence the body can have acceleration because of the change in direction of velocity although its speed may be uniform.

# Equations of motion:

***Q.No.1*: Give the equations of motion.**

***Ans*:** If "S, $v_i$, $v_f$" and "a" are the distance covered, initial and final velocities and uniform acceleration of a body, the time taken t are related by the following equations of motion:

$$v_f = v_i + at$$
$$S = v_i t + \frac{1}{2} a t^2$$
$$2aS = v_f^2 - v_i^2$$

- While applying these equations, "$v_i$" is taken as zero (0) when the body starts moving from rest. "$v_f$" is zero when a moving body finally comes to rest. "a" is positive when $v_f > v_i$. Similarly, "a" is negative when there is retardation (or deceleration) or when $v_f < v_i$.

***Important note***: these equations ***can only*** be applied to a body if it moves with a uniform acceleration along a straight line. Also, that the direction of initial velocity "$v_i$" is taken as a reference, any vector quantity whose direction is opposite to the direction of "$v_i$" is taken as negative. Based on this idea acceleration due to gravity is taken negative "**–** g" throughout projectile motion. Because in vertical direction acceleration is "g" directed downward, whereas initial velocity in vertical direction at the point of projection "$v_{oy}$" is upward. All equations of projectile motion are based on this idea.

# Motion in two dimensions:

***Q.No.1*: What is two-dimensional motion? Give some examples.**

***Ans*:** If a body moves in two directions simultaneously (along $x$- and $y$-axes), (or it moves in a plane) then its motion is called two-dimensional motion.

Motion along a ***circular path*** and ***projectile motion*** are common examples of two-dimensional motion.

# Projectile motion:

***Q.No.1*: What is projectile motion? Give examples of projectile motion.**

***Ans*:** If a body moves with a constant velocity along horizontal direction and at the same time falls freely under the action of gravity (i.e. it has a constant vertical acceleration "g"), then its motion is called projectile motion.

***Examples*:**

Motion of a shell fired from a gun, motion of a kicked football, jumping of a frog or a locust, motion of a shuttle cock in badminton etc.

Projectile motion is two dimensional, in one plane. Projectile motion in vertical and horizontal directions can be treated independent of each other.

***Q.No.2*: On what assumptions is the study of projectile motion based?**

***Ans*:** Following assumptions are made to make the study of projectile motion simple and easy to understand:

(*i*) There is no air resistance.

(Air resistance basically affects the projectile motion in horizontal direction, ***in other words***, no force is assumed to act on a projectile in horizontal direction).

(*ii*) Value of "g" throughout the projectile motion is assumed to be constant.

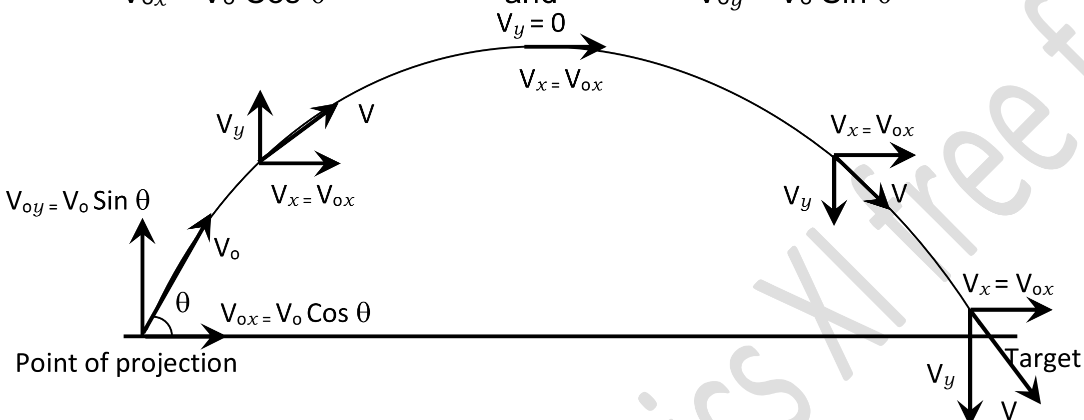
(*iii*) There is no effect of earth's rotation on projectile motion.

🗣(In reality rotation of the earth affects projectile motion when a projectile has to cover long distance to hit a target e.g. an intercontinental missile, which covers long distances, takes a long time during which the rotation of earth changes position of the target).

## ***Explanation:***

Let a projectile, such as a shell fired from a gun, be projected with initial velocity "$V_o$" making an angle "θ" with the horizontal. Assuming that the air resistance is negligible, its initial velocity "$V_o$" can be resolved into "$x$" and "$y$" components. Taking $x$-axis along the horizontal and $y$-axis along the vertical direction, $V_{ox}$ and $V_{oy}$ the $x$- and $y$-components of the initial velocity $V_o$ are given by:

$V_{ox} = V_o \cos\theta$ and $V_{oy} = V_o \sin\theta$




As we have assumed that there is no air resistance, which basically affects the motion of a projectile in horizontal direction, therefore, no force is assumed to act on the projectile in horizontal direction ($F_x = 0$). Therefore, there will be no acceleration in $x$-direction, ***In other words***, component of its velocity $V_x$ will remains constant throughout its motion.

🗣On the other hand, as the projectile (shell) has some weight, therefore, it will experience a constant downward force, in $y$-direction, (= its weight). Hence there will be a constant downward directed (in $y$-direction) acceleration "g". Since this acceleration is opposite to the direction of initial velocity "$V_{oy}$" in $y$ –direction, at the point of projection (upward), hence we will take it with a minus sign "– g" throughout the projectile motion.

Velocity of the projectile at any point will have two components "$V_x$" and "$V_y$". Velocity "V" with which it hits the target (or its net velocity at any instant) can be calculated with the help of "$V_x$" and "$V_y$" at that instant:

$$V = \sqrt{V_x^2 + V_y^2} \qquad \text{Where } V_x = V_{ox}$$

All equations of motion are independently applicable to projectile motion in horizontal as well as to vertical directions.

| Along $x$-axis. | Along $y$-axis. |
|---|---|
| $a_x = 0$ | $a_y = -g$ |
| $V_x = V_{ox} = V_o \cos\theta$ | $V_y = V_{oy} - g\,t$ |
| $x = V_{ox}\,t$ | $y = V_{oy}\,t - \frac{1}{2} g\,t^2$ |

## ***Time taken by projectile to reach the highest point "T".***

In vertical direction force of gravity acts on the projectile as a result of which the projectile has a constant vertical acceleration 'g', this constant acceleration is opposite to the initial velocity in vertical direction at the point of projection. Hence $a_y = -g$. Similarly, due to this acceleration, as the projectile rises its velocity decreases and becomes zero at the highest point, projectile will take some time "T" to reach the highest point. Hence In the vertical direction:

$$V_{oy} = V_o \sin\theta$$
$$a_y = -g$$
$$V_y = 0 \text{ (at the highest point)}$$

Time to reach the highest point T =?

On substituting these values in the following equation of motion, we get:

$$V_y = V_{oy} - g\,t$$
$$0 = V_{oy} - g\,T$$
$$T = \frac{V_{oy}}{g} = \frac{V_o \sin\theta}{g}$$

## ***Total time of flight "t".***

Projectile takes time T to reach the highest point it will take same time T to return to the ground. Hence total time of flight will be "t = 2T".

The total time of flight or ***in other words*** time for which a projectile remains in the air will be: t = 2T

$$\boxed{t = \frac{2V_{oy}}{g}} \quad \text{or} \quad \boxed{t = \frac{2V_o \sin}{g}}$$

- Total time taken by a projectile to move from point of projection to target depends upon the angle of projection "θ" and upon its velocity "$V_o$" with which it is projected.

- For a given projectile projected with a certain velocity "$V_o$" since the value of Sin θ depends on the value of "θ" and the value of Sin θ increases as "θ" is increased, therefore, larger the angle of projection "θ" longer is the time of flight.

## ***Maximum height reached "h":***

When a projectile reaches the highest point on its path its velocity component in $y$-direction "$V_y$" is zero, hence for maximum height:

$$V_{oy} = V_o \sin\theta$$
$$V_y = 0 \qquad \text{(at the highest point)}$$
$$a = -g$$
$$y = h$$
$$t = T = \frac{V_{oy}}{g}$$

On substituting the above data in the following equation, we get:

$$y = V_{oy}T - \tfrac{1}{2} g T^2$$
$$h = V_{oy} \times \frac{V_{oy}}{g} - \tfrac{1}{2} g \left\{\frac{V_{oy}}{g}\right\}^2$$
$$h = \frac{V_{oy}^{\,2}}{g} - \tfrac{1}{2} g \frac{V_{oy}^{\,2}}{g^2}$$
$$h = \frac{V_{oy}^{\,2}}{g} - \tfrac{1}{2}\frac{V_{oy}^{\,2}}{g}$$

$\therefore$ $h = \frac{1}{2}\frac{V_{oy}^2}{G}$ But $V_{oy} = V_o \sin\theta$ $\therefore$ $h = \frac{1}{2}\frac{V_o^2 \sin^2\theta}{g}$

From the above formula for maximum height reached by a projectile we can see that for a given projectile greater the angle of projection more is the height reached (also called vertical range).

## Range of a projectile "R":

The total horizontal distance covered by a projectile from the point of projection to the point where it lands (target point) is called its range R.

Since in the horizontal direction there is no acceleration (because there is no force in this direction), therefore, distance covered by a projectile in $x$-direction during any time "t" after the takeoff is given by: $x = V_{ox}\, t$

Hence for total distance covered from the point of projection to target or ***in other words*** for range $x$ = R.

$$\therefore \quad R = V_{ox}\, t$$

But $V_{ox} = V_o \cos\theta$ & $t = \frac{2 V_o \sin\theta}{g}$ (Total time of flight)

$$\therefore \quad R = V_o \cos\theta \times \frac{2 V_0 \sin\theta}{g} = \frac{2 V_o^2}{g} \sin\theta \cos\theta$$

But $2 \sin\theta \cos\theta = \sin 2\theta$ Hence range "R" of a projectile projected with a certain velocity "$V_o$", at an angle "$\theta$" to the horizontal is given by:

$$R = \frac{V_o^2}{g} \sin 2\theta$$

From the above formula we can see that the range of a projectile projected with a given initial velocity "$V_o$" depends upon the value of $\sin 2\theta$. Greater the value of $\sin 2\theta$ longer is the range of a projectile.

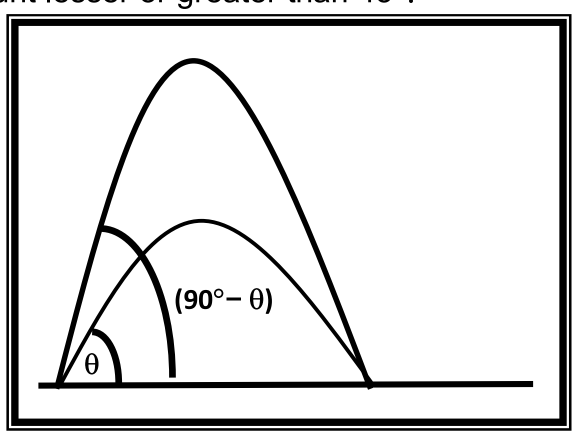
From the formula for range we can also see that range of a projectile will be equal for two angles of projection which are equal amount lesser or greater than 45°.

## Maximum range "$R_{max.}$":

Range of a given projectile is maximum when $\sin 2\theta$ is maximum. The maximum value of $\sin 2\theta$ is 1". This is possible when:

$2\theta = 90°$ (Because $\sin 90° = 1$)

***In other words:*** $\theta = 45°$




Range of a projectile will be maximum (i.e. it will cover maximum distance in horizontal direction) when it is projected at an angle of 45°.

Maximum range "$R_{max.}$" of a projectile is given by:

$$R_{max.} = \frac{V_o^2}{g} \qquad (\sin 2\theta = 1)$$

## Section–A Multiple-Choice Questions MCQs:

***Q.No.1***: To get a resultant of 10 m, two displacement vectors of magnitude 6 m and 8 m should be connected:
(a) Parallel (b) antiparallel
(c) At angle of 45° (d)perpendicular to each other

***Q.No.2***: The velocity of a particle at an instant is 10 m/s and after 5 sec, velocity of the particle is 20 m/s. Velocity 3 sec before in is:
(a) 8 (b) 4 (c) 6 (d) 7

***Q.No.3***:A ball is thrown upward with a velocity of 100 m/s. It will reach the ground after:
(a) 10 sec. (b) 20 sec. (c) 5 sec. (d) 40 sec

***Q.No.4***: Two projectiles are fired from the same point with the same speed at angles of 60° and 30 ° respectively. Which of the following is true?
(a) Their range will be the same.
(b) their maximum height will be same.
(c) Their landing velocity will be same.
(d) their time of flight will be same.

***Q.No.5***: The ratio of numerical values of average velocity and average speed of a body is always:
(a) Unity. (b) unity or less (c) unity or more.
(d) Less than unity.

***Q.No.6***: If the average velocity of a body becomes equal to the instantaneous velocity, body is said to be moving with:
(a) Uniform acceleration. (b) Uniform velocity.
(c) Variable velocity. (d) Variable acceleration.

***Q.No.7***:At the top of a trajectory of a projectile, the acceleration is:
(a) Maximum. (b) minimum. (c) zero (d) g

***Q.No.8***: At what angle the range of a projectile becomes equal to the height of projectile.
(a) 65° (b) 45° (c) 76° (d) 30°

***Q.No.9***: The angle at which dot product becomes equal to the cross product is:
(a) 65° (b) 45° (c) 76° (d) 30°

***Q.No.10***: If the dot product of two non-zero vectors vanish, the vectors will be:
(a) in the same direction. (b) opposite direction each other.
(c) perpendicular to each other. (d) zero.

***Q.No.11***: $\hat{j} \times \hat{j}$ is equal to: (***2003 Karachi Board***)
• $j^2$ • $j$ • one • zero

***Q.No.12***: The dot product of unit vector $\hat{i}$ & $\hat{k}$ is: (***2003 Karachi Board***)
• zero • 1 • −1 • $\hat{j}$

***Q.No.13***: If $\vec{F} = 4i - 2j$ and $\vec{d} = 3i + 4j$, the work done will be: (***2003 Karachi Board***)
• 4 joule • 8 joule • 2 joule • 12 joules

***Q.No.14***: If $\vec{a} \cdot \vec{b} = 0$ when $\vec{a} \neq 0$, $\vec{b} \neq 0$ the two vectors are: (***2004 Karachi Board***)
• parallel • opposite • perpendicular

***Q.No.15***: When $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, the angle between the vectors $\vec{A}$ and $\vec{B}$ is: (***2004 Karachi Board***)
• Zero • 45° • 90°

***Q.No.16***: If $\hat{i}, \hat{j}$ and $\hat{k}$ are the unit vectors along *x*-,*y*- and *z*-axes respectively, then $\hat{k} \times \hat{j}$: (***2009 Karachi Board***)
• $\hat{i}$ • $-\hat{i}$ • 1 • −1

***Q.No.17***: If a vector is divided by its own magnitude, the resulting vector is called: (***2009, 2003 Karachi Board***)
• Position vector • unit vector • null vector • free vector

***Q.No.18***: If $\vec{A} = a\hat{i}$ and $\vec{B} = b\hat{j}$, then $\vec{A} \times \vec{B}$ is equal to: (***2012 Karachi Board***)
• 0. • $ab\hat{k}$ • $-ab\hat{k}$ • None of these.

***Q.No.19***: Two forces act together on an object; the magnitude of their resultant is minimum when the angle between them is: (***2013 Karachi Board***)
• 0°. • 45° • 90° • 180°

***Q.No.20***: If $\vec{A} = 5\hat{i} + \hat{j}$ and $\vec{B} = 2\hat{k}$ Then $\vec{A} - \vec{B}$ is equal to: (***2014 Karachi board***)
• $5\hat{i} + \hat{j} + 2\hat{k}$ • $5\hat{i} - \hat{j} - 2\hat{k}$
• $5\hat{i} + \hat{j} - 2\hat{k}$ • $5\hat{i} - \hat{j} + 2\hat{k}$

***Q.No.21***: If $\vec{A} \cdot \vec{B} = 0$, $\vec{A} \times \vec{B} = 0$ and $\vec{A} \neq 0$ vector $\vec{B}$ is equal to:(***2016,14,05 Karachi board***)

• Equal to $\vec{A}$. • Zero. • Perpendicular to $\vec{A}$.
• Parallel to $\vec{A}$.

***Q.No.22*****:** The magnitude of product $\hat{k}\cdot(\hat{j}\times\hat{i})$
(***2017, 2016 Karachi Board***)

• zero • 1 • –1 • $|\hat{k}|$

***Q.No.23*****:** Two perpendicular vectors having magnitudes of 4 units and 3 units are added; their resultant has the magnitude of:
(***2018, 08,09 Karachi Board***)

• 7 units • 12 units • 25 units • 5 units

***Q.No.24*****:** If $\vec{F} = 3\hat{i}$, and $\vec{d} = 6\hat{j}$, the work done will be
(***2019 Karachi Board***)

• zero. • 2 • 9 • 18

***Q.No.25*****:** $(\hat{i}\times\hat{j})\cdot(\hat{j}\times\hat{i})$ is: (***2019 Karachi Board***)

• –1 • $\hat{k}$ • 1 • zero

***Q.No.26*****:** If $\vec{A}\cdot\vec{B} = 0$ , $\vec{A}\times\vec{B} = 0$ then $\vec{B}$ is:
(***2022 Karachi Board***)

• zero vector. • equal to $\vec{A}$.
• perpendicular to $\vec{A}$. • not parallel to $\vec{A}$.

***Q.No.27*****:** $\hat{i}\cdot(\hat{k}\times\hat{j})$ is equal to:
(***2022 Karachi Board***)

• 1 • –1 • 0 • $\hat{i}$

***Q.No.28*****:** If a vector has three rectangular components each equal to "a", magnitude of the vector will be: (***2022 Karachi Board***)

• $\sqrt{3a}$ • 3 a • $a^3$ • $\sqrt{3}$ a

***Q.No.29*****:** It is not a vector quantity:
(***2022 Karachi Board***)

• Force • Torque • Frequency • Weight

***Q.No.30*****:** When two bodies of unequal weights are dropped simultaneously from the same height, then:

• Heavier body will reach the ground earlier
• Lighter body will reach the ground earlier
• Both of them will reach the ground at the same time.

***Q.No.31*****:** Direction of retardation is:

• Same as the direction of motion.
• Opposite to the direction of motion.
• Perpendicular to the direction of motion.

***Q.No.32*****:** When a body moves with a constant speed in a circle: (***2005 Karachi Board***)

• its velocity is changing
• its acceleration is zero
• its acceleration is increasing
• its velocity is uniform.

***Q.No.33*****:** When a constant force is applied on a body, it moves with: (***2007 Karachi Board***)

• Constant speed.
• Constant velocity.
• Constant acceleration. • None

***Q.No.33*****:** A helicopter weighing 3920 N is moving up with a constant speed of 4 m/s. The force on the helicopter is: (***2012 Karachi Board***)

• 4720 N. • 3920 N. • 3924 N. • 3916 N.

***Q.No.34*****:** If the average and instantaneous velocities of a body are the same, the body will move with: (***2013 Karachi Board***)

• Variable velocity. • Uniform velocity.
• Variable acceleration • Uniform acceleration.

***Q.No.35*****:** A bus of weight 30000 N is moving with uniform velocity of 14 m/s. Its acceleration is
(***2017 Karachi Board***)

• 14 m/s. • zero.
• 7 m/s. • 9.8 m/s.

***Q.No.36*****:** If velocity of a body is decreasing, The direction of acceleration is:
(***2016 Karachi Board***)

• In the direction of velocity.
• Opposite to the direction of velocity.
• Perpendicular to the direction of velocity.
• 60° to the direction of velocity.

***Q.No.37*****:** If the time interval is very small ($\Delta t \to 0$), the rate of change of velocity of a bogy is called:
(***2019 Karachi Board***)

• average acceleration • Acceleration.
• Instantaneous acceleration
• constant acceleration

***Q.No.38*****:** A projectile is fired at an angle 'θ' with the horizontal. Its velocity will be minimum at:
(***2017 Karachi Board***)

• the point of projection. • the highest point.
• point of landing on the ground.
• all point of its path.

***Q.No.39*****:** A bullet is fired horizontally with 20 m/s, in the absence of air friction its horizontal velocity component after 2 s will be:
(***2018 Karachi Board***)

• 40 m/s. • 20 m/s. • 10 m/s. • 5 m/s

***Q.No.40*****:** The horizontal range of a projectile depends upon: (***2010 Karachi Board***)

• The angle of projection.
• The velocity of projectile.
• 'g' at the place. • All of them.

***Q.No.41*****:** A projectile is thrown at an angle of 30° with the horizontal having a certain initial velocity. It will have the same range if thrown with the same velocity at an angle of: (***2009 Karachi Board***)

• 45° • 60° • 75° • 15°

***Q.No.42***: Motion on a curved path when one component of velocity is constant and the other is variable is called: (***2004 Karachi Board***)

• Circular motion. • Projectile motion. • Vibratory motion.

***Q.No.43***: When the dot product of two vectors is negative then the angle between them is: (***2015 Hyderabad Board***)

(a) 0° (b) 90° (c) 180° (d) 360°

***Q.No.44***: A …….. vector can be displaced parallel to itself. (***2015 Hyderabad Board***)

(a) Position. (b) Unit (c) Null. (d) Free.

***Q.No.45***: ……… component of velocity remains same during the motion of a projectile. (***2015 Hyderabad Board***)

(a) Horizontal. (b) Vertical.
(c) Both (a) & (b) (d) None of these

***Q.No.46***: The path followed by a projectile is (***2016 Hyderabad Board***)

(a) Linear. (b) circular. (c) Parabola (d) Hyperbola.

***Q.No.47***: The dot product of i and j is: (***2018 Hyderabad Board***)

(a) k (b) one (c) Zero (d) None of these

***Q.No.48***: Scalar product of two vectors obeys: (***2018 Hyderabad Board***)

(a) Snell's law (b) commutative law.
(c) Associative law. (d) Both b and c.

***Q.No.49***: If $|\vec{A} \cdot \vec{B}| = |\vec{A} \times \vec{B}|$ then the angle between $\vec{A}$ and $\vec{B}$ is: (***2018 Hyderabad Board***)

(a) 0° (b) 45° (c) 60° (d) 90°

***Q.No.50***: If $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors , then $(\hat{i} \times \hat{j})$: **(*2025 Karachi Board*)**

• Zero • 1 • $\hat{j}$ • $\hat{k}$

## **Answers**:

(1) perpendicular to each other
(2) 6 m/s
(3) 20 sec.
(4) Their range will be the same.
(5) Unity.
(6) Uniform velocity.
(7) g
(8) 76°.
(9) 45°
(10) Perpendicular to each other.
(11) zero.
(12) zero.
(13) 4 joules.
(14) Perpendicular
(15) 90°
(16) $-\hat{i}$
(17) unit vector.
(18) $ab\hat{k}$.
(19) 180°.
(20) $5\hat{i} + \hat{j} - 2\hat{k}$.
(21) Zero.
(22) –1
(23) 5 units.
(24) zero.
(25) –1
(26) zero vector.
(27) –1
(28) $\sqrt{3}$ a
(29) Frequency
(30) the highest point.
(31) Same as the direction of motion.
(32) its velocity is changing
(33) Constant acceleration.
(34) Uniform velocity.
(35) zero.
(36) Opposite to the direction of velocity
(37) Instantaneous acceleration
(38) the highest point.
(39) 20 m/s.
(40) All of them.
(41) 6.0°
(42) Projectile motion.
(43) 180°
(44) Position.
(45) Horizontal.
(46) Hyperbola.
(47) Zero
(48) Both b and c.
(49) 45°
(50) $\hat{k}$

# ***Numericals:***

***Q.No.1*: A helicopter is ascending at a rate of 12 m/s. At a height of 80 m above the ground a package is dropped.**

**How long does the package take to reach the ground?**

***Data*:**

Initial velocity of the package =velocity with which helicopter is ascending $v_i$=12m/s

Net displacement of the package S = 80 m.

Time taken to reach the ground t =?

***Solution*:**

Initial velocity of the package is equal to the velocity with which helicopter is ascending, hence after being dropped the package will move upward for a short time and then fall downward. ***In other words***, initial velocity of the package is upward and the acceleration "g" and the net displacement "S" are in the downward direction (opposite to the direction of $v_i$), Hence:

$v_i = +12$ m/s  $a = -g$  &  $S = -80$ m

Distance covered by the package is given by:

$$S = v_i t + \frac{1}{2} a t^2$$

$$-80 = 12 \times t + \frac{1}{2} \times -9.8 \times t^2$$

$$-80 = 12t - 4.9t^2$$

$$4.9t^2 - 12t - 80 = 0$$

This is a quadratic equation its solution by quadratic formula is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ (quadratic formula)}$$

$$t = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 4.9 \times -80}}{2 \times 4.9}$$

$$t = \frac{12 \pm \sqrt{144 + 1568}}{9.8} = \frac{12 \pm \sqrt{1712}}{9.8}$$

$$t = \frac{12 \pm 41.98}{9.8}$$ Hence,

$$t = \frac{12 + 41.98}{9.8} \quad \text{OR} \quad t = \frac{12 - 41.98}{9.8}$$

$$t = \frac{53.38}{9.8} \quad \text{OR} \quad t = -\frac{29.38}{9.8}$$

t = 5.45 sec. OR t = – 2.99 sec.

Since time cannot be negative, therefore, the package takes ***5.45 sec.*** to reach the ground.

***Q.No.2*: Two tug boats are towing a ship, each exerts a force of 6000 N, and the angle between the two ropes is 60°.**

**Calculate the resultant force on the ship.**

***Data*:**

Magnitude of force $F_1$= 6000 N = $6\times10^6$ N

Magnitude of force $F_2$ = 6000 N = $6\times10^6$ N

Angle $\theta = 60°$

Magnitude of the resultant force F= ?

***Solution*:**

Let $\vec{F_1}$ and $\vec{F_2}$ be the force exerted by the two ropes making an angle "θ" with each other, then the magnitude of resultant "F", according to ***law of cosine*** is given by:

$$F = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos\theta}$$

$$F=\sqrt{(6\times10^3)^2+(6\times10^3)^2+2\times6\times10^3\times6\times10^3 \cos 60}$$

$$F=\sqrt{36\times10^6+36\times10^6+72\times10^6 \times 0.5}$$

$$F=\sqrt{108\times10^6}$$ ***F = 10392 N***

Magnitude of the resultant force acting on the ship is ***10392 N***.

***Q.No.3*: A car starts from rest and moves with a constant acceleration. During the 5th second of its motion, it covers a distance of 36 m, calculate:**

**(*i*) The acceleration of the car.**

**(*ii*) The total distance covered by the car during this time.**

***Data*:**

Initial velocity $v_i = 0$

Distance covered during 5th second S=36m

Acceleration a =?

Total distance covered in 5 s, $S_5$ =?

***Solution*:**

(*i*) Let the total distance covered by the car in 5 seconds be $S_5$, given by:

$S_5 = v_i t + \frac{1}{2} a t^2 = 0 + \frac{1}{2} a (5)^2$

$S_5 = 12.5\, a$ …………… (*i*)

Distance $S_4$ covered in 4 sec. is given by:

$S_4 = v_i t + \frac{1}{2} a t^2 = 0 + \frac{1}{2} a (4)^2$

$S_4 = 8\, a$ ……………….. (*ii*)

Distance covered during 5Th second will be:

$S_5 - S_4 = S$

$12.5\, a - 8\, a = 36$

$$4.5\,a = 36 \qquad a = \frac{36}{4.5}$$

$$\boxed{a = 8\ m/s^2}$$

Acceleration of the car is ***8 m/s²***.

(*ii*) Total distance covered by the car in 5 sec. is given by:

$$S_5 = v_i t + \tfrac{1}{2} a t^2$$

$$S_5 = 0 + \tfrac{1}{2} \times 8 \times (5)^2 \qquad \boxed{S_5 = 100\ m}$$

Total distance covered by the car is ***100m***

***Q.No.4*: Show that range of a projectile at complimentary angles is same.**

***Ans***: Range of a projectile is given by:

$$R = \frac{V_o^2 \sin 2\theta}{g}$$

From the formula for range of a projectile we can see that for a given value of "$V_o$" range of a projectile depends upon the value of Sin 2θ. This formula shows that range of a projectile will be <u>equal for two angles of projection</u> which are equal amount lesser or greater than 45°, since:

(45°+ α) and (45°− α) are any two angles of projection which are equal amount "α" greater or lesser than 45° for which range of the projectile will be equal. These angles are ***complimentary angles*** i.e. sum of these angles is 90°.

***For example,*** range of a given projectile will be equal at 60° and 30° because 30°<45°< 60° (30° is 15° less than 45° and 60° is 15° greater than 45° and 30°+ 60° = 90°).

Similarly range of the projectile will be equal at 15° and 75° angles of projection, because 15° < 45° < 75° (15° is 30° less than 45° and 75° is 30° greater than 45° and 15°+ 75° = 90°).

***Q.No.5*: At what angle the range of a projectile becomes equal to the height of projectile?**

***Ans***: Range of projectile is $R = \frac{V_o^2 \sin 2\theta}{g}$

Maximum height reached $h = \frac{V_o^2 \sin^2\theta}{2g}$

$R = h$ if $\sin 2\theta = \frac{\sin^2\theta}{2}$

This happens when θ = 76°. Hence at θ = 76°

Sin 2×76 = Sin 152 = 0.5

$$\frac{\sin^2\theta}{2} = \frac{\sin^2 76}{2} = \frac{(\sin 76)^2}{2} = \frac{(0.9703)^2}{2} = \frac{0.9415}{2} = 0.5$$

***Q.No.5*: A mortar shell is fired at a ground level target 500 meter distance with an initial velocity of 90 m/s. What is its launch angle?**

***Data***:

| | |
|---|---|
| Horizontal range | $R_H = 500$ m |
| Initial velocity | $V_o = 90$ m/s |
| Launch angle | θ = ? |

***Solution***:

Horizontal range of a projectile is given by:

$$R_H = \frac{V_o^2 \sin 2\theta}{g}$$

$$500 = \frac{(90)^2 \sin 2\theta}{9.8}$$

$$\sin 2\theta = \frac{500 \times 9.8}{90 \times 90}$$

$$\sin 2\theta = \frac{4900}{8100} = 0.6049$$

$$2\theta = \sin^{-1} 0.6049$$

$$2\theta = 37.2° \qquad \boxed{\theta = 18.6°}$$

But range of a projectile is same for two launch angles which are equal amount greater or lesser than 45°. Hence the second possible launch angle will be: 90° –18.6° = 71.4°.

$$\boxed{\text{Launch angle} = 71.4°}$$

***Q.No.6*: Find the angle between**

$$\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$$

***Solution***:

Angle between vectors $\vec{A}$ and $\vec{B}$ is given by: $\theta = \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{AB}$

$$\theta = \cos^{-1}\frac{(2\hat{i} + 2\hat{j} - \hat{k})\cdot(6\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{(2)^2+(2)^2+(-1)^2}\sqrt{(6)^2+(-3)^2+(2)^2}}$$

Since $\hat{i}\cdot\hat{j} = 0$, $\hat{j}\cdot\hat{k} = 0$ and $\hat{k}\cdot\hat{i} = 0$

$$\therefore \quad \theta = \cos^{-1}\frac{12\,\hat{i}\cdot\hat{i} - 6\hat{j}\cdot\hat{j} - 2\hat{k}\cdot\hat{k}}{\sqrt{4+4+1}\sqrt{36+9+4}}$$

Since $\hat{i}\cdot\hat{i} = 1$, $\hat{j}\cdot\hat{j} = 1$ and $\hat{k}\cdot\hat{k} = 1$

$$\therefore \quad \theta = \cos^{-1}\frac{12 - 6 - 2}{\sqrt{9}\sqrt{49}}$$

$\therefore \quad \theta = \cos^{-1}\dfrac{4}{3\times7}$

$\therefore \quad \theta = \cos^{-1}\dfrac{4}{21} = \cos^{-1} 0.1905$

$\boxed{\theta = 79.01^\circ}$

The angle between vectors is **79°.**

**Q.No.7: Find the work done in moving an object along a vector: $\vec{r} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ if applied force is: $\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$**

**Solution:**

Work done is the dot product of applied force F and displacement $\vec{r}$. Therefore,

$\text{Work} = \vec{F}\cdot\vec{r}$

$\text{Work} = (2\hat{i} - \hat{j} - \hat{k}).(3\hat{i} + 2\hat{j} - 5\hat{k})$

Since $\hat{i}\cdot\hat{j} = 0,\ \hat{j}\cdot\hat{k} = 0$ and $\hat{k}\cdot\hat{i} = 0$

$\text{Work} = 6\,\hat{i}\cdot\hat{i} - 2\,\hat{j}\cdot\hat{j} + 5\,\hat{k}\cdot\hat{k}$

But $\hat{i}\cdot\hat{i} = 1,\ \hat{j}\cdot\hat{j} = 1$ and $\hat{k}\cdot\hat{k} = 1$

$\text{Work} = 6 - 2 + 5 \qquad \boxed{\text{Work} = 9}$

**Q.No.8:**

**If vectors $\vec{A} = a\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{B} = \hat{i} + a\hat{j} + \hat{k}$ are perpendicular to each other, then find the value of "a".**

**(*2018 Karachi Board*)**

**Solution:**

$\vec{A}\cdot\vec{B} = 0 \qquad (\text{When } \vec{A}\perp\vec{B})$

$(a\hat{i} + \hat{j} - 2\hat{k})\cdot(\hat{i} + a\hat{j} + \hat{k}) = 0$

But $\hat{i}\cdot\hat{j} = 0,\ \hat{j}\cdot\hat{k} = 0$ and $\hat{k}\cdot\hat{i} = 0$

$\therefore \quad a\,\hat{i}\cdot\hat{i} + a\,\hat{j}\cdot\hat{j} - 2\,\hat{k}\cdot\hat{k} = 0$

$a + a - 2 = 0$

OR $\quad 2a - 2 = 0 \qquad \boxed{a = 1}$

**Q.No.9:**

**If $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$**

**Find a unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$.**

**(*2017,16,14,11,2,1999 Karachi Board*)**

**Data:**

$\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$

$\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$

Unit vector ⊥ to $\vec{A}$ and $\vec{B}$ $\quad \hat{U} = ?$

**Solution:**

By definition: $\quad \vec{A}\times\vec{B} = |\vec{A}\times\vec{B}|\,\hat{U}$

$\hat{U}$ is a unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$. But

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$\vec{A}\times\vec{B} = \hat{i}(-6\times-1-3\times-3) + \hat{j}(-3\times4 - 2\times-1)$

$\quad + \hat{k}(2\times3 - 4\times-6)$

$\vec{A}\times\vec{B} = \hat{i}(6+9) + \hat{j}(-12+2) + \hat{k}(6+24)$

$\vec{A}\times\vec{B} = 15\hat{i} - 10\hat{j} + 30\hat{k}$

$|\vec{A}\times\vec{B}| = \sqrt{(15)^2 + (-10)^2 + (30)^2}$

$|\vec{A}\times\vec{B}| = \sqrt{225+100+900} = \sqrt{1225} = 35$

$\therefore \quad \hat{U} = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

$\hat{U} = \dfrac{15\hat{i} - 10\hat{j} + 30\hat{k}}{35} = \dfrac{15}{35}\hat{i} - \dfrac{10}{35}\hat{j} + \dfrac{30}{35}\hat{k}$

Hence a unit vector perpendicular to $\vec{A}$ as well as $\vec{B}$ will be:

$$\boxed{\hat{U} = \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}}$$

**Q.No.10: If one of the rectangular components of force 50 N is 25 N. Find the value of the other.** **(*2010 Karachi Board*)**

**Data:**

Let $\quad F = 50$ N. and $F_x = 25$ N

Then $\quad F_y = ?$

**Solution:**

Since $F_x = F\cos\theta$ OR $25 = 50\cos\theta$

$\cos\theta = \dfrac{25}{50} = 0.5$ OR $\theta = \cos^{-1} 0.5$

$\underline{\theta = 60^\circ}$

The other component is:

$F_y = F\sin\theta = 50\sin 60^\circ$

$F_y = 50 \times 0.866$ **$F_y = 43.3$ N**

***Q.No.11*: Two forces of magnitude 10 N and 15 N are acting at a point. The magnitude of their resultant is 20 N. Find the angle between them (*2004, 1993 Karachi Board*)**

***Data*:**

Magnitude of $F_1 = 10$ N
Magnitude of $F_2 = 15$ N
Magnitude of resultant F = 20N
Angle between $F_1$ & $F_2$ $\theta$ =?

***Solution*:**

According to law of Cosine, the magnitude of the resultant is given by:

$F^2 = F_1^2 + F_2^2 + 2 F_1 F_2 \cos\theta$

Where $\theta$ is the angle between $F_1$ & $F_2$.

$(20)^2 = (10)^2 + (15)^2 + 2 \times 10 \times 15 \times \cos\theta$

$400 = 100 + 225 + 300 \cos\theta$

$400 - 325 = 300 \cos\theta$

$\cos\theta = \frac{75}{300} = 0.25 \quad \therefore \quad \theta = \cos^{-1} 0.25$

**$\theta = 75.53°$**

***Q.No.12*: Two forces of equal magnitude are acting at a point, find the angle between the two forces when the magnitude of resultant is also equal to the magnitude of either of these forces.**

***Data*:** (***2003 Karachi Board***)

Force $F_1$ = Force $F_2$
Magnitude of their resultant F = magnitude of $F_1$ or $F_2$
Angle between $F_1$ and $F_2$, $\theta$ =?

***Solution*:**

Magnitude of resultant of any two forces when angle ***between*** them is $\theta$, according to law of Cosine, is given by:

$F = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos\theta}$

Since $F_1 = F_2 = F$, therefore,

$\therefore \quad F = \sqrt{F^2 + F^2 + 2 F F \cos\theta}$

Squaring both sides we get:

$F^2 = 2 F^2 + 2 F^2 \cos\theta$

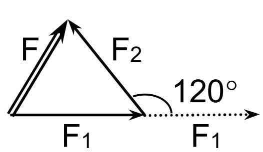
$F^2 - 2 F^2 = 2 F^2 \cos\theta$

$\therefore \quad - F^2 = 2 F^2 \cos\theta$

$\cos\theta = \frac{-F^2}{2 F^2}$

$\cos\theta = -\frac{1}{2}$

$\theta = \cos^{-1} -\frac{1}{2}$




$\therefore$ **$\theta = 120°$**

$\therefore$ Angle between the two forces is ***120°***.

***Q.No.13*: Two possible angles to hit a target by a mortar shell fired with initial velocity of 98 m/s are 15° and 75°. Calculate the range of projectile and the minimum time required to hit the target? (*2004,1995 Karachi Board*)**

***Data*:**

Initial velocity $V_o = 98$ m/s
Angles for same range $\theta = 15°$&$75°$
Range of projectile R = ?
Minimum time required t = ?

***Solution*:**

Range of a projectile is given by:

$R = \frac{V_o^2}{g} \sin 2\theta$

$R = \frac{(98)^2}{9.8} \sin 2\times15°$ (range at 15°)

$R = \frac{9604}{9.8} \sin 30°$

$\therefore \quad R = \frac{9604 \times 0.5}{9.8}$ **$R = 490$ m**

Range of the projectile is ***490 m*** for both the angles. The projectile will take minimum time to hit the target when it is fired at **smaller** of the two possible angles, hence

$T = \frac{2 V_o \sin\theta}{g}$

$T = \frac{2 \times 98 \times \sin 15°}{9.8}$ **$T = 5.176$ sec**

The projectile takes minimum time of ***5.176 sec.*** at 15° to hit the target.

***Q.No.14*:**

**$\vec{A} = 3\hat{i} + \hat{j} - 2\hat{k}$ & $\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$**

**Find the projection of $\vec{A}$ on $\vec{B}$.**

**(*2013 Karachi Board*)**

***Solution*:**

By definition:

$\vec{B} \cdot \vec{A} = B$ (projection of $\vec{A}$ on $\vec{B}$)

Hence:

Projection of $\vec{A}$ on $\vec{B} = \frac{\vec{B} \cdot \vec{A}}{B}$

$\vec{B} \cdot \vec{A} = (3\hat{i} + \hat{j} - 2\hat{k}) \cdot (-\hat{i} + 3\hat{j} + 4\hat{k})$

$\vec{B} \cdot \vec{A} = -3\, \hat{i} \cdot \hat{i} + 3\, \hat{j} \cdot \hat{j} - 8\, \hat{k} \cdot \hat{k}$

$\vec{B} \cdot \vec{A} = -3 + 3 - 8$

$\vec{B} \cdot \vec{A} = -8$

Similarly: $B = \sqrt{(-1)^2 + (3)^2 + (4)^2}$

$B = \sqrt{1 + 9 + 16} = \sqrt{26}$

$\therefore$ **Projection of $\vec{A}$ onto $\vec{B}$ = $\frac{-8}{\sqrt{26}}$**

***Q.No.15***: **A body starts from rest and moves with constant acceleration 10 m/sec$^2$, how much distance will it travel in the 4th sec of its motion.** (***2019 Karachi Board***)

***Data***:

Initial velocity $v_i = 0$

Acceleration $a = 10\ m/s^2$

Distance covered during 4th second S = ?

***Solution***:

Distance traveled in 4 seconds $S_4$:

$S_4 = v_i t + \frac{1}{2} a t^2$

$S_4 = 0 + \frac{1}{2} 10 (4)^2$

$S_4 = 80$ m …………… (*i*)

Distance traveled in 3 sec. $S_3$:

$S_3 = v_i t + \frac{1}{2} a t^2$

$S_3 = 0 + \frac{1}{2} 10 (3)^2$

$S_3 = 45$ m ……………….. (*ii*)

Distance traveled during 4th second will be:

$S = S_4 - S_3 = 80 - 45 =$ ***35 m***

Distance traveled during 4th sec is ***35 m.***

***Q.No.16***: **Two forces $\vec{F_1} = 3\hat{i} - 2\hat{j} + 5\hat{k}$ and $\vec{F_2} = \hat{i} + 6\hat{j} + 2\hat{k}$ act on a body which is displaced along $\vec{r} = 4\hat{i} - \hat{j} + 3\hat{k}$. Calculate work done on the body.** (***2022 Karachi Board***)

***Solution***:

Net force acting on the body $\vec{F} = \vec{F_1} + \vec{F_2}$

$\vec{F} = (3\hat{i} - 2\hat{j} + 5\hat{k}) + (\hat{i} + 6\hat{j} + 2\hat{k})$

$\vec{F} = 4\hat{i} + 4\hat{j} + 7\hat{k}$

$Work = \vec{F} \cdot \vec{r} = (4\hat{i} + 4\hat{j} + 7\hat{k}) \cdot (4\hat{i} - \hat{j} + 3\hat{k})$

$Work = 16\ \hat{i}\cdot\hat{i} - 4\ \hat{j}\cdot\hat{j} + 21\ \hat{k}\cdot\hat{k}$

Work = 16 – 4 + 21 **Work = 33 Joules.**

***Q.No.17***: **At what angle the horizontal range of projectile becomes equal to its vertical range? Prove mathematically.** (***2025 Karachi Board***)

***Data***:

Angle $\theta = ?$

Horizontal range $R_H$ = Vertical range = Maximum height "h" reached

***Solution***:

Range of projectile is $R_H = \frac{V_o^2 \sin 2\theta}{g}$

Maximum height reached $h = \frac{V_o^2 \sin^2\theta}{2g}$

But $R_H = h$

$\frac{V_o^2 \sin 2\theta}{g} = \frac{V_o^2 \sin^2\theta}{2g}$

OR $\sin 2\theta = \frac{\sin^2\theta}{2}$

This happens when $\theta = 76°$. Hence at $\theta = 76°$

$\sin 2\times76 = \sin 152 = 0.5$

$\frac{\sin^2\theta}{2} = \frac{\sin^2 76}{2} = \frac{(\sin 76)^2}{2} = \frac{(0.9703)^2}{2} = \frac{0.9415}{2} = 0.5$

At an angle of **76°** horizontal range of projectile becomes equal to its vertical range.

**Unit. 3**.....

# Dynamics.

***Dynamics*** is the branch of mechanics that deals with motion of a body with reference to force.

## ***Newton's laws of motion:***

### ***First law of motion:***

According to Newton's first law of motion:

***"A body tends to continue its state of rest or of uniform motion unless a force is applied on it".***

Hence in the absence of any external force a body at rest will remain at rest and a body in motion will continue its motion along a straight line with uniform speed.

In reality we see that a body in motion comes to rest after covering some distance. This is so because frictional forces, such as air resistance, contact friction etc. bring the body to rest. If all these frictional forces are eliminated, however they may be small, then the body will continue its motion with uniform velocity.

- First law of motion is also known as ***law of inertia***.

### ***Inertia:***

***Inertia is the property of matter by virtue of which it tends to oppose any change in its state of rest or of uniform motion.***

Hence if a body is at rest it will tend to remain at rest forever unless a force is applied to move it, similarly if a body is in motion it will tend to continue its motion with uniform velocity unless a force is applied to change its velocity.

***Inertia of a body depends upon its mass*** greater the mass of a body larger is its inertia and therefore, more difficult it is to stop it if it in motion or to move it if it is at rest.

Hence ***mass of a body is a measure of its inertia***.

### ***Second law of motion:***

According to second law of motion:

***"When an unbalanced force acts on a body it produces acceleration in the body in its own direction such that the magnitude of acceleration is directly proportional to the magnitude of force and inversely proportional to mass of the body".***

If '$\vec{F}$' is the unbalanced force, acceleration produced is '$\vec{a}$' and 'm' is mass of the body, then: $\vec{a} \propto \vec{F}$ and $\vec{a} \propto 1/m$ $\therefore$ $\vec{a} \propto \frac{\vec{F}}{m}$ $\quad a = k\frac{F}{m}$ ( But k=1)

$$\therefore \quad \vec{F} = m\vec{a}$$

Direction of acceleration '$\vec{a}$' produced in the body is always ***same*** as the direction of unbalanced force $\vec{F}$ (unbalanced force is the resultant or vector sum of all the forces acting on the body).

***Unit of force*:** Unit of force can be defined with the help of second law of motion, according to which: $F = m\,a$

Hence the S.I unit of force will be 'kg-m/$s^2$' which is commonly known as ***Newton***.

***" Force is said to be one Newton (1N) if it produces an acceleration of 1 m/$s^2$ in a body of mass 1 kg".***

***Q No.1*: Can a balanced force produce acceleration in a body?**

***Ans*:** No. Acceleration is produced only by unbalanced force (i.e. . In this case, the body either remains at rest or moves with a uniform velocity.

## ***Third law of motion*:**

According to Newton's third law of motion:

***"To every action there is an equal and opposite reaction".***

Action and reaction are two forces exchanged between two bodies when the interact with each other.

***Q.No.3*: Do action reaction cancel each other since they are equal and opposite forces?**

***Ans*:** Two equal and opposite forces acting on the ***same body*** cancel each other. Although action and reaction are equal and opposite forces but they act ***on two different bodies***. Action is the force exerted by first body on the second whereas reaction is an equal and opposite force applied by the second on first. Hence they ***do not*** cancel each other.

# ***Momentum*:**

Momentum may be defined as:

***'The product of mass and velocity of a body'.***

Hence: Momentum = mass × velocity

$$\text{Momentum} = m\,\vec{v}$$

Momentum of a body depends upon its ***mass*** and ***velocity***.

Momentum of a moving body actually represents the quantity of motion present in the body.

- Momentum tells us how difficult or how easy it is to stop a moving body. It is difficult to stop a body having large momentum. (***In other words,*** it is difficult to stop a ***fast moving*** and a ***heavy*** body).
- Momentum is a ***vector quantity***; its direction is ***same*** as the direction velocity of the body.

## ***S.I unit and dimensions of momentum*:**

The ***unit of momentum*** is the product of units of mass and velocity.

Hence the S.I unit of momentum is kg-m/s which is same as ***N-s***.

***Dimensions*** of momentum are $[MLT^{-1}]$.

***Q.No.1*: What is the relation between momentum and second law of motion?**

***Ans*:** When a force acts on a body it changes its momentum such that the rate of change in momentum of the body is equal to the force acting on it. This is the relation between momentum and second law of motion.

***Q.No.2*: Derive a relation between momentum of a body and second law of motion.**

***Ans*:** Consider a body of mass 'm' moving with a certain velocity '$v_i$'. Let a force 'F' be applied on it for time '$\Delta$ t' such that the velocity of the body changes and becomes '$v_f$'.

Therefore: Initial momentum = $m v_i$
Final momentum = $m v_f$
Change in momentum = $m v_f - m v_i$

$\therefore$ Rate of change in momentum = $\frac{m v_f - m v_i}{\Delta t}$

But $\frac{v_f - v_i}{\Delta t}$ = a (Acceleration of the body)

Hence, Rate of change of momentum of a body = m a

But m a = F (Force)

$\therefore$ Rate of change of momentum of a body = Force acting on the body

$$F = \frac{m v_f - m v_i}{\Delta t}$$

In vector form: $\vec{F} = \frac{m \vec{v}_f - m \vec{v}_i}{\Delta t} = \frac{\vec{\Delta p}}{\Delta t} = \frac{\text{change in momentum}}{\text{Time}}$

***"When a net (or unbalanced) force acts on a body it changes its momentum such that the rate of change of momentum of the body is equal to the force".***

## ***Impulse***:

Impulse of a force is the product of force '$\vec{F}$' and the short interval of time '$\Delta t$" for which it acts.

- It is a ***vector quantity***.
- It is also equal to the change in momentum of the body. Hence

$$\boxed{\vec{F}\,\Delta t = \vec{\Delta p}}$$

## ***Law of conservation of linear momentum***:

***Q.No.1: State law of conservation of linear momentum?***

***Ans***: According to law of conservation of momentum:

***"If no external force acts on a system of two or more interacting bodies the total linear momentum of the system remains constant."***

***It means that*** when bodies interact (collide with each other, exerting action reaction forces upon each other) in a system on which no external force is acting, momentum of individual bodies may change. Some bodies may gain and some may loose, but the total momentum of the system before collision is equal to the total momentum after collision.

## ***Proof of law of conservation of momentum***:

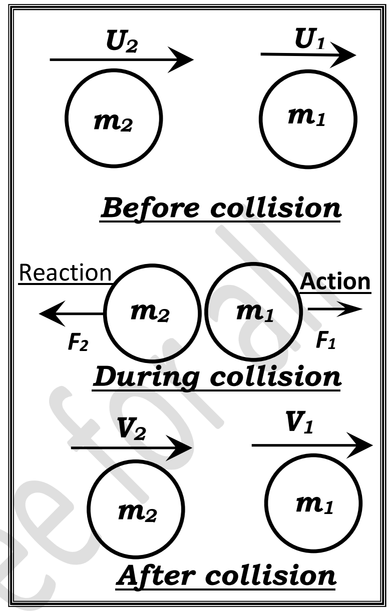
***Q.No.2: State and prove law of conservation of momentum?***

***Ans***: Consider a system of two colliding bodies of masses '$m_1$' and '$m_2$' moving with velocities '$U_1$' and '$U_2$' respectively in the same direction, such that no external force acts on them '$m_2$' is behind '$m_1$' and is moving faster than $m_1$. There will be a collision between the two bodies. Let their velocities after collision be '$V_1$' and '$V_2$' respectively.

Therefore, when the bodies come in contact with each other only action and reaction forces are exchanged between them.

Let '$\Delta t$' be the time for which the bodies come in contact with each other during the collision, and '$F_{12}$' be the force acting on '$m_1$' (exerted by $m_2$), then:

$$F_{12} = \frac{m_1 V_1 - m_1 U_1}{\Delta t} \qquad \text{(Action force)}$$




Similarly force '$F_{21}$' acting on $m_2$ (exerted by $m_1$) is given by:

$$F_{21} = \frac{m_2 V_2 - m_2 U_2}{\Delta t} \text{ (reaction force)}$$

Their velocities after collision be '$V_1$' and '$V_2$' respectively.

Since the system is isolated (***It means that*** no external force acts on the two bodies), therefore, when the bodies come in contact with each other <u>only action and reaction forces</u> are exchanged between them.

Let '$\Delta t$' be the time for which the bodies come in contact with each other during the collision, and '$F_1$' be the force acting on '$m_1$' (exerted by $m_2$), then:

$$F_1 = \frac{m_1 V_1 - m_1 U_1}{\Delta t} \qquad \text{(action force)}$$

Similarly force '$F_2$' acting on $m_2$ (exerted by $m_1$) is given by:

$$F_2 = \frac{m_2 V_2 - m_2 U_2}{\Delta t} \qquad \text{(reaction force)}$$

According to third law of motion, action and reaction are equal and opposite, hence

$$\vec{F}_{12} = -\vec{F}_{22} \quad \text{OR} \quad \frac{m_1 V_1 - m_1 U_1}{\Delta t} = -\frac{(m_2 V_2 - m_2 U_2)}{\Delta t}$$

$$m_1 V_1 - m_1 U_1 = -m_2 V_2 + m_2 U_2$$

$$m_1 U_1 + m_2 U_2 = m_1 V_1 + m_2 V_2$$

Left hand side of the above equation gives us the ***total momentum of the colliding bodies before*** and right-hand side gives us their ***total momentum after collision***.

∴ ***Total momentum before collision*** = ***Total momentum after collision***

***In other words***, If no external force acts on a system of colliding bodies the total momentum always remains constant (Or Momentum of a system is said to be ***<u>conserved</u>***). Momentum of individual bodies may change, body/bodies may gain and body/bodies may loose but if no external force acts on a system then total momentum of the system remains constant.

## ***<u>Elastic collisions</u>*:**

***<u>Q.No.3</u>: What are elastic collisions?***

***<u>Ans</u>*:** Elastic collisions are those in which ***<u>momentum</u>*** as well as ***<u>kinetic energy</u>*** of the system is conserved. ***<u>It means that</u>*** during an elastic collision:

- Total momentum before collision = Total momentum after collision.
- Total kinetic energy before collision= Total kinetic energy after collision.

(Elastic collision takes place between ***hard*** bodies having ***smooth*** surface)

## ***<u>Inelastic collisions</u>*:**

***<u>Q.No.4</u>: What are inelastic collisions?***

***<u>Ans</u>*:** Inelastic collisions are those in which momentum of the colliding bodies ***<u>may be conserved</u>*** but kinetic energy is ***<u>not</u>*** conserved. ***<u>In other words</u>***:

- Total momentum before collision may be equal to total momentum after collision.
- Total K.E before collision ***<u>is not equal</u>*** to total K.E after collision.

🗣During an Inelastic collisions kinetic energy of the system is converted into ***other forms of energy*** (mostly into heat energy).

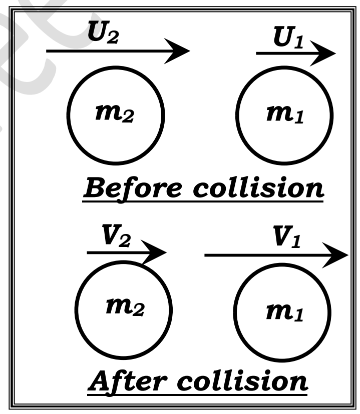
- Inelastic collision takes place between ***soft*** and ***rough*** bodies; the kinetic energy is converted partly into heat and partly used to change shape of the colliding bodies.
- When two cars collide, they dent each other (***in other words*** they change each other's shape). For making a dent on the other car each car has to do work on the other using its own kinetic energy. Also, bodies of colliding cars get slightly hot.

***Q.No.5*: What is one dimensional collision?**

***Ans***: One dimensional collision is that in which the colliding bodies move along the same line before and after the collision.

## ***Elastic collision in one dimension*:**

***Q.No.6*: Two spherical non-rotating bodies of masses '$m_1$' and '$m_2$' moving with velocities '$U_1$' and '$U_2$' respectively collide elastically. Assuming that both the bodies move along the same straight line before and after the collision, that passes through their centers, derive expressions for their final velocities '$V_1$' and '$V_2$'.**




***Ans***: Consider two solid spherical non-rotating bodies of masses '$m_1$' and '$m_2$' moving with velocities '$U_1$' and '$U_2$' respectively along the same line which passes through their centers. Let the body of mass "$m_2$" be behind "$m_1$" and $U_2>U_1$, then there will be a collision between the two bodies (this collision is one dimensional, because the two bodies move along the same straight line which passes through their centers before and after the collision). Let "$V_1$" and "$V_2$" be their velocities after collision.

Assuming the collision to be elastic (for which momentum and kinetic energy both must be conserved), on applying law of conservation of momentum we get:

Total momentum before collision = Total momentum after collision

$$m_1U_1 + m_2U_2 = m_1V_1 + m_2V_2$$
$$m_1U_1 - m_1V_1 = m_2V_2 - m_2U_2$$
$$m_1(U_1 - V_1) = m_2(V_2 - U_2) \quad \ldots\ldots (i)$$

Similarly on applying law of conservation of kinetic energy we get:

Total initial kinetic energy = Total final kinetic energy

$$\tfrac{1}{2}m_1U_1^2 + \tfrac{1}{2}m_2U_2^2 = \tfrac{1}{2}m_1V_1^2 + \tfrac{1}{2}m_2V_2^2$$
$$\tfrac{1}{2}m_1U_1^2 - \tfrac{1}{2}m_1V_1^2 = \tfrac{1}{2}m_2V_2^2 - \tfrac{1}{2}m_2U_2^2$$
$$\tfrac{1}{2}m_1(U_1^2 - V_1^2) = \tfrac{1}{2}m_2(V_2^2 - U_2^2)$$
$$m_1(U_1 - V_1)(U_1 + V_1) = m_2(V_2 - U_2)(V_2 + U_2) \quad \ldots\ldots (ii)$$

***Derivation of "$V_1$":***

Divide equation (*ii*) by the corresponding sides of equation (*i*).

$$\frac{m_1(U_1 - V_1)(U_1 + V_1)}{m_1(U_1 - V_1)} = \frac{m_2(V_2 - U_2)(V_2 + U_2)}{m_2(V_2 - U_2)}$$
$$U_1 + V_1 = V_2 + U_2 \quad \ldots\ldots (iii)$$
$$\therefore \quad V_2 = U_1 + V_1 - U_2$$

Substituting the above value of "$V_2$" in equation (*i*), we get:

$$m_1(U_1 - V_1) = m_2(U_1 + V_1 - U_2 - U_2)$$
$$m_1U_1 - m_1V_1 = m_2U_1 + m_2V_1 - 2m_2U_2$$

$$m_1V_1 + m_2V_1 = m_1U_1 - m_2U_1 + 2m_2U_2$$

$$V_1(m_1 + m_2) = (m_1 - m_2)U_1 + 2m_2U_2$$

$$\therefore \quad \boxed{V_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}U_1 + \frac{2m_2}{(m_1 + m_2)}U_2}$$

***Derivation of "$V_2$":***

From equation (*iii*) we have: $V_2 = U_1 + V_1 - U_2$

On substituting the value of "$V_1$", we get:

$$V_2 = U_1 + \frac{(m_1 - m_2)}{(m_1 + m_2)}U_1 + \frac{2m_2U_2}{(m_1 + m_2)} - U_2$$

$$V_2 = U_1\left\{1 + \frac{(m_1 - m_2)}{(m_1 + m_2)}\right\} + U_2\left\{\frac{2m_2}{(m_1 + m_2)} - 1\right\}$$

$$V_2 = U_1\left\{\frac{(m_1 + m_2) + (m_1 - m_2)}{(m_1 + m_2)}\right\} + U_2\left\{\frac{2m_2 - (m_1 + m_2)}{(m_1 + m_2)}\right\}$$

$$\boxed{V_2 = \frac{2m_1}{(m_1 + m_2)}U_1 + \frac{(m_2 - m_1)}{(m_1 + m_2)}U_2}$$

# ***Friction:***

***Friction is a force which opposes motion of a body.***

Force of friction exists between a body and the surface on which the body slides/rolls or between the body and the viscous medium through which it moves.

Hence force of friction is always against the direction of motion of a body and it acts parallel to the surfaces in contact.

## ***Origin of friction (Cause of friction).***

***Classical physics:***

According to classical physics when two surfaces are in contact, their irregular surfaces are inter-locked. Hence to move one surface over the other an extra force called ***force of friction*** is needed.

***Modern physics:***

According to modern physics, when two surfaces are in contact, intermolecular force of attraction between molecules of the two surfaces resist the motion of one surface over the other thus causing ***force of friction***.

## ***Types of friction:***

- Friction between the body and the surface when the body is at ***rest*** relative to the surface and tends to move is called ***static friction***.
- Force of friction between a body and a surface relative to which it is in motion is called ***kinetic or dynamic friction***.
  Kinetic friction is slightly ***less*** than the static friction.
- When a body slides on a surface the corresponding force of friction between the body and the surface on which it slides is called ***sliding friction***.
- When a body rolls on a surface the corresponding force of friction between the body and the surface on which it rolls is called ***rolling friction***.

🗣Force of friction depends upon ***roughness*** of surfaces in contact, their ***temperature***, their ***material***, their ***area of contact*** etc. Hence rolling friction is less than the sliding friction because area of contact between surface and the rolling body will be less.

🗣Force of friction is a ***self-adjusting force.*** Hence when a gradually increasing force is applied on a body at rest it is balanced by the force of friction because of its self-adjusting nature and the body still remains at rest, until a certain limit is reached. If the applied force is increased beyond this limit the body starts moving with acceleration (in accordance with second law of motion). The maximum force of friction between the body and the surface up to which it remains at rest is called ***limiting friction***.

Force of friction "$f$" is directly proportional to the normal reaction "R" or the force with which the body presses the surface. $f \propto R$

$$\boxed{f = \mu R}$$

Where "$\mu$" is the constant of proportionality it is known as ***coefficient of friction***. Coefficient of friction is of two types:

(*i*) Coefficient of static friction $\mu_s$.

(*ii*) Coefficient of kinetic friction $\mu_k$.

$\mu_k < \mu_s$ because force of friction between the body and the surface when the body is in motion is less than when it is at rest.

❖ Force of friction can be decreased by either making the surfaces smooth, by using a lubricant, by using ball bearings or by making a fast-moving body oblong in shape.

***Q.No.1**: Why is force of friction called a **self-adjusting** force?*

***Ans**:* When the force applied on a stationary body is increased from a very low value, force of friction also increases so that the body still remains at rest, until a maximum value of force of friction (called limiting friction) is reached. If the applied force is increased beyond the limiting friction the body starts moving with acceleration. In other words, before moving, force of friction adjusts its value equal and opposite to the applied force so that the body remains at rest, hence, it is called ***self-adjusting force***.

***Q.No.2**: What is coefficient of friction?*

***Ans**:* Coefficient of friction is the ratio of force of friction "$f$" to the normal reaction "R".

Coefficient of friction is of two types:

(i) Coefficient of static friction $\mu_s$. (At rest)

(ii) Coefficient of kinetic friction $\mu_k$. (In motion) $\boxed{\mu_s > \mu_k}$

***Q.No.3**: What is the S.I unit of coefficient of friction?*

***Ans**:* Since coefficient of friction is a ratio of two similar quantities, hence, it does not have any unit.

## ***Angle of friction:***

Angle of friction is the angle between force of friction "$f$" and the normal reaction "R".

If "$\alpha$" is the angle of friction, then $\tan\alpha = \mu$

Where "$\mu$" is the coefficient of friction. It means that tangent of angle of friction is also called **coefficient of friction**.

## ***Angle of repose:***

Angle of repose is the ***minimum angle*** made by an inclined surface with the horizontal at which a body placed on the inclined surface just starts sliding.

## ***Relation between angle of repose "θ" and angle of friction "α":***

Relation between angle of repose $\theta$ and the corresponding angle of friction $\alpha$ is that

$$\theta = \alpha$$

## ***Section–A Multiple-Choice Questions MCQs*:**

***Q.No.1*: The rate of change of linear momentum of a body is called:
(a) Linear force. (b) Angular force.
(c) Power. (d) Impulse.

***Q.No.2***: The term mass refers to the same physical concept as:
(a) Weight. (b) Inertia.
(c) Force. (d) Acceleration

***Q.No.3***: Which one of the following force is also called as self-adjusting force?
(a) Frictional force. (b) Tension.
(c) Weight. (d) Thrust.

***Q.No.4***: The laws of motion show the relationship between:
(a) Velocity and acceleration.
(b) Mass and velocity.
(c) Mass and acceleration.
(d) Force and acceleration

***Q.No.5***:: The motion of a rocket in space is according to the law of conservation of:
(a) Energy. (b) Linear momentum.
(c) mass. (d) Angular momentum.

***Q.No.6***: A bomb of mass 12 kg. initially at rest explodes into two pieces of masses 4 kg. and 8 kg. The speed of 8 kg. mass is 6 m/s. The kinetic energy of 4 kg. mass is:
(a) 32 J (b) 48 J (c) 114 J (d) 288 J

***Q.No.7***: If momentum is increased by 20% then K.E increases by:
(a) 44% (b) 55% (c) 66% (d) 77%

***Q.No.8***: The kinetic energy of mass 2 kg. and momentum of 2 Ns is:
(a) 1 J (b) 2 J (c) 3 J (d) 4 J

***Q.No.9***: For the same kinetic energy, the momentum is maximum for:
(a) an electron. (b) a proton.
(c) a deuteron. (d) alpha particle.

***Q.No.10***: A 3 kg. bowling ball experiences a net force of 15 N. What will be its acceleration.
(a) 35 $m/s^2$. (b) 7 $m/s^2$. (c) 5 $m/s^2$. (d) 35 $m/s^2$.

***Q.No.11***: The unit of linear momentum is:
(***2003, 2004 Karachi Board***)
• N/s • Ns • Js • J/s

***Q.No.12***:The rate of change of linear momentum is equal to:
(***2004 Karachi Board***)
• Acceleration. • force. • torque

***Q.No.13***: In an elastic collision of two bodies:
(***2008 Karachi Board***)
• Kinetic energy is conserved
• Momentum is conserved.
• Both K.E and momentum are conserved.

***Q.No.14***:The property of fluids due to which they resist their flow is called:
**(*2012 Karachi Board*)**
• Static friction. • Coefficient of friction.
• Viscosity. • Terminal velocity.

***Q.No.15***: A helicopter weighing 3920 N is moving up with a constant speed of 4 m/s. The force on the helicopter is:
**(*2012 Karachi Board*)**
• 4720 N • 3920 N. • 3924 N. • 3916 N.

***Q.No.16***: Kinetic friction is always:
(***2016 Karachi Board***)
• Greater than static friction.
• Equal to static fiction.
• Less than static fiction. • Zero.

***Q.No.17***: If 'F' be the limiting friction and 'R' be the normal reaction, then the coefficient of static friction will be equal to:
(***2019, 2015 Karachi Board***)
• F / R • R / F • F R • 1 / F R

***Q.No.18***: If the average velocity of a body is equal to instantaneous velocity, the body is said to be moving with:
(***2025 Karachi Board***)
• Uniform velocity. • Variable velocity
• Uniform acceleration • Variable acceleration

***Q.No.19***: This force is called "Self-adjusting force". (***2025 Karachi Board***)
• Tension. • Frictional force
• Weight. • Thrust

***Answers*:**

(1) Linear force.
(2) Inertia.
(3) Frictional force.
(4) Force and acceleration
(5) Linear momentum.
(6) 288 J
(7) 44%
(8) 1 J
(9) alpha particle.
(10) 5 $m/s^2$.
(11) Ns
(12) force.
(13) Both K.E and momentum are conserved.
(14) Viscosity.
(15) 3920 N.
(16) Less than static fiction.
(17) F / R
(18) Uniform velocity.
(19) Frictional force

# Numericals:

***Q.No.1*: A car weighing 9800 N is moving with a speed of 40 km./h. on the application of the brakes it comes to rest after traveling a distance of 50 m. Calculate the average retarding force.**

***Data*:**

Weight of the car W = 9800 N.

Initial velocity $v_i = 40\ km/h. = \frac{40 \times 1000}{60 \times 60} \frac{m}{s} = 11.11 m/s$

Final velocity $v_f = 0$

Distance covered S = 50 m.

Average force F =?

***Solution*:**

$$2\,a\,S = v_f^2 - v_i^2$$

$$2 \times a \times 50 = (0)^2 - (11.11)^2$$

$$100\,a = -123.457$$

$$a = -\frac{123.457}{100} = \boldsymbol{-1.23457\ m/s^2}$$

If 'm' is mass of the car, then: W = m g

$$\therefore \quad m = \frac{W}{g} = \frac{9800}{9.8} = \boldsymbol{1000\ kg.}$$

F = m a (second law)

$$\therefore \quad F = 1000 \times -1.23457 \qquad \boxed{F = -1234.57N.}$$

***Negative sign*** shows that the retarding force acting on the car is ***1234.57N***.

***Q.No.2*: A helicopter weighs 3920 newton. Calculate the force on it if it is ascending at the rate of 2 m/s². What will be the force on it if it is moving up with a constant speed of 4 m/s?**

***Data*:**

Weight W = 3920 N.

Upward acceleration $a = 2\ m/s^2$.

Constant speed v = 4 m/s.

Force on the helicopter F =?

***Solution*:**

In the first case helicopter is moving upward with an acceleration of 2 m/s², therefore, the upward lifting force "F" exerted by its engine and blades is greater than its weight "W" (downward). Hence,

Unbalanced force = F – W

F – W = ma (by second law)

But $m = \frac{W}{g} = \frac{3920}{9.8} = \boldsymbol{400\ kg}$.

$$\therefore \quad F - 3920 = 400 \times 2$$

$$F - 3920 = 800$$

$$F = 3920 + 800 \qquad \boxed{F = 4720\ N}$$

In the second case, the helicopter moves up with a constant speed, therefore, it's acceleration 'a' will be zero

$\therefore \quad F - W = ma \qquad F - W = 0$

$\therefore \quad F = W \qquad \boxed{F = 3920\ N.}$

***Q.No.3*: A 100 gm. bullet is fired from a 10 kg. gun with a speed of 1000 m/s. What is speed of recoil of the gun?**

***Data*:**

Mass of bullet m =100 gms = 0.1kg

Mass of the gun M = 10 kg.

Speed of the bullet $V_1$=1000 m/s.

Speed of the gun $V_2$ =?.

***Solution*:**

According to law of conservation of momentum:

Total initial momentum = Total final momentum

Since the gun and the bullet both are initially at rest, therefore, total initial momentum is zero. Applying law of conservation of momentum we get:

$$0 = m\,V_1 + M\,V_2$$

$$0 = 0.1 \times 1000 + 10\,V_2$$

$$10\,V_2 = -100$$

$$\therefore \quad V_2 = -\frac{100}{10} \qquad \boxed{V_2 = -10\ m/s.}$$

***Negative sign*** shows that after firing the bullet, the gun moves backward (opposite to the direction of motion of the bullet). This backward movement of the gun after firing is called recoil of the gun.

***Q.No.4*: A machine gun fires 10 bullets per second into a target. Each bullet weighs 20 gm. and had a speed of 1500 m/s. Find the force necessary to hold the gun in position.**

***Data*:**

Mass of each bullet m = 20 gm. = 0.020 kg

Number of bullets fired per sec. = 10

Velocity of each bullet after firing V=1500 m/s

Force necessary to hold the gun in position F=?

***Solution*:**

Machine gun and the bullets form one system. Before firing both are at rest, their total initial momentum is zero. To conserve momentum their total momentum after firing must also be zero. This is possible only if the machine gun has equal and opposite momentum (after firing) to that of the bullet. ***In other words***, the machine gun must recoil (move backward).

Force necessary to hold the gun in position is equal to the reaction force exerted by the bullets on the gun. This force is also equal to the rate of change of momentum of the bullets. (velocity of bullets before firing U = 0). Hence,

F = rate of change of momentum of 10 bullets.

$$F = \frac{\text{change in momentum of 10 bullets}}{\text{Time}}$$

$$\therefore \quad F = \frac{10\,(m\,V - m\,U)}{t}$$

$$F = \frac{10\,(0.02\times1500 - 0.02\times0)}{1} \quad (t = 1s)$$

$$F = 10 \times 0.02 \times 1500 \qquad \boxed{F = 300\ N}$$

To hold the machine gun in position while firing 10 bullets per sec. ***300 N*** force will be required.

***Q.No.5*: A 50 gm. bullet is fired into a 10 kg block that is suspended by a long cord so that it can swing as a pendulum. If the block is displaced so that its center of gravity rises by 10 cm. What was the speed of the bullet?**

***Data*:**

Mass of the bullet m = 50 gm. =0.050 kg

Mass of the block M = 10 kg.

Height through which C.G rises $h$=10 cm=0.10 m

Velocity with which bullet collides the block $U_1$=?

***Solution*:**

Since the gun is initially at rest, therefore, its initial momentum is zero, and initial momentum of bullet is $mU_1$. After collision the bullet lodges itself into the block and both swing as a pendulum. Let $V_2$ be the velocity with which the bullet-block system swings immediately after the collision, then total final momentum will be $mV_2 + MV_2$ or $(m + M)V_2$. Applying law of conservation of momentum we get:

Total initial momentum = Total final momentum

$$m\,U_1 = (m + M)\,V_2$$

$$0.05\,U_1 = (0.05 + 10)\,V_2$$

$$\therefore \quad 0.05\,U_1 = 10.05\,V_2 \quad \ldots\ldots\ldots(i)$$

K.E of bullet-block system after collision changes into its P.E as its C.G rises.

K.E lost = P.E gained

$$\therefore \quad \tfrac{1}{2}(m + M)\,V_2^2 = (m + M)\,g\,h$$

$$\tfrac{1}{2}(0.05 + 10)V_2^2 = (0.05 + 10)\ 9.8\times0.1$$

$$\tfrac{1}{2}\ \cancel{10.05}\ V_2^2 = \cancel{10.05} \times 9.8 \times 0.1$$

$$V_2^2 = 2\times0.98 \quad \text{OR} \quad V_2^2 = 1.96$$

$$\therefore \quad V_2 = \sqrt{1.96} \qquad \mathbf{V_2 = 1.4\ m/s.}$$

Immediately after collision the bullet-block system will move with a velocity of 1.4 m/s. On substituting the value of '$V_2$' in equation ($i$) we get:

$$0.05\,U_1 = 10.05\,V_2 \quad \ldots\ldots\ldots(i)$$

$$0.05\,U_1 = 10.05 \times 1.4$$

$$\therefore \quad U_1 = \frac{10.05 \times 1.4}{0.05} \qquad \boxed{U_1 = 281.4\ m/s}$$

Speed of the bullet just before collision with the block was ***281.4 m/s*.**

***Q.No.6*: A 70 gram ball collides with another ball of mass 140 gram. The initial of the first ball is 9 m/s to right while the second ball is at rest. If the collision were perfectly elastic, what would be the velocity of two balls after the collision?**

***Data*:**

Mass of first ball $m_1$ = 70 g = 0.07 kg

Initial velocity of first ball $u_1$ = 9 m/s (right)

Mass of second ball $m_2$=140 g= 0.140 kg

Initial velocity of second ball $u_2$= 0

Final velocities of both balls $v_1$ & $v_2$ =?

***Solution*:**

Velocity $v_1$ of first after collision is given by:

$$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}\,u_1 + \frac{2\,m_2}{(m_1 + m_2)}\,u_2$$

$$v_1 = \frac{(0.07 - 0.140)}{(0.07 + 0.140)}\times 9 + \frac{2 \times 0.140}{(0.07 + 0.140)} \times 0$$

$$v_1 = -\frac{0.07}{0.210} \times 9 + 0 \qquad \boxed{v_1 = -3\ m/s}$$

∴ Negative sign shows that, after an elastic collision with heavier second ball, the first ball moves opposite to its initial direction of motion i.e. to ***left***.

Velocity $v_2$ of second ball after collision is given by:

$$v_2 = \frac{2\,m_1}{(m_1 + m_2)}\,u_1 + \frac{(m_2 - m_1)}{(m_1 + m_2)}\,u_2$$

$$v_2 = \frac{2 \times 0.07}{(0.07 + 0.140)} \times 9 + \frac{(0.140 - 0.07)}{(0.07 + 0.140)}\times0$$

$$v_2 = \frac{2 \times 0.07}{0.21} \times 9 + 0 \qquad \boxed{v_2 = 6\ m/s}$$

Velocity of second ball after collision is ***6 m/s. to right*.**

***Q.No.7*: A truck weighing 2500 kg and moving with a velocity of 21 m/s collides with a stationary car weighing 1000 kg. The truck and car move together after the impact. Calculate their common velocity?**

***Data*:**

Mass of the truck $m_1$ = 2500 kg

Initial velocity of the truck $u_1$ = 21 m/s

Mass of the car $m_2$ =1000 kg

Initial velocity of the car $u_2$ = 0

Final velocity of both $v_1 = v_2 = v$ = ?

***Solution*:**

Applying law of conservation of momentum:

Total initial momentum = Total final momentum

$$m\,u_1 + m\,u_2 = m_1\,v_2 + m_2\,v_2$$

Sine after collision both move together, therefore, $v_1 = v_2 = v$

$$m\,u_1 + m\,u_2 = (m_1 + m_2)\,v$$

$$2500\times21 + 1000\times0 = (2500 + 1000)v$$

$$52{,}500 + 0 = 3{,}500\,v$$

$$v = \frac{52{,}500}{3{,}500} \qquad \boxed{v = 15\ m/s}$$

Speed of both after collision is ***15 m/s.***

***Q.No.8*: A stone is dropped from the peak of a hill. It covers a distance of 30 meters in the last second of its motion. Find the height of the peak.** (***2013 Karachi Board***)

***Data*:**

Initial speed $v_i = 0$.

Distance covered during the last second = 30 m.

Height =Total distance covered $S_t$ = ?

***Solution*:**

Let 't' be the total time taken by the stone to reach the ground, then height of the cliff or ***in other words***, total distance covered by the stone is given by: $S_t = v_i\,t + \frac{1}{2}\,a\,t^2$

$$S_t = 0 \times t + \tfrac{1}{2} \times 9.8 \times t^2$$

$$S_t = 4.9\,t^2 \ldots\ldots\ldots\ldots(i)$$

Distance covered $S_{(t-1)}$ during (t –1) sec. is given by:

$$S_{(t-1)} = v_i\,(t-1) + \tfrac{1}{2}\,a\,(t-1)^2$$

$$S_{(t-1)} = 0 \times (t-1) + \tfrac{1}{2} \times 9.8\times(t-1)^2$$

$$S_{(t-1)} = 0 + 4.9\,(t^2 - 2t + 1)$$

$$S_{(t-1)} = 4.9\,t^2 - 9.8t + 4.9 \ldots.(ii)$$

But distance covered during the last second is:

$S_t - S_{(t-1)} = 30$ m. Therefore,

$$\cancel{4.9\,t^2} - \cancel{4.9\,t^2} + 9.8\,t - 4.9 = 30$$

$$9.8\,t = 30 + 4.9$$

$$t = \frac{34.9}{9.8} = 3.56 \text{ second.}$$

Height of the cliff will be equal to the total vertical distance covered (during t sec.):

$$S_t = 4.9\,t^2$$

$$S_t = 4.9\,(3.56)^2 \qquad \boxed{S_t = 62.14\ m}$$

Height of the cliff is ***62.14 m.***

***Q.No.9*: A car is waiting at a traffic signal. As it turns green, the car starts ahead with a constant acceleration of 3 m/s$^2$. At the same time a bus traveling with a constant speed of 20 m/s overtakes and passes the car.**

**(a) How far beyond its starting point the car overtakes the bus?**

**(b) What will be the velocity of the car at that time?** (***2013 Karachi Board***)

***Data*:**

Initial velocity of the car $v_i = 0$

Acceleration of the car $a = 3m/s^2$

Constant speed of bus $v = 20$ m/s.

(a) Distance beyond the starting point at which car overtakes the bus S = ?

(b) Final velocity of the car $v_f$ =?

***Solution*:**

(a) Let "t" be the time after which car overtakes the bus. Distance covered by the car in time "t": $S_1 = v_i\,t + \frac{1}{2}\,a\,t^2$

$$S_1 = 0 \times t + \tfrac{1}{2} \times 3 \times t^2$$

$$S_1 = 1.5\,t^2 \ldots\ldots\ldots\ldots(i)$$

Distance covered by the bus during the same time: $S_2 = v_i\,t + \frac{1}{2}\,a\,t^2$

$S_2 = 20 \times t$ (bus is travelling with uniform speed, a = 0)

$$S_2 = 20\,t \ldots\ldots\ldots\ldots(ii)$$

When the car overtakes the bus at that moment $S_1 = S_2$ OR $1.5\,t^2 = 20\,t$

$$1.5\,t = 20$$

$$t = \frac{20}{1.5} = 13.33 \text{ second}$$

The car overtakes the bus after **13.33 second** at a distance beyond the starting point, given by: $S_1 = 1.5\,t^2$

$$S_1 = 1.5 \times (13.33)^2$$

$$S_1 = 1.5 \times 177.78$$

$$\boxed{S_1 = 266.67\ m}$$

(b) velocity of car when it overtakes the bus:

$$v_f = v_i + at$$

$$v_f = 0 + 3 \times 13.33 \qquad \boxed{v_f = 40\ m/s}$$

***Q.No.10*: An electron in a vacuum tube starting from rest is uniformly accelerated by an electric field so that it has a speed of $6\times10^6$ m/s after covering a distance of 1.8 cm.**

**Find the force acting on the electron.**

**Take mass of electron as $9.1\times10^{-31}$ kg.**

***Data***:

Initial velocity $v_i = 0$.

Final velocity $v_f = 6\times10^6$ m/s

Distance covered S = 1.8 cm= 0.018 m

Mass of the electron $m = 9.1\times10^{-31}$ kg

Force F = ?

***Solution***

$$2\,a\,S = v_f^2 - v_i^2$$

$$2 \times a \times 0.018 = (6\times10^6)^2 - (0)^2$$

$$0.036\,a = 36\times10^{12}$$

$$a = \frac{36\times10^{12}}{0.036} \qquad \boldsymbol{a = 10^{15}\ m/s^2.}$$

According to second law of motion

$$F = m\,a = 9.1\times10^{-31} \times 10^{15}$$

$$\boxed{F = 9.1\times10^{-16}\ N}$$

Accelerating force (such a force speeds up a body) acting on the electron is ***9.1×10 ⁻¹⁶ N***.

***Q.No.11*: A bullet having a mass 0.005 kg is moving with a speed of 100 m/s. It penetrates into a bag of sand and is brought rest after moving 25 cm. into the bag. Find the decelerating force on the bullet. Also calculate the time in which it is brought rest.**

***Data***:

Mass of bullet m = 0.005 kg.

Initial speed $v_i = 100$ m/s.

Final speed $v_f = 0$

Distance covered S = 25 cm. = 0.25 m

Decelerating force F =?

Time taken t =?

***Solution***:

$$2\,a\,S = v_f^2 - v_i^2$$

$$2 \times a \times 0.25 = (0)^2 - (100)^2$$

$$0.50\,a = -10000$$

$$a = -\frac{10000}{0.50}$$

$$\boldsymbol{a = -20000\ m/s^2.}$$

According to second law of motion \

$$F = m\,a$$

$$F = 0.005 \times -20000$$

$$\boxed{F = -100\ N}$$

***Negative sign*** shows that the force acting on the bullet is decelerating (or retarding) in nature, such a force slows down a body i.e. it produces deceleration. Therefore, the magnitude the decelerating force acting on the bullet is ***100 N***. Since,

$$v_f = v_i + a\,t$$

$$0 = 100 - 20000\,t$$

$$20000\,t = 100$$

$$t = \frac{100}{20000} \qquad \boxed{t = 0.005\ sec}$$

The bullet takes ***0.005 sec.*** to come to rest.

***Q.No.12*: A machine gun fires 10 bullets per second into a target. Each bullet weighs 30 gm. and had a speed of 1500 m/s.**
**Find the force necessary to hold the gun in position. (*2025 Karachi Board*)**

***Data***:

Mass of each bullet m = 30 gm. = 0.030 kg

Number of bullets fired per sec. = 10

Velocity of each bullet after firing V=1500 m/s

Force necessary to hold the gun in position F=?

***Solution***:

Machine gun and the bullets form one system. Before firing both are at rest, their total initial momentum is zero. To conserve momentum their total momentum after firing must also be zero. This is possible only if the machine gun has equal and opposite momentum (after firing) to that of the bullet. ***In other words***, the machine gun must recoil (move backward).

Force necessary to hold the gun in position is equal to the reaction force exerted by the bullets on the gun. This force is also equal to the rate of change of momentum of the bullets. (velocity of bullets before firing U = 0). Hence,

F = rate of change of momentum of 10 bullets.

$$F = \frac{\text{change in momentum of 10 bullets}}{\text{Time}}$$

$$\therefore \qquad F = \frac{10\,(m\,V - m\,U)}{t}$$

$$F = \frac{10\,(0.03\times1500 - 0.03\times0)}{1} \quad (t = 1s)$$

$$F = 10 \times 0.03 \times 1500 \qquad \boxed{F = 450\ N}$$

To hold the machine gun in position while firing 10 bullets per sec. ***450 N*** force will be required.

## *Do you know that.......*

There are **four** **FUNDAMENTAL FORCES** in nature:

- Strong force.
- Electromagnetic force.
- Gravitational force.
- Weak force.

- **Strong force,** this is the strongest force in nature, it keeps nucleons (protons and neutrons) togather in a nucleus. It is a **short range** force. Its range is even shorter than diameter of a heavy nucleus.
  This force is about **100 times stronger than the electromagnetic force**.
  Field particles **Gluons** are exchanged in this force.
- **Electromanetic force,** this force is basically an interaction between electric and magnetic fields. Electromagnetic force is responsible for the formation of atoms and molecules, in general whole chemistry is due to this force.
  Field particles **photons** are exchanged in this force.
  This force is about **$10^{36}$ times stronger than the gravitational force**.
- **Gravitational force,** exists between all bodies having mass .It is a **long range** force, it means that this force acts over very long distance.
  Field particles **Gravitons** are exchanged in this force.
- **Weak force**, Weak force exists during certain nuclear reactions such as in beta decay. This force is responsible for the stability of atoms. It is a short range force.

Field particles **W and Z bosons** are exchanged in this force.

***Unit.4***..... ✍

# Rotational and circular motion

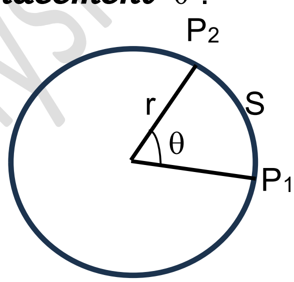
## Angular motion:

When a body either spins or moves along a circular path its motion is called ***angular motion*** (***or circular motion***).

## Angular displacement "θ":

Angle subtended by a body or a point at the center of its circular path is called its ***angular displacement***, represented by θ.

Let a body be moving along a circular path of radius "r" and $P_1$ and $P_2$ be its instantaneous positions. Length of the arc between points $P_1$ and $P_2$ represents the ***linear displacement*** "S" of the body, whereas the angle subtended by the arc at the center of the circular path represents its ***angular displacement*** "θ".




### Unit of angular displacement:

#### Radian:

The S.I unit of angular displacement "θ" is ***radian***.

- Angular displacement "θ" is said to be one radian (1 rad.) if length of the arc "S" is equal to radius "r" of the circular path. Hence this unit of angle can be defined as the ratio of length of the arc S and radius r of the circle.

$$\theta = 1 \text{ radian} \quad \text{when } S = r$$

$$\therefore \quad \theta = \frac{S}{r} \qquad \boxed{S = r\,\theta}$$

This equation is valid only when "θ" is measured in radian.

#### Degree

Most commonly used unit for the measurement of angle is ***degree***.

- If circumference of a circle is divided in 360 equal parts then the angle subtended by each part at the center of the circle is called **one degree 1°**, ***In other words*** for one complete revolution, angle subtended at the center of the circle will be 360°.

- Since "θ" is the ratio of two similar quantities (S and r), therefore, it is a ***dimensionless*** quantity.

## ***Relation between degree and radian*:**

For one complete revolution of a body, the angle subtended at the center of the circle is 360° whereas the distance covered by the body along the circular path is equal to the circumference 2 π r of the circle. Therefore,

$S = 2\pi r$ & $\theta = 360°$ (for one complete revolution)

But $\theta = \frac{S}{r}$ Therefore, $\theta = \frac{2\pi \not{r}}{\not{r}}$

$\theta = 2\pi$ radian (for one complete revolution)

Therefore, $2\pi$ radian = 360°

1 radian = 57.296° or **1 radian = 57.3°**

Similarly, $1° = 2\pi/360° = 0.0175$ radian.

# ***Angular velocity "ω":***

Rate of change of angle at the center of the circular path is called ***angular velocity*** "ω" of the body.

OR it is the rate of change of angular displacement of a body.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

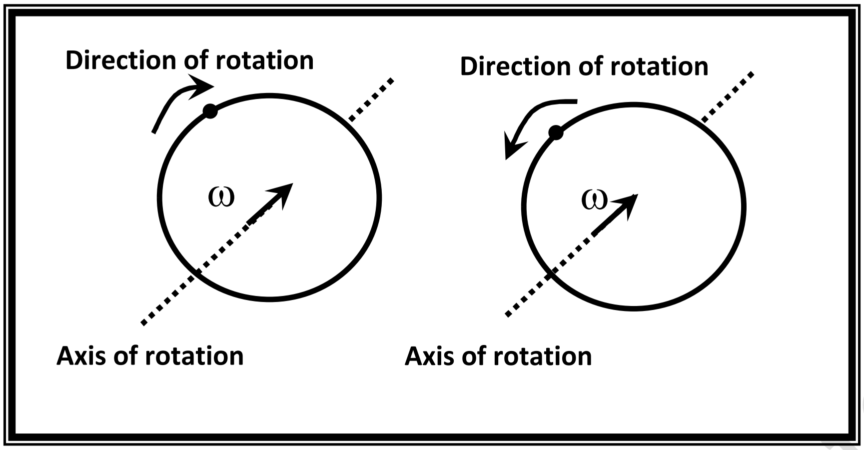
Angular velocity "ω" is a ***vector quantity***. Its direction is determined by ***right hand Rule.*** It is always directed ***along the axis of rotation*** and depends upon the direction of angular motion of the body.

- If a body is moving in ***clockwise*** direction then "ω" will be directed ***into the paper away from the reader*** along the axis of rotation (or revolution).
- If a body is moving in ***anticlockwise*** direction then "ω" will be directed ***out of paper*** or ***towards the reader*** along the axis of rotation (or revolution).

***To find*** the direction of angular velocity 'ω' of a body, first of all curl the fingers of your right hand in the direction of motion of the body (clockwise or anticlockwise) about the axis of rotation then the stretched thumb will point in the direction of angular velocity. This is also known as ***right hand rule***.

- S.I unit of angular velocity "ω" is ***rad. /sec*** (***radian per second***) or $s^{-1}$.
- Angular velocity is sometimes called ***angular frequency***.

***Note that*** every point on a rotating object has the ***same*** angular velocity where as its linear velocity at any point will be along the tangent (also called tangential velocity) to the circular path and its value will be proportional to distance of the point from axis of rotation of the body.




***Average angular velocity "$\omega_{av}$":***

It is the total angular displacement divided by total time taken

OR It is the angular velocity of a body during any interval of time.

$$\omega_{av} = \frac{\Delta \theta}{\Delta t}$$

Where '$\Delta \theta$' is the total angular displacement of the body during any time '$\Delta t$".

***Instantaneous angular velocity "$\omega_{inst.}$":***

It is the ***rate of angular displacement*** of a body at a particular instant or at a particular point.

OR It is the ***angular velocity*** of a body at a particular instant or at a particular point.

Instantaneous angular velocity of a body can be determined by finding the angular displacement of the body during a very short interval of time '$\Delta t$' just before or after the instant, and then dividing it by that short time interval. Time interval '$\Delta t$' is so short that it approaches to zero (it is not exactly zero) Hence:

$$\omega_{inst.} = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t}$$

***Uniform angular velocity*:**

If a body has equal angular displacement in equal intervals of time, however short the time interval may be, then it is said to have ***uniform angular velocity***.

When the angular velocity of a body is uniform its average and instantaneous angular velocities are equal, i.e.

$$\omega_{av} = \omega_{inst.} \qquad \omega = \text{uniform}$$

***In other words***, if a body is moving with a uniform angular velocity the angular velocity of the body will be equal whether it is measured in a long or a short time interval.

# ***Angular acceleration*:**

Rate of change of angular velocity of a body is called its angular acceleration.

$$\vec{\alpha} = \frac{\Delta \vec{\omega}}{\Delta t}$$

Where '$\Delta \omega$' is the change in angular velocity in time '$\Delta t$'

If '$\omega_i$' is the initial and '$\omega_f$' is the angular speed of a body after some time '$\Delta t$", then the magnitude of its angular acceleration '$\alpha$' is given by:

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t}$$

- Angular acceleration "$\alpha$" is a ***vector quantity***.
- S.I unit of angular acceleration is ***rad./s²***.
- Angular acceleration of a body is said to be 1 rad./$s^2$ if its angular velocity changes by 1 rad./s in 1 second.

***In other words***, angular motion of a body is said to be accelerated if its angular velocity is changing (increasing or decreasing).

## *Average angular acceleration*:

It is the total change in angular velocity divided by total time taken

OR

It is the angular acceleration of a body during any interval of time.

$$\alpha_{av} = \frac{\Delta \omega}{\Delta t}$$

Where '$\Delta \omega$' is the total change in angular velocity of the body during any time '$\Delta t$".

## *Instantaneous angular acceleration*:

It is the angular acceleration of a body at a particular instant or at a particular point.

OR it is the rate of change of angular velocity during a very short interval of time or at a particular point.

- Instantaneous angular acceleration of a body can be determined by finding the change in angular velocity '$\Delta \omega$' during a very short interval of time '$\Delta t$' just before or after the instant, and then dividing it by that short time interval. '$\Delta t$' is so short that it approaches to zero (it is not exactly zero) Hence:

$$\alpha_{inst.} = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$$

## *Uniform angular acceleration*:

If there is equal change in the angular velocity of a body in equal intervals of time, however short the time interval may be, then it is said to have ***uniform angular acceleration***.

- When the angular acceleration of a body is uniform its average and instantaneous angular accelerations are equal, i.e.

$$\alpha_{av} = \alpha_{inst.} \qquad \alpha = \text{uniform}$$

***In other words***, if a body has a uniform angular acceleration then there will be equal change in its angular velocity in equal intervals of time whether it is measured in a long or a short time interval.

## *Relation between linear and angular quantities:*

Consider a point moving along a circular path of radius 'r', such that it covers a linear distance '$\Delta S$' along an arc which makes an angle '$\Delta\theta$' at the center of circular path in time '$\Delta t$" or ***in other words***, angular displacement of the body is '$\Delta\theta$' in time '$\Delta t$".

Hence: $$\Delta S = r\,\Delta\theta$$

On dividing both sides of this equation by Δt (time taken)

$$\frac{\Delta S}{\Delta t} = r \frac{\Delta \theta}{\Delta t}$$

But $\frac{\Delta S}{\Delta t} = V$ (Linear velocity of the body)

And $\frac{\Delta \theta}{\Delta t} = \omega$ (Angular velocity of the body)

Therefore,

$$\boxed{V = r\,\omega}$$

***It means that***, at any instant linear velocity 'V' is equal to 'r' times angular velocity 'ω' of the body.

***Note that*** at any given instant angular velocity 'ω' is directed along the axis of rotation (or axis of revolution) and the linear velocity 'V' is directed along the tangent to the circular path at that instant. This linear velocity is also called ***tangential velocity*** (Since it is directed along the tangent to the circular path).

If the angular velocity of a rotating body is changing such that Δω is change in its angular velocity in time Δt and ΔV is the corresponding change in linear velocity of a point on the rotating body, then: $\Delta V = r\,\Delta\omega$

On dividing both sides of this equation by Δt, we get:

$$\frac{\Delta V}{\Delta t} = r \frac{\Delta \omega}{\Delta t}$$

$$\boxed{a = r\,\alpha}$$

***It means that***, at any instant linear acceleration 'a' (also called tangential acceleration) of a point on a rotating body is equal to 'r' times its angular acceleration 'α'.

***<u>Q</u>: Why do points on a disc, rotating with uniform angular speed, lying away from the axis of rotation, move with a higher speed than points lying close to it, although both have the same angular speed?***

***<u>Ans</u>:*** Linear speed 'V' of any point on a rotating body at any instant in terms of its angular speed 'ω' is given by: $V = r\,\omega$

This formula shows that if a disk is rotating with a uniform angular speed ω, The angular speed of all points on it will be equal because they will all subtend equal angles at the center in equal intervals of time.

Since their linear speed 'V' depends not only upon their angular speed 'ω' but also upon radius 'r' of their circular path. Therefore, their linear speed will be ***<u>different</u>***.

Points on a rotating body which are **<u>close</u>** to the axis of rotation will have **<u>smaller</u>** linear speed 'V' because distance 'r' of these points from the axis of rotation (radius of their circular path) is **<u>small</u>**.

Points lying **<u>away</u>** from the axis of rotation of a rotating disk move with a **<u>higher</u>** linear speed.

## ***Equations of angular motion:***

Corresponding to equations of uniformly accelerated linear motion there are equations for angular motion when angular acceleration of the body is uniform.

| Linear motion | Angular motion |
|---|---|
| $V_f = V_i + a\,t$ | $\omega_f = \omega_i + \alpha\,t$ |
| $S = V_i\,t + \frac{1}{2}\,a\,t^2$ | $\theta = \omega_i\,t + \frac{1}{2}\,\alpha\,t^2$ |
| $2\,a\,S = V_f^2 - V_i^2$ | $2\,\alpha\,\theta = \omega_f^2 - \omega_i^2$ |

## ***Centripetal acceleration $a_c$:***

Acceleration of a body directed ***towards the center*** of a circular path is called centripetal acceleration, its magnitude is given by:

$$a_c = \frac{v^2}{r}$$

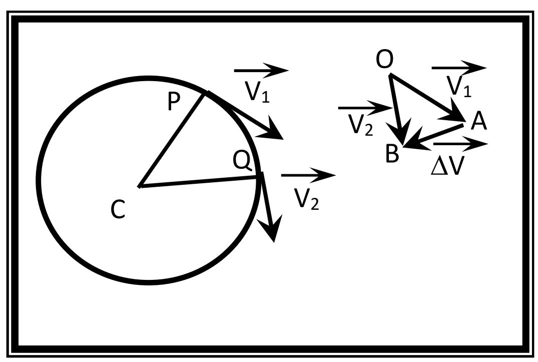
Where "r" is radius of the circular path, and "$v$" is the linear speed of the body.

## ***Derivation of Centripetal acceleration $a_c$:***

When a body moves along a circular path it's linear velocity at any instant is directed along the ***tangent*** to the circular path at that instant. Hence as the body continues its motion the direction of its linear velocity continuously changes. ***It means that*** motion of the body is accelerated. Acceleration produced in the body is due to a change only in the ***direction of its linear velocity***, it is called ***centripetal acceleration***.

Since K.E of a body depends upon the **magnitude** and **not upon the direction** of linear velocity therefore, ***kinetic energy*** of the body ***does not change***.

Let $\vec{V_1}$ and $\vec{V_2}$ be the velocities of a point at two successive positions after an interval of time '$\Delta t$' and '$\Delta\theta$' be the angular displacement at the center of circular path of radius 'r' during that time. If the body is moving with a uniform speed then the magnitudes of $\vec{V_1}$ and $\vec{V_2}$ will be equal but the direction of these velocities will be different (each along the tangent at two different points along the same circular path). Hence these two are different velocities. But their magnitude $V_1 = V_2 = V$.




The change in linear velocity $\vec{\Delta V}$ can be determined by joining the tails of $\vec{V_1}$ and $\vec{V_2}$.

Line joining the heads represents '$\vec{\Delta V}$', directed from the head of $\vec{V_1}$ to the head of $\vec{V_2}$ (According to head to tail rule $\vec{V_2}$ is the final velocity and $\vec{V_1}$ is the initial velocity, $\vec{V_2} = \vec{V_1} + \vec{\Delta V}$)

Since the change in velocity '$\vec{\Delta V}$' is directed ***towards the center of the circular path,*** therefore, acceleration is also ***directed towards the center***. This acceleration is called ***centripetal acceleration***.

Triangles OAB and CPQ are similar. Therefore:

$$\frac{AB}{PQ} = \frac{OA}{CP}$$

But $AB = \Delta V$, $PQ = \Delta S$, $OA = V$ & $CP = r$

$$\frac{\Delta V}{\Delta S} = \frac{V}{r}$$

$$\Delta V = \frac{V}{r}\Delta S$$

Divide both sides of the above equation by $\Delta t$:

$$\frac{\Delta V}{\Delta t} = \frac{V}{r}\frac{\Delta S}{\Delta t}$$

But $\frac{\Delta V}{\Delta t} = a_c$. & $\frac{\Delta S}{\Delta t} = V$

Therefore, the magnitude of centripetal acceleration $a_c$ is given by:

$$\boxed{a_c = \frac{V^2}{r}}$$

Direction of centripetal acceleration '$a_c$' is same as the direction of change in velocity '$\overrightarrow{\Delta V}$', Which is directed towards the center of the circular path, therefore, '$a_c$' is also directed towards the center of the circular path.

🗣 ***Note that*** the centripetal acceleration is due to a ***change only in the direction of linear velocity***, it is ***not due to any other reason***.

🗣 ***Also note that*** whenever a body moves along a circular path with whatever speed (with a constant or a variable speed) it will always have centripetal acceleration due to a change in direction of its linear velocity.

## ***Centripetal force $F_c$:***

Force required to move a body along a circular path is called centripetal force or ***in other words***, it is the force required to produce centripetal acceleration in a body. This force is called ***centripetal*** because it is always directed ***towards the center*** of circular path followed by a body.

🗣 Centripetal force is always required when a body moves along a circular path. If this force is not provided the body will not move along a circle or part of a circle. If centripetal force is removed during circular motion of a body it will fly off the circular path and will move along the tangent to the circle at the point from where this force is removed.

🗣 In case of our ***solar system,*** our beautiful little earth revolves around the sun because the gravitational force of the sun on earth provides the necessary centripetal force.

🗣 In case of the ***moon,*** moon revolves around the earth because the gravitational force of the earth on moon provides the necessary centripetal force.

🗣 In case of ***an atom,*** negatively charged electron revolves around the positively charged nucleus because the attractive Coulomb force of the nucleus provides the necessary centripetal force.

🗣 In case of ***a stone tied at the end of a string when whirled*** the stone revolves along a circular path because tension in the string provides the necessary centripetal force.

Centripetal force acting on a body is given by:

$$\boxed{F_c = \frac{m v^2}{r}}$$

$$\boxed{F_c = m r \omega^2} \qquad (v = r\omega)$$

$$\boxed{F_c = \frac{4\pi^2 m r}{T^2}} \qquad (\omega = \frac{2\pi}{T})$$

Where 'r' is radius of the circular path, and 'v' is its linear speed, 'ω' is its angular speed and 'T' is the period of revolution (time to complete one revolution) of the body.

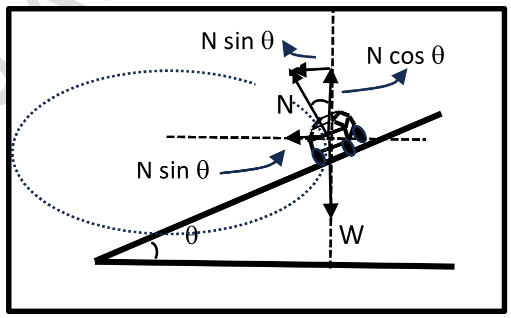
## ***Motion of a car on a curved flat road:***

Consider a car taking a turn on a curved flat road. Curved road is a part of a big circular path. We know that when any object moves along a circle or part of a circle ***centripetal force must be provided to it***. In this case friction between tires of the car and road surface provides the necessary centripetal force. Magnitude of this force depends upon many factors such as **road conditions** (i.e. its roughness, icy/ rainy conditions), upon **condition of tires**, **speed of the car** etc. The car cannot travel at high speed and may skid if the road is smooth/icy/rainy and tires are worn out. ***In other words***, motion of a car on a flat curved road depends mainly upon force of friction. If friction is ***less*** than required the car may skid.

## ***Banked curve:*** (**Karachi Board 2024**)

Banked curve is a curved road whose outer side is raised and makes some angle "θ" with the ground.

Purpose of making a banking curve is to decrease dependence of moving vehicles on friction.




Assuming that there is no force of friction between road surface and tires of the car, then forces acting on the car on a banked curve are its weight W = mg (vertically down ward) and normal reaction "N" acting on the car, perpendicular to the road, and makes a certain angle "θ" with the horizontal.

Resolving N into components we see that:

Component N Sin θ provides the centripetal force $\frac{mv^2}{r}$, necessary to move the car on banked curved road without skidding, and

Component N Cos θ balances weight W = mg of the car.

$$N \sin\theta = \frac{mv^2}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

$$N \cos\theta = mg \quad \ldots\ldots\ldots\ldots\ldots (ii)$$

Dividing equation (*i*) by (*ii*) we get:

$$\frac{\cancel{N}\sin\theta}{\cancel{N}\cos\theta} = \frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g} \qquad (\text{But } \frac{\sin\theta}{\cos\theta} = \tan\theta)$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\boxed{\theta = \tan^{-1}\left[\frac{v^2}{rg}\right]}$$

This formula gives us banking angle "θ" for ideal banked road (road free of friction) in terms of speed "v" of the car, radius "r" of the curved road.

- It is use-full for designing and constructing good quality banked curved roads on a high way. This formula is also used in designing banked curve for racing cars and motor bicycles etc.
- For ***high speed*** and ***sharp curved road*** banking angle must be ***large***. Hence for each banking angle and radius of the circular path there is a maximum speed limit with which a car can travel without skidding. If speed of the car is more than this speed limit car will move along a larger circle, on the other hand if speed of the car is below this speed limit it will move along a smaller circular curve.
- ***Note that*** the banking angle "θ" does not depend on mass "m" of the car.
- ***Also note that*** this derivation is for frictionless road surface, if there is friction it will also help to take a sharp turn with high speed on the banked road.

Using the above formula for banking angle maximum speed with which a car can be driven on a banked curve can also be calculated by rearranging the formula. Speed "v" is given by:

$$v^2 = r\,g\,\tan\theta$$

$$\boxed{v = \sqrt{r\,g\,\tan\theta}}$$

***Example*:**

Calculate the speed at which a car can be driven safely on a 100 m curve banked at 25°. (Assuming the road to be frictionless)

***Solution*:**

$v = \sqrt{r\,g\,\tan\theta}$ OR $v = \sqrt{100\times9.8\times\tan 25°} = \sqrt{456.98} =$ **21.4 m/s= 77 km/h.**

## ***Orbital velocity*:** (**Karachi Board 2024**)

Velocity with which a satellite orbits (revolves around) a heavenly body is called its **orbital velocity**.

Orbital velocity of a satellite orbiting any heavenly body at the center can be calculated by applying Newton's law of universal gravitation. Heavenly body at the center can be earth, moon, sun etc. and the satellite may be earth (revolving around the sun), moon (revolving around the earth), an artificial satellite (revolving around the earth) etc.

Gravitational force exerted on satellite by the body at the center provides centripetal force necessary to keep the satellite in orbit.

According to Newton's law of gravitation, force $F_G$ with which body at the center attracts the satellite is given by:

$$F_G = G\frac{m\,M}{R^2}$$

Where m is mass of the satellite., M is mass of the central body.

'G' is ***universal gravitational constant*** ($G = 6.67\times10^{-11}$ N-m²/kg²).

R radius of the orbit (or distance between centers of satellite and the central body)

Centripetal force $F_c$ required to keep the satellite in orbit is given by:

$$F_c = \frac{m\,V^2}{R}$$

Since $F_c = F_G$ Therefore, $\frac{\cancel{m}\,V^2}{\cancel{R}} = G\frac{\cancel{m}\,M}{R^2}$

$$V^2 = \frac{G\,M}{R} \quad \text{OR} \quad \boxed{V = \sqrt{\frac{G\,M}{R}}}$$

With the help of above formula orbital speed of any satellite can be calculated, provided we know mass M of body at the center and distance R between centers of the two bodies (G is the universal gravitational constant).

***Note that*** the orbital velocity V of the satellite at any instant is directed along the tangent to its circular path at that instant.

***Period of revolution "T":***

Time in which a satellite completes one revolution around the central body is called its period of revolution T.

In one revolution the satellite covers a linear distance of "$2\pi R$" equal to the circumference of the orbit. Hence

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

Therefore,

$$V = \frac{2\pi R}{T}$$

$$V^2 = \frac{4\pi^2 R^2}{T^2}$$

But

$$V^2 = \frac{GM}{R}$$

Therefore,

$$\frac{4\pi^2 R^2}{T^2} = \frac{GM}{R}$$

$$\boxed{T^2 = \frac{4\pi^2 R^3}{GM}}$$

With the help of this formula time in which a satellite completes one revolution around the central body (its period of revolution) can be calculated, this is also known as KEPLER'S law.

## ***Inertia:***

It is the property of a body to resist a ***change in its state of rest*** or of ***uniform linear motion***. It means that if a body is at rest it tends to remain at rest and if it is in motion it tends to continue its motion without any change in its velocity. Inertia of a body depends only upon its ***mass***.

Hence it is ***difficult*** to move a ***heavy*** body if it is at rest and if it is in motion then it is ***difficult*** to ***speed it up*** or ***slow it down***.

It is ***easy*** to move a ***light*** body if it is at rest, easy to slow it down or speed it up a light body. Heavy body has ***more inertia*** but light body has ***less inertia***. In other words, due to inertia a body tends to resist linear acceleration.

Hence inertia is for translational or linear motion.

## ***Moment of inertia:***

***Moment of inertia*** is actually rotational inertia ***it means that*** it is the property of a body to resist its angular acceleration or it shows how difficult or easy it is to change angular velocity of a body.

- If moment of inertia of a rotating body is ***large*** it will be ***difficult*** to change (increase or decrease) its angular velocity or it will strongly resist any change in its angular velocity.
- If moment of inertia of a rotating body is ***small*** it will be ***easy*** to change (increase or decrease) its angular velocity.

***Moment of inertia*** is for angular or circular motion it is also called rotational inertia. It depends upon ***size and shape***, ***mass***, ***distribution of mass*** of a body and upon the ***location of axis of rotation***.

If $m_1$, $m_2$ ,$m_3$ …. are mass of particles at perpendicular distance $r_1$, $r_2$, $r_3$,... respectively from axis of rotation of the rotating body, then the moment of inertia "I" of the whole body is given by:

$$I = \Sigma m_i r_i^2 \qquad (i = 1,2,3 \ldots\ldots)$$

***It means that*** moment of inertia of the whole body is equal to the sum of products of mass and square of perpendicular distance of particles of the body from the axis of rotation. Moment of inertia is also known as ***rotational inertia***.

MKS unit of moment of inertia is **kg.m²**. It is a ***scalar*** quantity.

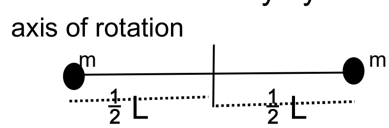
Dimensions of moment of inertia are $[M^1 L^2 T^0]$.

## ***Difference between inertia and moment of inertia;***

- Inertia is for ***linear (or translational) motion*** but moment of inertia is for ***rotational (or angular) motion*** of a body.
- Due to inertia bodies resist ***linear acceleration "a"*** but due to moment of inertia bodies resist ***angular acceleration "α"***.
- Inertia depends only upon ***mass*** of the body but moment of inertia depends not only upon ***mass*** but also upon the ***distribution of mass*** of the body, ***location of axis of rotation*** and the ***perpendicular distance*** of the rotating object from the axis of rotation.
- Bodies of same mass but of different shapes and sizes may have different moment of inertia.
- Momen of inertia is also called ***rotational inertia***.

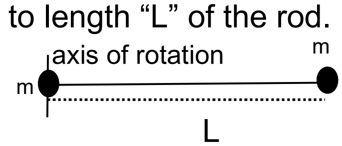
## ***Moment of inertia (or rotational inertia) of a two-particle system:***

Consider two rigid spherical bodies each of mass "m" connected at the ends of a rod of length "L" and of negligible mass. Let the rod rotates about its center so that the axis of rotation passes perpendicularly through its center. Under this condition perpendicular distance of each spherical body from the axis of rotation will be " $\frac{1}{2}$ L". Moment of inertia of this two-body system will be:




$$I = \Sigma m_i r_i^2 = m \times (\tfrac{1}{2} L)^2 + m \times (\tfrac{1}{2} L)^2$$
$$I = \tfrac{1}{2} m L^2$$

If the axis of rotation passes perpendicularly through one of rods ends i.e. if the rod rotates about one of its ends then perpendicular distance from axis of rotation of mass at this end will be zero whereas for the mass at the other end it will be equal to length "L" of the rod. Hence




$$I = \Sigma m_i r_i^2 = m \times (0)^2 + m \times L^2$$
$$I = m L^2$$

This example shows that moment of inertia depends also upon the location of axis of rotation.

## ***Moment of inertia of different bodies:***

- Moment of inertia of a ***solid cylinder*** of mass "M" and radius "R", rotating about its axis is: $I = \frac{1}{2} M R^2$
- Moment of inertia of a ***hollow cylinder*** of mass "M", inner radius "a" and outer radius "b" rotating about axis passing through its center is:

$$I = \frac{1}{2} M (a^2 + b^2)$$

- Moment of inertia of a ***solid sphere*** of mass "M" and radius "R" rotating about axis its center is: $I = \frac{2}{5} M R^2$
- Moment of inertia of a ***solid rod*** of mass "M" and length "L" rotating about one of its ends is: $I = \frac{1}{3} M R^2$

## ***Angular momentum*:**

Angular momentum is for rotational motion; it tells you how difficult it is to change angular velocity of a body.

Angular momentum "$\vec{L}$" is the cross product of position vector "$\vec{r}$" and linear momentum "$\vec{p}$".

$$\vec{L} = \vec{r} \times \vec{p}$$

- It is a ***vector quantity***. Its direction is always perpendicular to the plane containing $\vec{r}$ and linear momentum $\vec{p}$ and can be determined by right hand rule.
- Its MKS unit is **kg m²/s**.
- Its dimensions are [$\mathbf{M\,L^2\,T^{-1}}$]

Since linear momentum of a body is the product of its mass "m" and linear velocity "v" hence magnitude of angular momentum of a body about a certain axis of is given by:

$$L = m\,v\,r \sin\theta$$

"θ" is the angle between the position vector "$\vec{r}$" and linear momentum "$\vec{p}$" of the body.

If $\theta = 90°$ then, $L = m\,v\,r \sin 90°$

$L = m\,v\,r$

But $v = r\,\omega$, Where ω is the angular velocity of the body. Hence,

$L = m\,r^2\,\omega$

But $m\,r^2 = I$, where "I" is moment of inertia of the body. In other words,

$L = I\,\omega$

We know that angular velocity ω of all the points on a rotating body is same whatever their location may be. Angular momentum, therefore, depends upon its moment of inertia. Higher the moment of inertia of a body higher will be its angular momentum and more difficult it will be to stop a rotating/spinning/rolling body.

## ***Law of conservation of Angular momentum*:**

Law of conservation of angular momentum states that:

**"*If net torque acting on a body is zero the angular momentum of the body remains constant*"**.

## ***Torque or moment of force*:**

Torque "τ" or is moment of force the turning effect of a force produced in a body about a certain axis. It is a ***vector*** quantity. It is equal to the vector or cross product of position vector "$\vec{r}$" relative to a certain axis of rotation and force "$\vec{F}$".

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- Direction of torque is determined by ***right hand rule***. Its direction is same as the direction of $\vec{r} \times \vec{F}$.
- Torques which produce anticlockwise turning in a body with respect to the observer are

directed outward (by right hand rule) are taken as ***positive*** whereas torques which produce clockwise turning (directed inward) are taken as ***negative***.

Magnitude of torque acting on a body is given by:

$$\tau = r\,F\,\text{Sin}\,\theta$$

Where "$\theta$" is the angle between position $\vec{r}$ and force $\vec{F}$.

Magnitude of torque acting on a body is ***maximum*** when applied force $\vec{F}$ is perpendicular to the position vector $\vec{r}$, In other words angle $\theta$ is 90°.

## ***Relation between torque and moment of inertia:***

According to Newton's second law of motion:

$$F = m\,a$$

But linear acceleration "a" and angular acceleration "$\alpha$" are related by:

$$a = r\,\alpha$$

Note that this formula is valid when the body is moving along a circular path with angular acceleration "$\alpha$" in this case linear acceleration "a" will be directed along tangent to the circular path. In other words, linear acceleration in the above formula is also called tangential acceleration. Substituting the relation between linear and angular accelerations in second law we get:

$$F = m\,r\,\alpha$$

Multiplying both sides by "r" we get:

$$r\,F = m\,r^2\,\alpha$$

But: $r\,F = \tau$ (torque acting on the body)

and $m\,r^2 = I$ (moment of inertia of the body)

Hence: $$\tau = I\,\alpha$$

If "I" is constant, then

$$\tau \propto \alpha$$

This is the form of Newton's second law for angular/circular motion. It shows that when a net torque "$\tau$" acts on a body it produces an angular acceleration "$\alpha$" in it such that torque is directly proportional to the angular acceleration produced. In this case the constant of proportionality is "I" moment of inertia of the body.

## ***Section–A Multiple-Choice Questions MCQs:***

***Q.No.1*: One radian is about:
(a)25° (b) 45° (c) 37° (d) 57°

***Q.No.2***: Wheel turns with constant angular speed then:
(a) each point on its rim moves with constant velocity.
(b) each point on its rim moves with constant acceleration
(c) the wheel turns through equal angles in equal times.
(d) the angle through which wheel turns in each second increases as time goes on.
(e) the angle through which wheel turns in each second decreases as time goes on.

***Q.No.3***: The rotational inertia of a wheel about its axle does not depend upon its:
(a) diameter. (b) distribution of mass.
(c) mass. (d) speed of rotation.

***Q.No.4***: A force with a given magnitude is to be applied to a wheel. The torque can be maximized by:
(a) applying force near the axle radially outward from the axle.
(b) applying force near the rim radially outward.
(c) applying force near the axle parallel to a tangent to the wheel.
(d) applying force at the rim tangent to the rim.

***Q.No.5***: An object rotating about a fixed axis, "I" is its rotational inertia and "α" is its angular acceleration. It
(a) is the definition of torque.
(b) is the definition of rotational inertia.
(c) is the definition of angular acceleration.
(d) follows directly from Newton's second law

***Q.No.6***: The angular momentum vector of earth about its rotation axis, due to its daily rotation is directed:
(a) tangent to the equator towards east.
(b) tangent to the equator towards west.
(c) north. (d) towards the sun.

***Q.No.7***: A stone of 2 kg is tied to a 0.50 m long string and swung around a circle at angular velocity of 12 rad/s. The net torque on the stone about the center of the circle is:
(a) 0 N.m (b) 6 N.m (c) 12 N.m (d) 72 N.m

***Q.No.8***: A man with his arms at his sides, is spinning on a light frictionless turntable. When he extends his arms:
(a) his angular velocity increases.
(b) his angular velocity remains same.
(c) his angular velocity decreases.
(d) his angular momentum remains same.

***Q.No.9***: A space station revolves around the earth as a satellite, 100 km above the earth surface. What is the net force on an astronaut at rest inside the space station?
(a) equal to her weight on earth.
(b) a little less than her weight on earth.
(c) less than half her weight on earth.
(d) Zero (she is weightless)

***Q.No.10***: If the external torque acting on a body is zero, then it
(a) angular momentum is zero.
(b) angular momentum is conserved.
(c) angular acceleration is maximum.
(d) rotational motion is maximum.

***Q.No.11***: If ω is the angular speed of a particle moving in a circle of radius 'r', the centripetal acceleration will be: (***2004 Karachi Board***)
• ω r • ω $r^2$ • $\omega^2$ r

***Q.No.12***: Every point on a rotating body has the same: (***2004 Karachi Board***)
• Linear velocity. • Angular velocity.
•Linear acceleration.

***Q.No.13***: When a body moves with a constant speed in a circle:
(***2005 Karachi Board***)
• Its velocity is changing.
•Its acceleration is zero.
• Its acceleration is increasing.
• Its velocity is uniform.

***Q.No.14***: The rate of change of angular momentum with respect to time:
(***2007 Karachi Board***)
• Force. • Angular velocity
• Angular acceleration. • Torque

***Q.No.15***: Angle subtended at its center by an arc whose length is equal to its radius is:
(***2008 Karachi Board***)
• 37.3° • 47.3° • 57.3° • 67.3°

***Q.No.16***: The unit of angular velocity is:
(***2015 Karachi Board***)
• radian/cm. • meter/sec.
• radian/sec. • radian/$sec^2$.

***Q.No.17***: One radian is equal:
(***2025 Karachi Board***)

- 0.017° • 35.7° • 57.3° • 0.117°

***Q.No.18***: The turning effect of a force about the axis of rotation is called:

(***2025 Karachi Board***)

- Momentum.
- Inertia.
- Orbital velocity.
- Torque.

## ***Answers***:

(1) 57°
(2) The wheel turns through equal angles in equal time.
(3) Speed of rotation.
(4) Applying force at the rim tangent to the rim.
(5) follows directly from Newton's second law.
(6) towards the sun.
(7) 6 N.m
(8) his angular velocity decreases.
(9) Zero (she is weightless)
(10) angular momentum is conserved.
(11) $\omega^2 r$
(12) Angular velocity.
(13) Its velocity is changing.
(14) Torque.
(15) 57.3°
(16) radian/sec.
(17) 57.3°
(18) Torque.

# ***Numericals:***

***Q.No.1*: A car mechanic applies a force of 800 N to a wrench for the purpose of loosening a bolt. He applies the force which is perpendicular to the arm of the wrench. The distance from the bolt to the mechanic's hand is 0.40 m.**

**Find out the magnitude of torque applied?**

***Data*:**

$F = 800\ N \quad r = 0.40\ m \quad \tau = ?$

***Solution*:**

$\tau = F\, r\, \sin\theta \qquad (\theta = 90°)$

$\tau = 800 \times 0.40 \times \sin 90° = \mathbf{320\ Nm}$

Magnitude of torque applied is **320 Nm**.

***Q.No.2*: A car accelerates uniformly from rest and reaches a speed of 22 m/s in 9 s. If the diameter of a tire is 58 cm. Find:**
**(a) the number of revolutions the tire makes during this motion, assuming no slipping.**
**(b) the final rotational speed of the tire in revolution per second.**

***Data*:**

$V_i = 0, \quad V_f = 22\ m/s, \quad t = 9\ s,$

$d = 58\ cm = 0.58m \quad r = 0.29\ m$

(a) No. of revolutions =?

(b) Final rotational speed $\omega_f$ =?

***Solution*:**

(a) Circumference of a wheel

$= 2\pi r = 2\times22/7\times0.29 = 1.82\ m$

But $v_f - v_i = a\,t \qquad 22 - 0 = a\times9$

$a = 22/9 = 2.44\ m/s^2$

Distance travelled:

$S = v_i t + \frac{1}{2} a t^2$

$S = 0 + \frac{1}{2} \times 2.44\,(9)^2 = 99\ m.$

$\text{No. of revolutions} = \frac{\text{distance traveled}}{\text{Circumference of a wheel}}$

No. of revolutions = 99/1.82=***55 revolutions***.

(b) $v_f = r\,\omega_f, \quad \omega_f = \frac{v_f}{r} = \frac{22}{0.29} = \mathbf{75.86\ rad/s}$

There are $2\pi$ radians in one revolution.
Hence No. of revolutions/s in 75.86 rad/s

$\omega_f = \frac{75.86}{2\pi} =$ ***12 revolutions/s***

(a) Number of revolutions are ***55 revolutions***

(b) Final rotational speed is ***75.86 rad/s***

= ***12 revolutions/s***

***Q.No.3*: An ordinary workshop grindstone has a radius of 7.5 cm. and rotates 6500 rev/min.**

**(a) Calculate the magnitude of centripetal acceleration of its edge in $m/s^2$ and convert it into multiples of g.**

**(b) What is the linear speed of a point on its edge?**

***Data*:**

Radius $r = 7.5\ cm. = 0.075\ m$

Rotational speed = 6500 rev/min.

$= \frac{6500}{60}$ rev/s.

(a) Centripetal acceleration $a_c = ?$

(b) Linear speed $V = ?$

***Solution*:**

(a) Centripetal acceleration in terms of angular speed is given by:

$a_c = r\,\omega^2$

$\therefore \quad a_c = 0.075 \times \frac{(6500)^2}{(60)^2} = \mathbf{880.2\ m/s^2}.$

Centripetal acceleration of the edge is **880.2 m/s²**.

(b) Linear speed of a point on the edge is given by: $V = r\,\omega$

$\therefore \quad V = 0.075 \times \frac{6500}{60} = \mathbf{8.125\ m/s}.$

Linear speed of point on the edge is **8.125 m/s**.

***Q.No.4*: A satellite is orbiting the Earth with an orbital velocity of 3200 m/s.**

**What is the orbital radius?**

***Data*:**

Orbital velocity $V = 3200\ m/s$

Orbital radius $R = ?$

Mass of the earth $M = 5.98\times10^{24}\ kg$

Universal gravitational constant

$G = 6.67\times10^{-11}\ N\text{-}m^2/kg^2$

***Solution*:**

Since: $V^2 = \frac{G\,M}{R} \quad \text{OR} \quad R = \frac{G\,M}{V^2}$

$\therefore \quad R = \frac{6.67\times10^{-11} \times 5.98\times10^{24}}{(3200)^2}$

$\mathbf{R = 3.895 \times 10^{7}\ m}$

Orbital radius **$3.895 \times 10^{7}$ m.**

***Q.No.5*: A satellite is to orbit the earth at a height of 100 km (approximately 60 miles) above the surface of the earth. Determine the speed, acceleration and orbital period of the satellite.**

**(Given $M_{earth} = 5.98\times10^{24}$ kg, $R_{earth} = 6.37\times10^{6}$ m)**

***Data*:**

Height $h = 100$ km. $= 10^5$ m

$R_{earth} = 6.37\times10^6$ m.

Radius of orbit $R = R_{earth} + h$

$= 6.37\times10^6+10^5 = 6.47\times10^6$ m

$M_{earth} = 5.98\times10^{24}$ kg.

Speed $V = ?$

Acceleration $a = ?$

Orbital period $T = ?$

Universal gravitational constant

$G = 6.67\times10^{-11}$ N-m²/kg²

***Solution*:**

$$V^2 = \frac{GM}{R} \quad \text{OR} \quad V = \sqrt{\frac{GM}{R}}$$

$$V = \sqrt{\frac{6.67\times10^{-11}\times 5.98\times10^{24}}{6.47\times10^6}} = \sqrt{61648.563}$$

🗣 **V = <u>7851.7 m/s</u>. OR <u>7.851×10³ m/s</u>.**

Since the satellite is moving along a circular path therefore, it must have centripetal acceleration, given by:

🗣 $a = \frac{V^2}{R} = \frac{(7.851\times10^3)^2}{6.47\times10^6} = \mathbf{9.53\ m/s^2}$.

Orbital period can be found by:

$$T^2 = \frac{4\pi^2 R^3}{GM} = \frac{4\pi^2\times(6.47\times10^6)^3}{6.67\times10^{-11}\times5.98\times10^{24}}$$

$T^2 = 26,806,836$

$T = \sqrt{26,806,836} = 5177.53\ s = \frac{5177.53}{3600}\ h = \mathbf{1.44h}$

🗣 Orbital period is **<u>1.44 h</u>**.

***Q.No.6*: A thin disk with a 0.3 m diameter and a total moment of inertia of 0.45 kg.m² is rotating about its center of mass. There are three rocks with masses of 0.2 kg on the outer part of the disk. Find the total moment of inertia of the system.**

***Data*:**

Diameter d=0.3 m, Radius r = d/2 = 0.15 m

Moment of inertia of disk I = 0.45 kg.m²

Mass of each rock m = 0.2 kg.

Total number of rocks n = 3

Total moment of inertia =?

***Solution*:**

Moment of inertia of each rock $= m r^2$

$= 0.2\,(0.15)^2 = 4.5\times10^{-3}$ kg.m².

Total moment of inertia of rocks $= 3(4.5\times10^{-3})$

Total moment of inertia of rocks$=1.35\times10^{-2}$ kg.m²

∴ Total moment of inertia of the disk = moment of inertia of disk + moment of inertia of rocks

Total moment of inertia of disk $= 0.45+1.35\times10^{-2}$

**= <u>0.4635 kg.m²</u>**

🗣 Total moment of inertia of disk is **<u>0.4635 kg.m²</u>.**

***Q.No.7*: What is the ideal banking angle for a gentle turn of 1.20 km radius on a highway with a 105 km/h speed limit (about 65 mil/h) assuming everyone travels at the limit?**

***Data*:**

Radius of highway r =1.20 km.= 1200 m

Speed limit $v = 105$ km/h $= \frac{105\times1000}{60\times60}$ m/s

$= 29.17$ m/s.

Ideal banking angle $\theta = ?$

***Solution*:**

Banking angle is given by:

$$\theta = \tan^{-1}\frac{v^2}{rg}$$

$$\theta = \tan^{-1}\frac{(29.17)^2}{1200\times9.8} = \tan^{-1} 0.0723$$

$\theta = \mathbf{4.14°}$

🗣 Ideal banking angle is **<u>4.14</u>°**.

***Q.No.8*: A 1500 kg car moving on a flat, horizontal road negotiates a curve as shown in fig. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.**

***Data*:**

Mass of the car m = 1500 kg

Radius of the curve r = 35.0 m

Coefficient of static friction $\mu_s = 0.523$

Max. safe speed v =?

***Solution*:**

In this case the centripetal force is provided by force of friction between road and tires of the car, therefor,

Force of friction = centripetal force

$$\mu_s R = \frac{m v^2}{r}$$

But normal reaction R = m g (weight of car)

$$\mu_s \cancel{m} g = \frac{\cancel{m} v^2}{r}$$

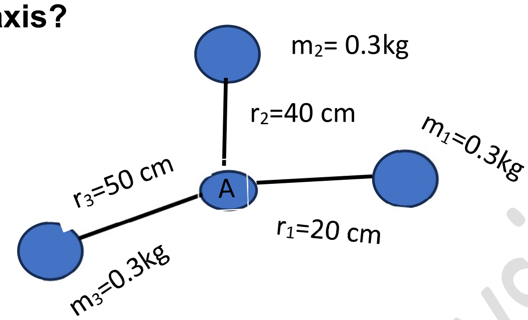
$$v^2 = \mu_s g r = 0.523 \times 9.8 \times 35$$

$$v^2 = 179.389$$

$$v = \sqrt{179.389} = \mathbf{13.394\ m/s.}$$

🗣 Maximum safe speed the car can have is **<u>13.394 m/s</u>.**

***Q.No.9*: A system of points shown in fig. Each particle has same mass of 0.3 kg and they all lie in the same plane. What is the moment of inertia of the system about given axis?**




***Data*:**

Mass $m_1 = m_2 = m_3 = 0.3$ kg.

Distance $r_1 = 20$ cm. = 0.20 m.

$r_2 = 40$ cm. = 0.40 m.

$r_3 = 50$ cm. = 0.50 m.

Moment of inertia of the system = ?

***Solution*:**

Let the axis of rotation be passing through "A", perpendicular to the plane of paper. Hence:

Total moment of inertia of the system

$= m_1 (r_1)^2 + m_2 (r_2)^2 + m_3 (r_3)^2$

$= 0.3 (0.2)^2 + 0.3 (0.4)^2 + 0.3 (0.5)^2$

$=$ **<u>0.135 kg.m²</u>.**

🗣 Total moment of inertia of the system is **<u>0.135 kg.m²</u>.**

***Q.No.10*:(a) What is the angular momentum of a 2.9 kg uniform cylindrical grinding wheel of radius 20 cm when rotating 1550 rpm?**

**(b) How much torque is required to stop it in 6 s?**

***Data*:**

(a) Angular momentum L = ?

Mass of the wheel M = 2.9 kg.

Radius of the wheel R =20 cm = 0.20 m

Angular speed $\omega$ =1550 rpm.

$= 1550 \times 2\pi/60$ rad./s = 162.32 rad./s

(Angular speed "$\omega$" is given in rpm "revolution per minute", angle subtended in each revolution is $2\pi$ radian. ∴ 1 rpm = $2\pi$ radian in 60 second)

(b) Torque required $\tau$ =?

Time to stop t = 6 s.

***Solution*:**

(a) Angular momentum is given $L = I\,\omega$

But $I = \frac{1}{2} M R^2$ For cylindrical wheel

Therefore, $L = \frac{1}{2} M R^2 \omega$

$L = \frac{1}{2}\, 2.9 \times (0.20)^2 \times 162.32 =$ **<u>9.414 kg-m²/s</u>.**

(b)Since torque "$\tau$" is the rate of change of angular momentum, and we have to stop the rotating grinding wheel, therefore, its final angular momentum "$L_f$" must be taken as zero.

$$\tau = \frac{\Delta L}{\Delta t} = \frac{L_f - L_i}{\Delta t} = \frac{0 - 9.414}{6} = \mathbf{-1.569\ N\text{-}m}$$

🗣 Angular momentum is **<u>9.414 kg-m²/s</u>** and **<u>–1.569 N-m</u>** toque will be required to stop it.

***Q.No.11*: Determine the angular momentum of the earth**

**(a) about its rotation axis (Assuming the earth as a uniform sphere).**

**(b) in its orbit around the sun (Take Earth as a particle orbiting the sun).**

**The earth has mass $6\times10^{24}$ kg and radius $6.4\times10^{6}$ m. and is $1.5\times10^{8}$ km from the sun.**

***Data*:**

Angular momentum of the earth L =?

(a) about its rotation axis =?

(b) about its orbit around the sun =?

Mass of the earth $M = 6\times10^{24}$ kg

Radius of the earth $R_e = 6.4\times10^{6}$ m.

Distance between earth and the sun

$R = 1.5\times10^{8}$ km.$= 1.5\times10^{8}\times10^{3}$ m$=1.5\times10^{11}$m

***Solution*:**

(a) Angular momentum (in its rotation axis): $L = I\,\omega$

But $I = \frac{2}{5} M R^2$ (For sphere) and $\omega = \frac{2\pi}{T}$

Where T is the time in which earth completes one rotation about its axis (24 hours)

$= 24 \times 60 \times 60$ s = 86400 s.

$L = \frac{2}{5} M R_e^2 \times \frac{2\pi}{T}$

$L = \frac{2 \times 6\times10^{24} \times (6.4\times10^6)^2 \times 2\pi}{5 \times 86400}$

**$L = 7.149 \times 10^{33}$ kg-m²/s.**

(b) Angular momentum of earth in its orbit around the sun $L = I\,\omega$

But $I = MR^2$ and $\omega = \frac{2\pi}{T}$

Where "T" is the time in which earth completes one revolution around the sun

$= 365 \times 24 \times 60 \times 60 = 31{,}536{,}000$ s.

and "R" is radius of earth orbit (distance between sun and center of earth)

$= R_e + R = 6.4\times10^6 + 1.5\times10^{11} = 1.5\times10^{11}$m.

$L = I\,\omega = \frac{MR^2 \times 2\pi}{T} = \frac{6\times10^{24}\,(1.5\times10^{11})^2 \times 2\pi}{31{,}536{,}000}$

**$L = \underline{2.69\times10^{40}}$ kg-m²/s.**

🗣 Angular momentum of earth in its rotation axis is **$\underline{7.149 \times 10^{33}}$ kg-m²/s** and in its orbit around the sun is **$\underline{2.69\times10^{40}}$ kg-m²/s.**

***Q.No.12*: A car is traveling on a flat circular tack of radius 200 m at 30 m/s and has a centripetal acceleration $a_c = 4.5$ m/s².**

**(a) If mass of the car is 1000 kg what frictional force is required to provide the acceleration?**

**(b) If the coefficient of static friction '$\mu_s$' is 0.8, What is the maximum speed at which the car can circle the track?**

***Data*:**

| | |
|---|---|
| Radius of the track | r = 200 m |
| Speed of the car | v = 30 m/s |
| Centripetal acceleration $a_c$ | = 4.5m/s² |
| Mass of the car | m = 1000 kg |
| (a) Frictional force | $f$ =? |
| (b) Coefficient of static friction | $\mu_s$=0.8 |
| Maximum speed | v =? |

***Solution*:**

(a) The car is travelling on a circular track, centripetal force required is provided by force of friction between car tires and the road, therefore:

$f = F_c$ but $F_c = m\,a_c$

$\therefore$ $f = m\,a_c$

$f = 1000 \times 4.5$ **$f = 4500$ N**

***Alternate method*:**

$f = \frac{m v^2}{r} = \frac{1000\,(30)^2}{200} = \frac{1000 \times 900}{200}$

**$f = 4500$ N**

(b) Maximum force of friction between tires and the road when coefficient of static friction is '$\mu_s$' is given by:

$f = \mu_s R = \mu_s m g = 0.8 \times 1000 \times 9.8 = 7840$ N

This max. force of friction provides centripetal force required for moving the car along the circular track with maximum speed, hence:

$f_{max.} = \frac{m\,v_{max.}^2}{r}$

$7849 = \frac{1000 \times v_{max.}^2}{200}$

$v_{max.}^2 = \frac{7849 \times 200}{1000} = 1568$

$v_{max.} = \sqrt{1568}$ **$v_{max.} = 39.6$ m/s**

🗣 The car can circle with a maximum speed of ***39.6 m/s***. If the car exceeds this speed it will skid out of the circular track.

***Q.No.13*: Tarzan swinging on a win of length 4 m in a vertical circle under the influence of gravity. When the win makes an angle of 20° with the vertical, Tarzan has a speed of 5 m/s.**

**Find his centripetal acceleration at this instant. (*2016, 2013 Karachi Board*)**

***Data*:**

| | |
|---|---|
| Length of the wine = Radius of the circular path | r = 4 m. |
| Angle with vertical | $\theta$ = 20°. |
| Tangential speed | v = 5 m/s. |
| Centripetal acceleration | $a_c$ =? |

***Solution*:**

Centripetal acceleration of Tarzan is given by:

$a_c = \frac{v^2}{} = \frac{(5)^2}{} = \frac{25}{}$

r 4 4

$a_c = 6.25\ m/s^2$.

🗣 Centripetal acceleration of Tarzan is ***6.25 m/s²***.

***Q.No.14***: **A string 1 m long would break when its tension is 69.6 N.**

**Find the greatest speed at which a ball of mass 2 kg. can be whirled with the string in a vertical circle.**

**(*1996 Karachi Board*)**

***Data***:

Length of the string = radius of the circle $r = 1m.$

Tension in the string $T = 69.6N$

Mass of the ball $m = 2\ kg.$

Maximum speed of the ball $V = ?$

***Solution***:

When the ball is whirled in a vertical circle tension in the string will be maximum when the ball is at the lowest level from the ground, in this position tension 'T' in the string will be vertically upward and weight 'mg' of the ball will be vertically downward. The unbalanced force (upward, towards the center) which provides the necessary centripetal force to move the ball in circle will be T– mg, therefore: $T - mg = \frac{m V^2}{r}$

$$69.6 - 2 \times 9.8 = \frac{2 V^2}{1}$$

$$69.6 - 19.6 = 2 V^2$$

$$50 = 2 V^2$$

$$V^2 = 25$$

$$V = \sqrt{25}$$

$$V = 5\ m/s.$$

🗣 The greatest speed with which the ball can be whirled is ***5 m/s***.

**Unit. 5**..... ✍

# Work, energy and power

## Work:

***Q.No.1*: What is work?**

***Ans*:** Work is said to be done by a force when it displaces a body in its own direction.

OR

Work is the scalar or dot product of force "$\vec{F}$" and displacement "$\vec{d}$" Hence it follows laws obeyed by scalar product of vectors.

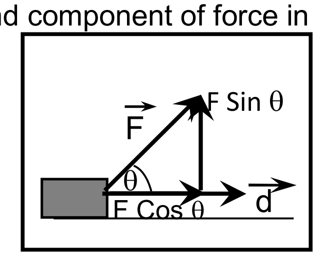
$$\boxed{Work = \vec{F} \cdot \vec{d}}$$

The magnitude of work is given by:

$$Work = F\,d\,\cos\theta$$

Where "θ" is the angle between force "$\vec{F}$" and displacement "$\vec{d}$".

***In other words***, work done is equal to the product of displacement and component of force in the direction of displacement. Hence if a force makes some angle with the direction of displacement then:




$$Work = (F\cos\theta)\,d$$

Where F Cos θ is the component of force in the direction of displacement.

***Q.No.2*: Is work a vector quantity?**

***Ans*:** Work is a ***scalar quantity*** because it does not have a direction, although it is the product of two vectors, force "$\vec{F}$" and displacement "$\vec{d}$".

***Q.No.3*: When is work done *maximum*?**

***Ans*: *Maximum*** amount of work is done when force "$\vec{F}$" is in the direction of displacement "$\vec{d}$" (or direction of motion, when angle between them θ = 0°).

$$Work = F\,d\,\cos\theta$$

$$Work = F\,d\,\cos 0^\circ$$

$$\boxed{Work_{max.} = F\,d} \qquad (\cos 0^\circ = 1)$$

***Q.No.4*: When is work done *zero*?**

***Ans*:** Work done is zero when force "$\vec{F}$" is perpendicular to displacement "$\vec{d}$" i.e. the angle between them is 90°.

$$Work = F\,d\,\cos\theta$$

$$Work = F\,d\,\cos 90^\circ$$

$$\boxed{Work_{min.} = 0} \qquad (\cos 90^\circ = 0)$$

- When you walk on a level floor carrying a brick you do no work on the brick because the vertical force supporting the block has no component in the direction of horizontal motion.
- When a body moves along a circular path, force responsible for its circular motion is

called centripetal force. work done by the centripetal force on the body is **<u>zero</u>**.

- When a body slides along a surface, work done by the normal reaction acting on the body perpendicular to the plane is zero.

In all these examples Force "$\vec{F}$" & displacement "$\vec{d}$" are perpendicular to each other, $\theta = 90°$.

(***<u>Note that</u>*** work may be done by other force/s but not by the forces mentioned above)

***<u>Q.No.5</u>: When is work done <u>negative</u>?***

***<u>Ans</u>*:** Work done is <u>negative</u> when a force is <u>opposite</u> to the direction of displacement i.e. the angle between them is 180°.

$$\text{Work} = F\,d\,\text{Cos}\,\theta$$

$$\text{Work} = F\,d\,\text{Cos}\,180°$$

$$\boxed{\text{Work} = -F\,d} \qquad (\text{Cos}\,180° = -1)$$

Work done by force of friction is negative, because force of friction is opposite to the direction of motion (Or direction of displacement) of the body.

***<u>Q.No.6</u>: When a body is <u>lifted</u> what is the sign of work done <u>on the body</u> by:***

***(1) lifting force. (2) gravitational force.***

***<u>Ans</u>*:** When a body is lifted:

(1) Work done <u>on</u> the body by lifting force is <u>positive</u> because the direction of lifting force is <u>same</u> as the direction of vertical displacement of the body (both are upward) i.e. the angle "$\theta$" between the direction of force and displacement is 0°.

(2) Work done <u>on</u> the body by the gravitational force of the earth is <u>negative</u> because the direction of gravitational force (downward) is <u>opposite</u> to the direction of vertical displacement (upward) of the body i.e. the angle "$\theta$" between the direction of force and displacement is 180°.

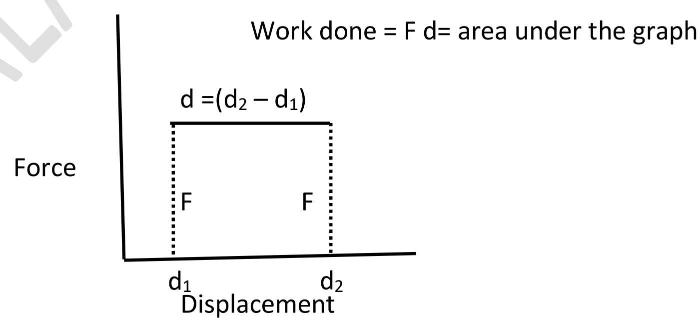
***<u>Q.No.7</u>: What is the <u>S.I unit of work</u> and its dimensions?***

***<u>Ans</u>*:** The M.K.S or S.I unit of work is "Joule J" (Basically it is N. m).
Work done is said to be "1 J" if a force of "1 N" displaces a body through "1 m".

- Dimensions of work are $[ML^2T^{-2}]$.

# ***<u>Work from Force - displacement graph</u>:***

Shape of a graph between force $\vec{F}$ and displacement $\vec{d}$, depends upon whether the force in magnitude and direction throughout the displacement is constant or not.
If magnitude of the force is constant and it is in the direction of displacement then the graph between force and displacement will be a straight line parallel to displacement axis. In this case the amount of work done will be equal to the <u>area under the straight-line</u> graph.



# ***Work done by a variable force*:**

(1) If the magnitude of force is variable (increasing or decreasing) and the direction of force remains the same throughout motion of the body, then shape of the graph between force and displacement depends upon the changes in the magnitude of force. In such a case the amount of work done can be determined first by dividing the entire displacement into a large number of small displacements, each displacement $\Delta d$ must be so small that the force has practically a constant value throughout that particular small displacement. Then work done during each small displacement is calculated. Sum of all these will give us the total amount of work done.

In this case amount of work done from the graph will be equal to the area under force-displacement curve (The shape of the curve depends upon the type of variations in the magnitude of force).

(2) If the variable force is such that both its magnitude and direction are continuously changing with time then in that case work done can be calculated by finding the work done during small displacements using magnitudes of force during that small displacement, displacement $\Delta d$ and angle between them and then finding their sum.

$$\text{Total work done} = \sum \vec{F} \cdot \vec{\Delta d}$$

🗣(***Note that*** when work is done ***on*** a body it gains and when work is done ***by*** a body it loses some form of energy).

# ***Energy*:**

***Q.No.1*: What is energy?**

***Ans*: *Energy is the ability of doing work*.**

If a body is capable of doing work it has energy.

The amount of energy possessed by a body will be equal to the amount of work which a body can do.

🗣If work is done on a body it gains an equal amount of some form of energy, and when work is done by the body it loses the same amount of energy.

🗣Energy is a ***scalar*** quantity.

🗣Energy has many forms such as mechanical, electrical, magnetic, heat, sound, nuclear, elastic energy etc. These forms of energy are inter-convertible; it means that one form of energy can be converted into another form.

***Q.No.2*: What is the *S.I unit of energy*, and dimensions of energy?**

***Ans*:** Unit of energy is same as the unit of work. S.I unit of energy is "***Joule***".

- Dimensions of energy are $[ML^2T^{-2}]$

# ***Kinetic energy (K.E)*:**

***Q.No.1*: What is kinetic energy?**

***Ans*:** Energy possessed by a body by virtue of its ***motion*** is called kinetic energy. When a body is in motion it is capable of doing work. This ability of doing work when a body is in motion is called kinetic energy.

***Q.No.2*: On what factors does the kinetic energy of a body depend?**

***Ans*:** K.E possessed by a body is directly proportional to its mass "m" and to the square of its velocity "$V^2$".

🗣***It means that*** K.E of the body changes in the same way and in the same proportion as its mass is changed i.e. if mass of a body is doubled its K.E will be doubled, if mass is halved its K.E will also be halved, if mass is increased by four times its K.E will also increase by four times and so on.

🗣 K.E of a body depends on square of its velocity, hence K.E of the body changes by square of number of times its velocity is changed. ***In other words***, if velocity of a body is doubled (increased by two times) it's K.E will increase by four times (square of two times), if its velocity is increased by three times (i.e. tripled) then it's K.E will increase by nine times (square of three), If velocity is decreased by two times its K.E will decrease by four times, and so on.

If mass and velocity of a body both are doubled then it's K.E will increase by eight times, because with mass it will increase by two times and with velocity it will increase by four times, hence K.E will increase by eight times. (Total number of times K.E changes is equal to the product of number of times it changes with a change in mass and velocity).

# Derivation:

***Q.No.3*: Derive a formula for kinetic energy (K.E).**

***Ans*:** Kinetic energy possessed by a body is equal to the amount of work which a moving body can do, or K.E gained by a body is equal to the amount of work done on it in moving it.

Consider a body of mass "m" thrown vertically upward with a certain velocity "$V_i$". Velocity of the body decreases as the body rises and becomes zero at the highest point.

Initial K.E given to the body at the point of projection is equal to the work done on it to throw it upward, this K.E is used up to do work against gravity as the body rises up. Hence to calculate K.E possessed by the body at the point of projection we will calculate the work done by the body against gravity.

Let "*h*" be the height reached by the body at which its velocity becomes zero (Or it is the vertical distance through which the body does work against gravity).

Work done by the body is given by:

$$\text{Work done by the body} = \vec{W} \cdot \vec{h}$$

Where "W" is the lifting force.

$\therefore$ $$\text{Work done by the body} = W\,h\,\cos\theta$$

$\therefore$ $$\text{Work done by the body} = W\,h\,\cos 0°$$

($\theta = 0°$ because lifting force and displacement are in the same direction)

$\therefore$ $$\text{Work done by the body} = W\,h \qquad (\cos 0° = 1)$$

$\therefore$ $$\text{Work done by the body} = m\,g\,h$$

In the vertical direction: $V_f = 0$ (at the highest point.

$V_i = V$ (at the point of projection)

$a = -g$

$S = h$ (height to which the body rises).

On substituting the above data in the following equation of motion, we get:

$$2\,a\,S = V_f^{\,2} - V_i^{\,2}$$

$$-2\,g\,h = (0)^2 - (V)^2$$

$$h = \frac{V^2}{2\,g}$$

But $$\text{Work done by the body} = m\,g\,h$$

$\therefore$ $$\text{Work done by the body} = m\,g\,\frac{V^2}{2\,g}$$

$\therefore$ $$\text{Work done by the body} = \tfrac{1}{2}\,m\,V^2$$

Since work done by the body to rise against gravity through a certain height "*h*" is equal to the amount of K.E possessed by it at the point of projection, hence:

$$\boxed{K.E = \tfrac{1}{2}\,m\,V^2}$$

# Gravitational potential energy (P.E):

***Q.No.1*: What is potential energy?**

***Ans*:** Energy possessed by a body by virtue of its ***position*** or ***configuration*** in the gravitational field is called gravitational potential energy.

Potential energy is of many types, such as gravitational P.E, electric P.E, elastic P.E, magnetic P.E, etc.

Gravitational P.E is the energy due to position of a body in the gravitational field. It is equal to the amount of work which the body can do while moving in the direction of gravitational field or the amount of work done on the body to move it against the direction of gravitational field.

But electric P.E is the energy possessed by a charged body due to its position in the electric field.

A body gains P.E when work is done on it but it loses P.E when work is done by it.

# ***Derivation:***

***Q.No.2: Derive a formula for gravitational potential energy?***

***Ans:*** Consider a body of mass "m" lifted through a certain vertical height "$h$", since the body is moved against gravity hence work has to be done on it. This work done on the body will be stored as the gravitational potential energy.

But work done on the body to lift it is (by the lifting force) = m g $h$

$$\therefore \qquad \boxed{P.E = m\,g\,h}$$

The above formula to calculate P.E of a body is valid only when the force "m g" applied to lift the body is constant throughout the displacement "$h$". This is possible only when "$h$" is small (of the order of tens of meters) but if the displacement is large (of the order of thousands of meters) then the force applied no longer remains constant throughout the displacement. Hence the above formula is not a general formula to calculate gravitational P.E it is applicable only to small ordinary vertical heights.

# ***Work done against the gravitational force:***

Weight (= mg) of a body is the *force* with which it is attracted by the earth. Work done in the gravitational field of the earth can be positive or negative, depending upon whether the body is allowed to move in the direction of gravity (downward) or it is moved against the direction of gravity (upward).

If a body is allowed to move freely in the direction of gravity (downward, allowed to fall freely) then work is done by the body. It will be positive, as can be seen:

$$\boxed{\text{Work done} = \vec{W} \cdot \vec{h}}$$

Where "$\vec{W}$" is weight of the body and "$\vec{h}$" is the vertical height between initial and final positions of the body, through which the body is allowed to move in the direction of gravity. Since W ∥ $h$ (θ = 0°), the magnitude of work done is given by:

Work done = W $h$ Cos θ

Work done = W $h$ Cos 0° (Cos 0° = 1)

Work done = W $h$

Work done = m g $h$

Similarly, if a body is lifted (moved upward) through vertical height "$h$" without accelerating it, then the work is done on the body and the amount of work done is given by:

Work done = W $h$ Cos θ

Work done = W $h$ Cos 180° (Cos 180°= −1)

Work done = − W $h$

Work done = − m g $h$

It shows that work done against gravity (by the gravitational force, weight) is negative.

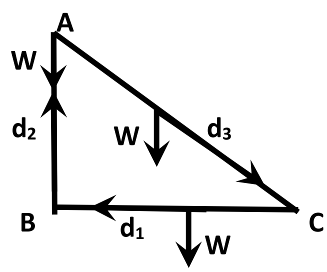
(***Note that*** when work is done ***on*** a body it gains and when work is done ***by*** a body it loses some form of energy).

***Q.No.3**: Prove that gravitational field is a conservative?* (***2016 Karachi Board***)

***Ans**: **Proof**:*

**Any force field is said to be a conservative field if work done in it is independent of path followed and the net amount of work done in a closed path is zero.**

To prove that the gravitational field is conservative, we will move a body in a closed path such as CBAC and calculate the net work done. The body is moved from C to B then B to A and finally from A to C. It means that initial and final positions of the body is point C. ***In other words***, the body is moved in a closed path CBAC.




W (= mg) is weight of the body.

$\text{Work}_{\text{C to B}} = W\, d_1 \cos 90^\circ = 0$ (angle between W and $d_1$ is 90°)

$\text{Work}_{\text{B to A}} = W\, d_2 \cos 180^\circ = -W\, d_2 = -m\, g\, d_2$ (angle between W & $d_2$ is 180°, Cos180° = –1)

$\text{Work}_{\text{A to C}} = W\, d_3 \cos\theta$ (But in triangle ABC, $d_3 \cos\theta = d_2$)

Therefore, $\text{Work}_{\text{A to C}} = W\, d_2 = m\, g\, d_2$

Hence network in closed path CBAC will be:

$$\text{Work}_{\text{CBAC}} = \text{Work}_{\text{C to B}} + \text{Work}_{\text{B to A}} + \text{Work}_{\text{A to C}}$$

$$\text{Net work}_{\text{CBAC}} = 0 - m\, g\, d_2 + m\, g\, d_2$$

$$\text{Net work done}_{\text{In closed path CBAC}} = 0$$

This shows that work done in the gravitational field is ***independent of path followed*** it depends only upon the vertical height $d_2$ (or *h*) between initial and final positions, also that ***net work done in a closed path is zero***. Hence gravitational field is a ***conservative field***.

Gravitational and electric fields are conservative fields.

***Note that*** in a conservative field work is always ***reversible***. For example, when we throw a ball up in the air, it slows down as its kinetic energy is used to do work against gravity, it is converted into the gravitational potential energy. But on the way down, the potential energy is used to do the same amount of work in moving the body, hence it is converted back into kinetic energy and the ball speeds up. If there is no air resistance, the ball will land as fast as it was thrown.

# General formula for gravitational potential energy.

***Q.No.4*: Derive a general formula for gravitational potential energy?**

***Ans*:** To derive a general formula for gravitational P.E we apply Newton's law of gravitation.

Consider a body of mass "m" lifted from point "1" to point "N" in the gravitational field of the earth through a **long distance**. Force between the body and earth over such a long distance will not be constant over such a long distance.

Hence to calculate the amount of work done to displace the body, we will divide the entire displacement into a large number of small displacements each must be so small that the magnitude of the force through that small displacement practically remains constant.

According to Newton's law of gravitation, the magnitude of force between the body and the earth at points 1 and 2 is given by:

$$F_1 = G\frac{m M_e}{r_1^2} \qquad F_2 = G\frac{m M_e}{r_2^2}$$

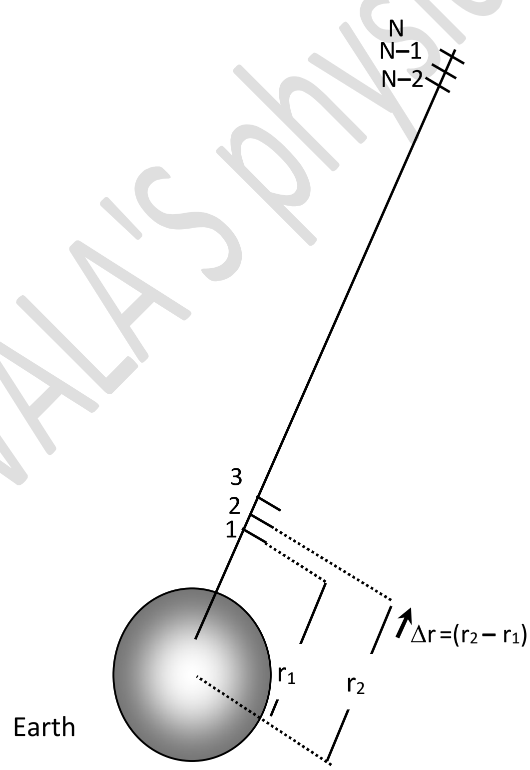
Where "$M_e$" is mass of the earth and "m" is mass of the body, $r_1$ and $r_2$ are the distances of points 1 and 2 respectively from the center of the earth.

Let "$\Delta r$" be the magnitude of displacement of the body from 1 to 2, then the average force between earth and the body between these points will be:

$$F_{av.} = \frac{F_1 + F_2}{2}$$

$$F_{av.} = \frac{G\frac{m M_e}{r_1^2} + G\frac{m M_e}{r_2^2}}{2} = \frac{Gm M_e\left\{\frac{1}{r_1^2} + \frac{1}{r_2^2}\right\}}{2}$$

$$F_{av.} = \frac{Gm M_e}{2}\left\{\frac{1}{r_1^2} + \frac{1}{r_2^2}\right\}$$




Work done to displace the body from 1 to 2 through "$\Delta r$" is given by:

$$\underset{1\to2}{\text{Work}} = F_{av.}\,\Delta r \quad \text{Where } \Delta r = (r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{1}{r_1^2} + \frac{1}{r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{r_2^2 + r_1^2}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

But $r_2 = r_1 + \Delta r$ $\therefore$

$$\underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{(r_1 + \Delta r)^2 + r_1^2}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$\therefore$

$$\underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{r_1^2 + 2r_1\Delta r + \Delta r^2 + r_1^2}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

Since $\Delta r$ is small $\therefore \Delta r^2$ can be neglected.

$$\underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{r_1^2 + 2r_1\Delta r + r_1^2}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{2r_1^2 + 2r_1\Delta r}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = \frac{Gm\,M_e}{2}\left\{\frac{2r_1(r_1 + \Delta r)}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = Gm\,M_e\left\{\frac{r_1(r_1 + \Delta r)}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = Gm\,M_e\left\{\frac{r_1(r_1 + r_2 - r_1)}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = Gm\,M_e\left\{\frac{r_1 r_2}{r_1^2\,r_2^2}\right\}(r_2 - r_1)$$

$$\underset{1\to2}{\text{Work}} = Gm\,M_e\left\{\frac{r_2}{r_1 r_2} - \frac{r_1}{r_1 r_2}\right\}$$

$$\underset{1\to2}{\text{Work}} = Gm\,M_e\left\{\frac{1}{r_1} - \frac{1}{r_2}\right\}$$

Since work done ***on*** a body to displace it against the gravitational force of the earth is equal to the P.E gained by it, therefore:

$$\underset{1\to2}{P.E} = Gm\,M_e\left\{\frac{1}{r_1} - \frac{1}{r_2}\right\}$$

Similarly:

$$\underset{2\to3}{P.E} = Gm\,M_e\left\{\frac{1}{r_2} - \frac{1}{r_3}\right\}$$

$$\underset{3\to4}{P.E} = Gm\,M_e\left\{\frac{1}{r_3} - \frac{1}{r_4}\right\}$$

$$\vdots \qquad\qquad \vdots$$

$$\underset{N-2\to N-1}{P.E} = Gm\,M_e\left\{\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}}\right\}$$

$$\underset{N-1\to N}{P.E} = Gm\,M_e\left\{\frac{1}{r_{N-1}} - \frac{1}{r_N}\right\}$$

The total P.E of the body between point 1 and N will be equal to the sum of the above equations and is given by:

$$\boxed{\underset{1\to N}{P.E} = Gm\,M_e\left\{\frac{1}{r_1} - \frac{1}{r_N}\right\}}$$

🗣The above equation actually gives us ***Change in potential energy*** of the body (in this case P.E gained) between any two points such as 1 and N in the gravitational field of the earth. It is equally valid for ***short*** as well as ***long*** distance, hence it is a general formula for P.E gained or lost between any two points in the gravitational field of the earth.

# Absolute potential energy:

***Q.No.5*: What is *absolute potential energy* of a body at a point?**

***Ans*:** Absolute potential energy of a body at a point in the gravitational field is ***the amount of work done to move the body from that point to a very far off point.***

A point which is very far from the center of the earth may be considered to be outside the gravitational field of the earth, hence the potential energy of a body at such a point will be zero.

Absolute potential energy of a body at point "1" can be calculated by assuming point "N" to be at infinity, hence on substituting $r_N = \infty$ in the following formula we get:

$$P.E_{1\to N} = Gm M_e \left\{ \frac{1}{r_1} - \frac{1}{r_N} \right\}$$

Absolute potential energy at point "1" is given by:

$$P.E_{abs.} = Gm M_e \left\{ \frac{1}{r_1} - \frac{1}{\infty} \right\}$$

But $\frac{1}{\infty} = 0$ $\quad\therefore$

$$\boxed{P.E_{abs.} = \frac{Gm M_e}{r_1}}$$

Since the gravitational force is attractive in nature, therefore, absolute potential energy of a body is given by: $\quad\therefore$

$$\boxed{P.E_{abs.} = -\frac{Gm M_e}{r_1}}$$

***It means that*** if a body is moved from a given point to infinite distance its potential energy increases and becomes zero at infinity (It is a negative quantity).

Absolute potential energy of a body of mass "m" at the surface of the earth is given by:

$$P.E_{abs.} = -\frac{Gm M_e}{R_e}$$

Similarly absolute potential of the body at any height "h" above the surface of the earth is given by:

$$P.E_{abs.} = -\frac{Gm M_e}{R_e + h}$$

$$P.E_{abs.} = -\frac{Gm M_e}{R_e\left\{1 + \frac{h}{R_e}\right\}}$$

$$P.E_{abs.} = -\frac{Gm M_e}{R_e}\left\{1 + \frac{h}{R_e}\right\}^{-1}$$

According to Binomial theorem:

$$\left\{1 + \frac{h}{R_e}\right\}^{-1} = 1 - \frac{h}{R_e} + \frac{h^2}{R_e^2} - \ldots\ldots\ldots$$

Since "h" is small as compared to "$R_e$", therefore, terms with higher powers of "h" are negligible,

$$\left\{1 + \frac{h}{R_e}\right\}^{-1} = \left\{1 - \frac{h}{R_e}\right\}$$

Therefore,

$$\boxed{P.E_{abs.} = -\frac{Gm M_e}{R_e}\left\{1 - \frac{h}{R_e}\right\}}$$

# ***Escape velocity***:

***Q.No.1*: What is escape velocity?**

***Ans*:** Escape velocity of a planet is the **minimum velocity** with which a body must be projected up so that it escapes gravity of that planet and never return back.

**Explanation**:

When we throw a body up, it rises against gravity of the earth and after reaching a certain height it starts falling back. Actually while throwing up we do some work on the body, this work done is stored in the body as kinetic energy. Due to this K.E the body now has the ability to do work against gravity. As the body rises up its K.E is used to do work against gravity. After reaching a certain height this K.E is totally used up. (K.E lost is stored as P.E) At this instant the body comes to rest for a moment and then it starts falling because of the gravitational pull.

If we keep on throwing the body up with more and more velocity, **in other words**, give it more K.E it rises higher each time it will return back to Earth.

If thrown vertically upward with a certain velocity, called **escape velocity**, the body escapes gravity of the earth. Thrown with such a high velocity body has now enough K.E just to overcome gravitational pull. It now follows a parabolic path and does not return to the earth.

## Derivation:

Minimum velocity required to escape the gravity (OR escape velocity) can be derived with the help of law of conservation of energy.

At the surface of earth total energy of the body will be equal to the sum of its initial K.E and Initial P.E. Similarly at the highest point its total energy will be equal to the sum of its final K.E and P.E. Hence according to law of conservation of energy:

$$\text{Total initial energy} = \text{Total final energy}$$

$$K.E_{\,initial} + P.E_{\,initial} = K.E_{final} + P.E_{\,final}$$

$$\frac{1}{2} m V_{initial}^{\;2} + \left(-\frac{Gm M_e}{R_e}\right)_{initial} = \frac{1}{2} m V_{final}^{\;2} + \left(-\frac{Gm M_e}{r}\right)_{final}$$

Where $M_e$ = mass of the earth. $R_e$ = radius of the earth. m = mass of the body

G = universal gravitational constant V = velocity of the body.

r = total distance from center of earth to highest point.

Potential energy is **negative** showing that it is attractive in nature.

If the body is thrown with escape velocity "$V_{esc}$" then reaching the highest point its K.E is totally used up ( $K.E_{final} = 0$) this will happen at infinite distance where its $P.E_{\,final}$ will also be zero, here the body will not have the ability to fall back.

Applying law of conservation of energy we get:

$$\frac{1}{2} m V_{esc}^{\;2} - \frac{G m M_e}{R_e} = 0 + 0$$

$$V_{esc}^{\;2} = 2\frac{G M_e}{R_e}$$

$$V_{esc} = \sqrt{\frac{2 G M_e}{R_e}}$$

This formula gives us escape velocity at the surface of earth ($M_e$ and $R_e$ are mass and radius of earth).

> **Example: Escape velocity for earth.**
> $V_{esc.} = \sqrt{2 G M_e / R_e}$
> $V_{esc.} = \sqrt{2\times 6.67\times10^{-11}\times 5.98\times10^{24} / 6.38\times10^{6}}$
> $V_{esc.}$ = **11182 m/s = 11.182 km/s.**
> Escape velocity on the surface of earth at different locations may be slightly different because average radius of the earth is used in this formula. Radius of earth is different at different locations, Earth bulges out at the equator, here radius of earth is more, flattens a little at poles, here radius of the earth is small.

This formula can be used to calculate escape velocity for any planet, using mass of the planet and its radius.

Escape velocities required for some planets using this formula are:

For earth: $V_{esc}$ **= 11.2 km./sec.** For moon: $V_{esc}$ **= 2.38 km./sec.**

For sun: $V_{esc}$ **= 618 km./sec.**

We can see from these values that escape velocity depends upon **mass** and **radius** of a planet. Acceleration due to gravity "g" on the surface of a planet can also be used to calculate its escape velocity. Since

$$g = \frac{G\,M}{R^2}$$

On substituting the value of g in escape velocity formula and then simplifying it we get:

$$V_{esc} = \sqrt{2\,g\,R}$$

- This formula can be used to calculate escape velocity of any planet provided the acceleration due to gravity "g" on its surface and its radius "R" are known.

Both the above formulae give us the same value of escape velocity for a planet.

If velocity of a body is less than the escape velocity the body will return back to the given planet. Hence escape velocity is the **minimum velocity required to escape the gravity of a planet**.

# Power:

***Q.No.1: What is power? Give its dimension.***

***Ans:*** ***"Rate of doing work or rate of transferring energy is called power".***
(or it is also equal to the rate of conversion of energy from one form to another).

OR

Power is the **scalar** or **dot product** of force $\vec{F}$ and velocity $\vec{v}$.

- Dimensions of power are $[M\,L^2\,T^{-3}]$.
- Power is a **scalar** quantity.

$$\text{Power} = \frac{\text{work done or energy transferred}}{\text{Time taken}}$$

$$\text{Power} = \frac{\vec{F}\cdot\vec{d}}{t}$$

But $\frac{\vec{d}}{t} = \vec{v}$

$$\boxed{\text{Power} = \vec{F}\cdot\vec{v}}$$

Average power is given by:

$$\text{Power}_{Av.} = \frac{\text{Work done}}{\text{Total time taken}}$$

$$\text{Power}_{Av.} = \frac{\vec{F}\cdot\vec{d}}{\Delta t}$$

OR

$$\text{Power}_{Av.} = \vec{F}\cdot\vec{V}_{Av.}$$

Instantaneous power "Power $_{Inst.}$" (power at a particular instant or at a particular time) can be calculated by imposing a limit on time taken, in this case time taken must be very short and must approach to zero (not equal to zero).

$$\text{Power}_{Inst.} = \vec{F}\cdot \lim_{\Delta t \to 0} \frac{\vec{d}}{\Delta t}$$

OR

$$\text{Power}_{Inst} = \vec{F}\cdot\vec{V}_{inst.}$$

Power is a ***scalar*** quantity.

***Q.No.2: What is the S.I unit of power?***

***Ans:*** Unit of power is "***watt***".

Power is said to be 1 watt when 1 joule of work is done in one second.

$$1\text{ watt} = \frac{1\text{ Joule}}{1\text{ second}} \qquad 1\text{ W} = 1\text{J/s}$$

1 kilowatt (kw) = $10^3$ w & 1 megawatt (Mw) = $10^6$ w

Since Power = $\frac{\text{Work or energy}}{\text{Time}}$ ∴ Work (Or energy) = Power × time

Hence the unit of work (or energy) can also be expressed as a product of unit of power and the unit of time.

If power is expressed in kilowatt (kw) and time in hours (h) then the unit of work (Or energy) will be ***kilowatt-hour (kwh)***:

1 kilowatt-hour (kwh) = 1 kw ×1h

1 kwh = 1000 w × 3600 s (1w = 1j/s)

1 kwh = 3.6 ×$10^6$ Joules.

1 kwh = 3.6 MJ (1 unit of electrical energy)

On commercial scale electrical energy is measured in "kwh".

(We can define 1 kwh (Or one-unit electrical energy) as the power delivered by an electrical machine of one kilowatt running continuously for one hour). As an example. If a pressing iron of 1 kw is continuously on for 1 h it will convert 3.6×$10^6$ Joules of electrical energy into heat energy.

# Law of conservation of energy:

***Q.No.1: State law on conservation of energy?***

***Ans*:** Energy can neither be created nor destroyed.

One form of energy can be converted into another and so on, but the conversion from one form to another is always such that the total amount of energy before and after the conversion is equal.

***Note that*** when a piece of paper is burnt we will get heat, light along with ashes and carbon dioxide. If we study this simple chemical reaction we will find that during this process ***energy is not created nor is the matter destroyed***.

We know that paper is obtained after chemically processing a certain plant. This plant grows using water and fertilizers from the soil and solar energy. (solar energy during photosynthesis, chemical energy during the formation of new bonds etc. are stored). **In other words**, energy was already stored in paper in different forms. This stored energy is converted into heat and light etc. when we burn it. Experiments show that the total energy stored comes out to be exactly equal to the energy obtained after burning process is complete. Similarly, matter is also converted from one state to another, such as from paper and oxygen of air into carbon dioxide and ashes etc. During this process new bonds are formed. (In general, combustion is an ordinary exothermic chemical process).

***Also note that*** energy can neither be created nor destroyed similarly matter can neither be created nor destroyed. ***It means that*** matter and energy cannot be converted into each other. These are old concepts and are valid under normal conditions.

According to Einstein, matter and energy are **interconvertible**. It means that matter can be converted into energy and vice versa, this conversion takes place according to Einstein's famous equation: $E = m c^2$

In **fission and fusion reactions**, applied in atom bomb and hydrogen bomb, tremendous amount of energy is released due to the **conversion of mass into energy**. The amount of energy released reactions (fission or fusion) is enough to destroy a whole city. This energy can also be used for peaceful purpose by converting into some useful form such as electricity as is done in nuclear reactors.

In **pair production** electron - positron pair is produced by **converting energy into matter**. Both electron and positron are matter particles.

# Interconversion of K.E and P.E:

Consider a body of mass "m" held at a certain height "$h$" above the ground. At this point velocity of the body is zero, as it is at rest, hence its kinetic energy will also be zero. Relative to the ground it will have potential energy "mg$h$".

- Now if the body is released and allowed to fall freely, its velocity keeps on increasing as it falls, ***in other words,*** its kinetic energy will **increase**. But since it falls down its height "$h$" above the ground decreases and hence its potential energy also **decreases**. It is found that in the absence of any opposing force (such as air resistance), while falling down body will lose P.E and at the same time gain an equal amount of K.E. Hence P.E possessed by the body at the starting point is totally converted into K.E on reaching the ground. ***In other words***, it will have maximum velocity just before hitting the ground. Hence we can write:

P.E lost = K.E gained

If "V" is the velocity with which the body reaches the ground then:

$$m\,g\,h = \frac{1}{2}\,m\,V^2$$

- If a body is thrown vertically upward with velocity "V" (or thrown upward with some K.E) then while going up against gravity its velocity decreases i.e. its K.E energy decreases, it is used to do work against gravity due to which its P.E will increase. After reaching the highest point its velocity reduces to zero. Here it has maximum P.E and zero K.E. While going up body keeps on loosing its K.E and at the same time gains an equal amount of P.E. In this case"

K.E lost = P.E gained

$$\frac{1}{2}\,m\,V^2 = m\,g\,h$$

🗣While falling down **if there is an opposing force** (any frictional force, such as air resistance) then a part of P.E lost by the body will be used to do work against the opposing force. In this case:

P.E lost = K.E gained + work done against frictional force

$$m\,g\,h = \frac{1}{2}\,m\,V^2 + f\,h$$

Where "$f$" is the force of friction.

In this case the body will come down with **lesser speed**. (lesser K.E)

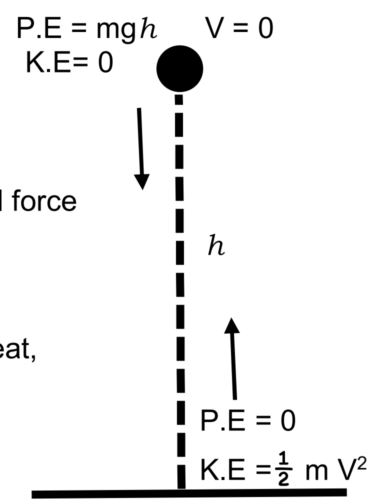
🗣While going up **if there is an opposing force** (any frictional force) then a part of K.E of the body will be used to do work against the opposing force. In this case:

K.E lost = P.E gained + work done against frictional force

$$\frac{1}{2}\,m\,V^2 = m\,g\,h + f\,h$$

In this case the body will rise to a **smaller height**.

**Note that:** energy used to do work in both the cases appears as heat, This heat is immediately taken away by the surrounding atmosphere.



## ***Section–A Multiple-Choice Questions MCQs:***

***Q.No.1***: Work done by centripetal force is always:

(a) Maximum. (b) Minimum.
(c) Zero. (d) None of these.

***Q.No.2***: A body of mass 5 kg. is moving with a momentum of 10 kg m/s. A force of 0.2 N acts on it in the direction of motion of the body for 10 sec. The increase in kinetic energy is:

(a) 2.8 J (b) 3.2 J (c) 3.8 J (d) 4.4 J

***Q.No.3***: Kinetic energy of a light body and a heavy body is same. Which body has maximum momentum?

(a) Light body. (b) Heavy body.
(c) Both have same momentum.
(d) none of them.

***Q.No.4***: Two bodies of masses 1 kg and 2 kg have equal momentum. The ratio of their kinetic energies is:

(a) 2:1 (b) 3:1 (c) 1:3 (d) 1:1

***Q.No.5***: A body falls from height "h". After it has fallen a height h/2, it will possess:

(a) Only potential energy.
(b) Only kinetic energy.
(c) half potential half kinetic.
(d) more kinetic less potential.

***Q.No.6***: Which of the following quantity can be multiplying force and velocity?

(a) acceleration. (b) power.
(c) torque. (d) work.

***Q.No.7***: The minimum velocity given to an object so that it emerges out from the gravitational field of earth is about:

(a)11.2 km/s. (b)15.3 km/s.
(c) 5 km/s. (d) 9.8 km/s

***Q.No.8***: When one joule of work is done on a body in one second, power of body is said to be:

(a) One watt. (b) 0.5 watt.
(c) zero. (d) 100 watts

***Q.No.9***: The absolute potential energy of an object depends on:

(a) Object's mass and height.
(b) Object's mass and speed
(c) Object's shape and size.
(d) Object's color and temperature.

***Q.No.10***: The escape velocity of a planet depends on which of the following factors?

(a) Mass of the planet only.
(b) Radius of the planet only.
(c) Both mass and radius of the planet.
(d) Density of the planet.

***Q.No.11***: The work done by a conservative field along a closed path is: **(*2008 Karachi Board*)**

• Positive. • Negative. • Zero. • None.

***Q.No.12***: When a body moves vertically, the work done will be:

• Positive • Negative •Zero • Maximum

***Q.No.13***: If speed of a body is halved, its kinetic energy becomes: **(*2011 Karachi Board*)**

• One fourth • Half •Three times • None of these

***Q.No.14***: The work done by a conservative force along a closed path is: **(*2011 Karachi Board*)**

• Positive • Negative • Zero • None of these

***Q.No.15***: A bucket of mass 10 kg. is moved downward in the gravitational field through a distance of 1 m. The work done in this case is equal to: **(*2012 Karachi Board*)**

•10 J • 98 J • – 98 J • 0.1 J

***Q.No.16***: The rate of doing work is zero when the angle between force and velocity is: **(*2012 Karachi Board*)**

• 0°. • 45°. • 180°. • 90°.

***Q.No.17***: A weight lifter consumes 500 J of energy to lift a load in 2 second. The power used by him is: **(*2013 Karachi Board*)**

•125 watt. • 250 watt.
• 500 watts. • 1600 watt

***Q.No.18***: If mass and speed both are doubled, the kinetic energy will be: **(*2014,05Karachi board*)**

• Double. • Four times.
• Six times. • Eight times.

***Q.No.19***: Kilowatt hour is the unit of: **(*2014 Karachi board*)**

• Energy. • Power. • Time. • Force.

***Q.No.20*: This one of the following is not the unit of power: (***2015 Karachi Board***)
- horse-power.
- joule/sec.
- kilowatt-hour.
- foot-pound/sec.

***Q.No.21*: Both kilowatt hour and electron volt are the units of: (***2017 Karachi Board***)
- power.
- energy.
- charge.
- angular momentum

***Q.No.22*: A 600N man runs up a stair of 4 m height in 3 seconds. The power needed is: (***2018 Karachi Board***)
- 240 watts.
- 350 watts.
- 450 watts.
- 800 watts.

***Q.No.23*: If velocity a body is doubled and its mass is reduced to one fourth, its kinetic energy will be: (***2022, 18,12 Karachi Board***)
- doubled
- unchanged
- halved
- four-fold

***Q.No.24*: one kilowatt-hour is equal to: (***2022, 2016 Karachi Board***)
- $3.6\times10^6$ J.
- $3.9\times10^6$ J.
- $3.6\times10^8$ J.
- $3.6\times10^9$ J.

***Q.No.25*: The dot product of force and velocity is: (***2022 Karachi Board***)
- Work,
- Power.
- Momentum.
- Energy.

***Q.No.26*: If $\vec{F} = 3\hat{i}$ and $\vec{d} = 6\hat{j}$, then work done will be: (***2022 Karachi Board***)
- $2\hat{j}$
- $3\hat{j}$
- $6\hat{j}$
- $0\hat{j}$

***Q.No.27*: A boy pushes a toy car on a horizontal floor with a force of 10N up to a displacement of 2m, work done by gravity on the car will be: (***2022 Karachi Board***)
- 20J
- 10J
- 5J
- 0

***Q.No.28*: A force acting on a body is perpendicular to its displacement, the work done is equal to: (***2022 Karachi Board***)
- Positive.
- Negative.
- Zero.
- Infinite.

***Q.No.29*: A field of force in which the work is independent of path is called a … field. (***2014 Hyderabad Board***)

(a) Conservative. (b) Non conservative.
(c) Energetic. (d) None of these.

***Q.No.30*: The work done on a body is zero, then angle between the applied force and displacement is: (***2015 Hyderabad Board***)

(a) 0° (b) 45° (c) 90° (d) 180°

***Q.No.31*: The dot product of force and velocity is ………(***2016 Hyderabad Board***)

(a) Work. (b) Power.
(c) Energy. (d)Torque.

***Q.No.32*: The rate of change of energy is called ………. (***2019 Hyderabad Board***)

(a) Work (b) Power
(c) Force (d) None of these

***Q.No.33*: Work done by centripetal force is: (***2025 Karachi Board***)
- Maximum.
- Negative.
- Zero.
- Infinite.

***Q.No.34*: The minimum velocity required for an object to escape earth's gravitational field is approximately: (***2025 Karachi Board***)
- 11.2 km/s.
- 5 km/s.
- 15.3 km/s.
- 9.8 km./s.

## <u>*Answers*</u>:

(1) Zero.
(2) 4.4 J
(3) Heavy body.
(4) 2:1
(5) Half potential half kinetic.
(6) Power.
(7) 11.2 km/s
(8) One watt.
(9) Object's mass and height.
(10) Both mass and radius of the planet.
(11) Zero.
(12) Negative
(13) One fourth
(14) Zero
(15) 98 J
(16) 90°
(17) 250 watts
(18) 250 watts
(19) Energy
(20) kilowatt-hour.
(21) energy.
(22) 800 watts.
(23) unchanged
(24) $3.6\times10^6$ J
(25) Power
(26) 0 $j$
(27) 0
(28) Zero
(29) Conservative.
(30) 90°
(31) Power
(32) Power.
(33) Zero.
(34) 11.2 km/s.

# Numericals:

**Q.No.1: A man pulls a trolley through 10 m by applying a force of 50 N which makes an angle of 60° with the horizontal. Calculate the work done by the man.**

**Data:**

Force applied $F = 50\ N$
Distance covered $d = 10\ m$
Angle between force and displacement $\theta = 60°$
Work done $W = ?$

**Solution:**

Work done is given by $Work = F\ d\ Cos\ \theta$

$Work = 50 \times 10 \times \cos 60°$

$Work = 500 \times 0.5$ **$Work = 250\ J$**

**Q.No.2: A 100 kg man runs up a long stair in 9.8 second. The vertical height of the stairs is 10 m. Calculate its power.**

**Data:**

Mass of man $m = 100\ kg$
Time taken $t = 9.8$ second
Vertical height $h = 10\ m$
Power $P = ?$

**Solution:**

As the man runs up the stairs he does work against gravity, rate of doing work is called power. But work done against gravity is given by:

Work against gravity = $m\ g\ h$

$$Power = \frac{Work}{Time} = \frac{m\ g\ h}{t}$$

$$Power = \frac{100 \times 9.8 \times 10}{9.8}$$ **$Power = 1000\ Watt$**

**Q.No.3: When an object is thrown upward it rises to a height "h". How high is the object in terms of "h" when it has lost one third of its original kinetic energy?**

**Data:**

Total height reached $= h$
Height at which the object looses one third it's K.E $x = ?$

**Solution:**

When an object is thrown upward it looses K.E and at the same time gains an equal amount of P.E, provided there is no air resistance. ∴ K.E lost = P.E gained

$$\therefore \tfrac{1}{2} m v^2 = m g h \quad \text{OR} \quad v^2 = 2 g h$$

Where "v" is the velocity with which the object was originally thrown upward.

Let "x" be the height at which the object losses one third its original K.E.

∴ Amount of K.E lost = P.E gained

$\tfrac{1}{3} \times$ (original K.E) = P.E gained

$$\tfrac{1}{3} \times (\tfrac{1}{2} m v^2) = m g x$$

On substituting the expression for $v^2$ in the above equation we get:

$$\tfrac{1}{3} \times (\tfrac{1}{2} m \times 2 g h) = m g x$$ **$x = \tfrac{1}{3} h$**

**Q.No.4: A 70 kg man runs up a hill through a height of 3 m in 2 second.**

**(a) How much work is done against gravitational field =?**

**(b) What is the average power output?**

**Data:**

Mass m = 70 kg. Height h = 3 m Time t = 2 s.

(a) work done against gravity = ?

(b) power P =?

**Solution:**

(a) Work done against gravity = $W\ h\ Cos\ \theta$

$W = mg$ is man's weight (downward)

Work done = $m\ g\ h\ Cos\ 180°$

(Force W is opposite to displacement h)

Work done = $70 \times 9.8 \times 3 \times -1 = -$ **2058 Joules,**

(b) $Power = \frac{work}{Time} = \frac{2058}{2} =$ **1029 watt**

**Q.No.5: A neutron travels a distance of 12 m in a time interval of $3.6 \times 10^{-4}$ second. Assuming its speed to be constant, find its K.E.**

**Mass of neutron = $1.7 \times 10^{-27}$ kg.**

**Data:**

Distance covered $S = 12\ m$
Time taken $t = 3.6 \times 10^{-4}$ second
Mass of neutron $m = 1.7 \times 10^{-27}$ kg.
Kinetic energy $K.E = ?$

**Solution:**

$$\text{Speed } v = \frac{distance}{Time} = \frac{12}{3.6 \times 10^{-4}} = 3.33 \times 10^4\ m/s$$

$$K.E = \tfrac{1}{2} m v^2 = \tfrac{1}{2} \times 1.7 \times 10^{-27} \times (3.33 \times 10^4)^2$$

**$K.E = 9.44 \times 10^{-19}$ Joules.**

**Q.No.6: A stone is thrown vertically upward so that it can reach a height of 10 m. Find the speed of stone when it is just 2 m above the ground.**

**Data:**

Height h = 10 m
Height above the ground = 2 m
Speed 2 m above ground v = ?

***Solution*:**

K.E with which stone must be thrown so that it reaches 10 m height can be found by:

K.E at the ground = P.E at the highest point

$\frac{1}{2} m v^2 = m g h$

$v = \sqrt{2 g h} = \sqrt{2 \times 9.8 \times 10} = \sqrt{196} = 14$ m/s

Hence speed with which stone is thrown up to reach 10 m height is **14 m/s.**

Considering the downward motion of stone:

$v_i = 0$ at 10 m height, $v_f = ?$ at 2m above ground

$a = g$, $S = 10 - 2 = 8$ m

Applying equation of motion: $2 a S = v_f^2 - v_i^2$

$2 \times 9.8 \times 8 = v_f^2 - (0)^2$

$156.8 = v_f^2$ $v_f = \sqrt{156.8} =$ **12.5 m/s.**

Speed 2 m above the ground will be **12.5 m/s.**

***Q.No.7*: The potential energy of a body at the top of a building is 200 Joule. When it is dropped its K.E just before striking the ground is160 Joule. Find work done against the air resistance.**

***Data*:**

P.E at the top = 200 J

K.E just before striking ground = 160 J

Work done against air resistance W =?

***Solution*:**

While falling:

P.E lost by the body = K.E gained by it + work done against air resistance.

200 = 160 + Work done

Work done against air resistance =200−160 = **40 J**

***Q.No.8*: Find the energy equivalent of 1 gram.**

***Data*:**

Mass m = 1 gram. = 0.001 kg

Energy E =?

***Solution*:**

According to Einstein's matter can be converted into energy and vice versa, according to following equation:

$E = m c^2$

$E = 0.001 \times (3\times10^8)^2$

$E =$ **$9 \times 10^{13}$ Joules**

Energy equivalence of 1 gram is

**$9 \times 10^{13}$ Joules.**

***Q.No.9*: A 1 kilowatt motor pump pumps water from ground to a height of 10 m. Find how much liters of water it can pump in one hour.**

**(Density ρ of water = 1000 kg/m$^3$)**

***Data*:**

| | |
|---|---|
| Power | P = 1 Kw = 1000 watt |
| Time | t = 1 h = 3600 sec. |
| Height | h = 10 m |
| Density of water | ρ = 1000 kg/m$^3$) |

Number of liters pumped= ?

***Solution*:**

Power P = $\dfrac{\text{work to pump water for 1 hour}}{\text{Time}}$

If "m" is mass of water pumped, then

OR P t = m g h

$1000 \times 3600 = m \times 9.8 \times 10$

$m = \dfrac{1000\times3600}{9.8 \times 10} = 36{,}735$ kg.

But density of water $\rho = \dfrac{\text{mass of water m}}{\text{Volume of water V}}$

$V = \dfrac{m}{\rho} = \dfrac{36735}{1000} =$ **36.735 m$^3$.**

But 1 m$^3$ = 1000 liter

Volume of water that can be pumped in one hour will be V = 36.735× 1000 = 36735 lit.

**= 3.67×10$^4$ liters**

***Q.No.10*: A rocket of mass 2 kg. is launched in air, when it attains height of 15 m the 400 Joules of its chemical fuel burns. Find speed of the rocket at maximum height.**

***Data*:**

| | |
|---|---|
| Mass of the rocket | m = 2 kg. |
| Vertical height attained | *h* = 15 m |
| Fuel burnt = Work done | W = 425 J |
| Velocity at maximum height | v = ? |

***Solution*:**

Due to the burning fuel work is done on the rocket and it moves upward against gravity with an increasing velocity. Hence it gains K.E as well as P.E as it rises:

Work done = K.E gained+ P.E gained

Work done = $\frac{1}{2} m v^2 + m g h$

$400 = \frac{1}{2} 2 \times v^2 + 2 \times 9.8 \times 15$

$400 = v^2 + 294$

$v^2 = 400 - 294 = 106$

$v = \sqrt{106}$ **v = *10.3 m/s***

Velocity of rocket at the highest point (15 m above the ground) is ***10.3 m/s***.

***Q.No.11*: A motor pumps water at the rate of 500 gm. /min to a height of 120 m. If the motor is 50% efficient then how much input electric power is needed?**

***Data***:

Mass of water pumped = 500 gm/min = 0.500 kg/60 sec.

Height h = 120 m. Efficiency E = 50%

Input power =?

***Solution***:

$$\text{Output power} = \frac{m\,g\,h}{t} = \frac{0.500\times9.8\times120}{60}$$

Output Power = 9.8 watt.

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}}\times100\%$$

$$\text{Input power} = \frac{\text{output power}}{\text{Efficiency}}\times100\%$$

$$\text{Input power} = \frac{9.8}{50}\times100\% = \mathbf{19.6\ Watt}$$

**19.6 Watt** (or approximately **20 Watt**) Input power will be needed.

***Q.No.12*****:Calculate the work done by a force given by $\vec{F} = 6\hat{i} + 8\hat{j} + 10\hat{k}$ in displacing a body from the position A to the position B. The position vectors of A and B are:**

$$\vec{r}_A = 4\hat{i} + 7\hat{j} + 4\hat{k}$$

$$\vec{r}_B = 9\hat{i} + 5\hat{j} + 7\hat{k}$$

(***1994 Karachi Board***)

***Solution***:

Displacement of the body is given by:

$$\Delta\vec{r} = \vec{r}_B - \vec{r}_A$$

$$\therefore \quad \Delta\vec{r} = (9\hat{i}+5\hat{j}+7\hat{k}) - (4\hat{i}+7\hat{j}+4\hat{k})$$

$$\Delta\vec{r} = 9\hat{i}+5\hat{j}+7\hat{k}-4\hat{i}-7\hat{j}-4\hat{k}$$

$$\Delta\vec{r} = 5\hat{i}-2\hat{j}+3\hat{k}$$

But Work done $= \vec{F}\cdot\Delta\vec{r}$

$$\therefore \text{Work done} = (6\hat{i}+8\hat{j}+10\hat{k})\cdot(5\hat{i}-2\hat{j}+3\hat{k})$$

Since $\hat{i}\cdot\hat{j} = 0$, $\hat{j}\cdot\hat{k} = 0$ and $\hat{k}\cdot\hat{i} = 0$

$$\therefore \quad \text{Work done} = 30\,\hat{i}\cdot\hat{i} - 16\,\hat{j}\cdot\hat{j} + 30\,\hat{k}\cdot\hat{k}$$

But $\hat{i}\cdot\hat{i} = 1$, $\hat{j}\cdot\hat{j} = 1$ and $\hat{k}\cdot\hat{k} = 1$

$\therefore$ Work done = 30 − 16 + 30

***Work done = 44 Units***

(If $\vec{F}$ and displacement $\Delta\vec{r}$ are expressed in M.K.S units then the unit of work done will be in ***Joules***.

***Q.No.13*****: An object weighing 98 N is dropped from a height of 10 m. It is found to be moving with a velocity of 12 m/s just before it hits the ground. How large was the frictional force acting upon it?** (***2012 Karachi Board***)

***Data***:

Weight W = mg = 98 N

Mass m = W/g = 98/9.8 = 10 kg.

Height h = 10 m.

Velocity V = 10 m/s.

Frictional force $f$ = ?

***Solution***:

Loss of P.E=K.E gained + work against friction

$$m\,g\,h = \tfrac{1}{2}mV^2 + fh$$

$$98\times10 = \tfrac{1}{2}\times10\times(12)^2 + f\times10$$

$$980 = 720 + 10f$$

$$980 - 720 = 10f$$

$$10f = 260$$

$$f = \frac{260}{10}$$

***f = 26 N***

***Q.No.14*****: A force $\vec{F} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ displaces a body through $\vec{r} = 2\hat{i} - \hat{k}$. Calculate the work done.** (***2012 Hyderabad Board***)

***Solution***:

Work $= \vec{F}\cdot\vec{r}$

OR Work $= (3\hat{i} + 4\hat{j} - 5\hat{k})\cdot(2\hat{i} - \hat{k})$

Since $\hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{i} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

Therefore,

Work $= (6\hat{i}\cdot\hat{i} + 5\hat{k}\cdot\hat{k})$

But $\hat{i}\cdot\hat{i} = \hat{k}\cdot\hat{k} = 1$

Work = 6 + 5

***Work = 11 Units***

***Q.No.15*****: A pump is needed to lift water through a height of 3 m at the rate of 600 gm/min. What must the minimum power of the pump be?** (***2015 Hyderabad Board***)

***Data***:

Height through which water is to be lifted h = 3 m

Rate at which water is to be lifted = 600 gm/min.

Power P = ?

***Solution***:

Since the rate of lifting water is 600 g/min.

Mass of water to be lifted in one min.is 600 g.

∴ Mass of water to be lifted in one second is 600/60 gm = 10 gm = 0.010 kg.

Rate of doing work to lift water = Rate at which it gains P.E.

$$= \frac{m\,g\,h}{t} \qquad (t = 1 \text{ sec})$$

$$= \frac{0.01 \times 9.8 \times 3}{1} \text{ J/s} = 0.294 \text{ J/s.}$$

But the rate of doing work is called power.

**Power needed to lift water = 0.294W**

***Unit. 6**….. ✍*

# Fluid statics

## *Fluids*:

There are three states of matter. States of matter that can flow are called ***fluids***. Since liquids and gasses can flow therefore, broadly speaking they are called **fluids**.
**Fluid statics** is a branch of physics in which we study properties of a stationary fluid (i.e. a fluid at rest).

### *Similarities and differences between physical properties of liquids and gasses*:

| ***Physical property*** | ***Gasses*** | ***Liquids*** |
|---|---|---|
| ***Volume.*** | No fixed volume, assumes volume of the container. | Fixed volume (although there is a slight change in volume when high external pressure is applied). |
| ***Shape.*** | No fixed shape. | No fixed shape. |
| ***Density.*** | Low density. | Higer density compared to gasses. |
| ***Compressibility*** | Highly compressible. | Low compressibility. |
| ***Viscosity.*** | Negligibly low viscosity. | Higher viscosity, some liquids are highly viscose. |

***Note that*** an ***ideal fluid*** is that which is ***incompressible***, it means that its volume cannot be changed by applying external pressure on its surface. Since density is mass per unit volume, therefore, density of an ideal fluid remains constant since its mass and volume both are not changing. ***In other words,*** gasses are not an ideal fluid, volume of liquids do not change appreciably on applying external pressure on their free surface, therefore, they can be considered as ideal fluids in ordinary sense.

## *Pascal's law*:

According to Pascal's law:

***"When pressure is changed at any point in a stationary fluid, this change in pressure is equally transmitted to all points in the fluid".***

## *Applications of Pascal's law*:

### (*1*) *Hydraulic brake system*:

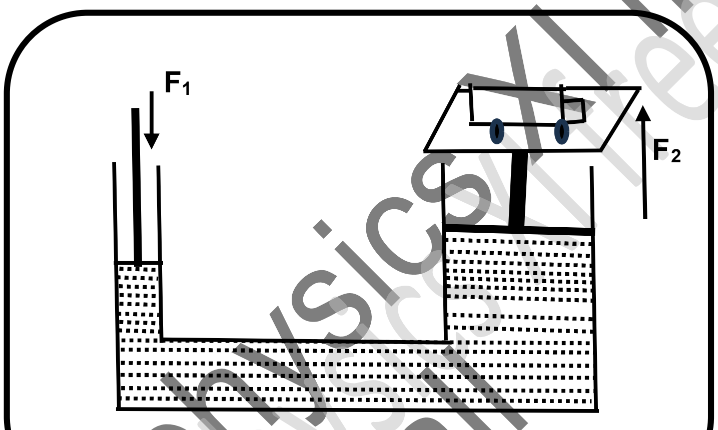
Hydraulic brake system of cars is an application of Pascal's law. Brake system of a car has a cylinder, called **master cylinder**, it is attached to the brake padel. When the driver presses the brake padel this increased pressure pushes the piston in the master cylinder. This increased pressure is transmitted equally to each brake cylinder provided with rear wheels in a front wheel drive car. Increased pressure is transmitted to entire braking system through the incompressible brake fluid (brake oil) filled in each cylinder and a strong metallic tube that connects all the cylinders with each other. In modern cars front wheels are provided with strong metallic disc rotating along with the wheel.

Increased transmitted pressure opens the brake pad which applies a strong force of friction to the rotating wheel drum, forcing the car to slow down. As soon as the brake padel is released pistons in all cylinders move back to their original positions, a spring pulls back the brake shoe and the car is again able to move as before.

**(*2*) *Hydraulic lift*:**

Pascal's law is applied to lift heavy objects such as a heavy car by applying a small force as in hydraulic lift.

Hydraulic lift has a U-shaped container filled with an ***incompressible*** liquid (a liquid that cannot be compressed on applying force on it, so that its volume does not change keeping it density constant. Real liquids show slight change in their volume on being compressed; hence this is true for an ideal liquid). Container has two cylinders at its two ends; both provided with pistons. One of the cylinders is narrower than the other cylinder which carries a platform on which a car can be parked easily.




***Hydraulic lift***



If a force $F_1$ is applied on the piston of smaller cylinder of area $A_1$, then pressure applied will be $P_1 = F_1/A_1$. Since the liquid is **incompressible**, therefore, according to Pascal's law, this increased pressure will be equally transmitted throughout the liquid. Pressure on piston of wider cylinder will be same as the pressure on smaller piston, but since this piston is of large area "$A_2$" therefore, force on this piston will also be large so that pressure on this piston is the same. If $F_2$ is the output force at piston of wider cylinder of area $A_2$ the pressure on it will be $P_2 = F_2/A_2$.

But input pressure $P_1$ = Out put pressure $P_2$

Therefore,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$\therefore$

$$\frac{F_2}{F_1} = \frac{A_2}{A\backslash_1}$$

$$\boxed{F_2 = \frac{A_2}{A_1} F_1}$$

We can see from this formula that the value of output force (Force required to lift heavy load i.e. a car) can be increased to any desired value by increasing the area of output (larger) piston that carries the car.

***Mechanical advantage of hydraulic lift*:** Mechanical advantage of any machine is the ratio of output force to input force (Load lifted to effort applied). It tells you about how many times the force required to lift a load will be **greater** than the applied force.

In this case:

$$\text{Mechanical advantage of a Hydraulic lift} = \frac{F_2}{F_1}$$

If mechanical advantage of a lift is 100 then that machine will be able to lift a 100 times heavier load, ***it means that*** this particular machine will be able to lift a 10000 Newton car by applying only 100 Newton on the smaller piston.

***Example*:** Let the radius of smaller piston be 25 cm. (= 0.25 m), radius of wider piston be 1 m. and the applied force is 90 N, then:

$$F_2 = \frac{A_2}{A_1} F_1 = \frac{\pi r_2^2}{\pi r_1^2} F_1$$

$$F_2 = \frac{r_2^2}{r_1^2} F_1 = \frac{(1)^2 \times 90}{(0.25)^2} = 16 \times 90 = \mathbf{1440\ N}$$

From this example you can see that output force can be increased by 16 times i.e. only 90 N input force will be needed to lift 1440 N load (this is for an ideal lift, free of friction, if there is friction then a part of input energy will be wasted to overcome friction).

Mechanical advantage of this lift will be Output force/ input force $= F_2/F_1 = 1440/90 = 16$

## ***Upthrust (or Buoyant force)***:

When a body is immersed in a fluid (any liquid or a gas) it experiences an upward force, exerted by the fluid, this force is known as ***upthrust or buoyant force*.**

Pressure in a fluid depends mainly upon its density and depth, as a body is immersed deeper into a fluid there will be a difference in pressure of the liquid between lower and upper surfaces of the body.

We know that pressure in a fluid is equal at all points and acts in all directions lying at the same depth, hence when a body is fully immersed into a fluid then pressure exerted by the fluid on the upper surface of the body is less than the pressure at its lower surface (Lower surface is deeper)

This difference in pressure results in an upward directed force acting on lower side of the body, called ***upthrust or buoyant force*.**

Due to upthrust body appears to be lighter when immersed in a fluid i.e. its apparent weight is less than its true weight.

Upthrust helps io support a body and tends to prevent it from sinking.

Since density of liquids is more than the density of gasses due to which liquids exert more pressure, ***in other words***, upthrust in liquids is much more than in gasses.

- If density of the object is ***less*** than the density of liquid, the object will ***float***
- If density of the object is ***equal*** to the density of liquid, the object will ***remain suspended in the liquid*** just like a fish.
- If density of the object is ***more*** than the density of the liquid the object will ***sink***, because the upthrust will be smaller than weight of the body and will not be enough to support it.

## ***Archimede's principle***:

According to Archimede's principle:

> ***"When an object is immersed into a liquid it experiences upthrust equal to weight of the liquid displaced."***

The magnitude of upthrust depends upon:

- ***Volume of the body immersed***, because more is the volume of the body immersed more is the volume of fluid displaced, more will be the upthrust.
- ***Density of the fluid*** more is the density of fluid more will be the upthrust.

Due to ***upthrust***, exerted by the liquid the object appears to loss some of its weight. Hence a body immersed in a liquid appears to be lighter. In other words:

> Volume of liquid displaced = volume of body immersed. If a body is fully immersed then the volume of liquid displaced is equal to total volume of the body but if a body is partially immersed then volume of liquid displaced will be equal to volume of the body inside the liquid.

Apparent weight of a body = its actual weight – upthrust

The magnitude of upthrust or buoyant force, according to Archimede's principle, is given by:

Buoyant force = density of fluid × Volume of fluid displaced × acceleration due to gravity.

$$\text{Buoyant force} = \rho V g$$

> Density $\rho$ = mass m / volume V
> mass of liquid displaced m = density of liquid $\rho$ × volume V of liquid displaced.
> $m = \rho \times V$
> Weight of liquid displaced (Buoyant force)= m g
> Weight of liquid displaced (Buoyant force)= $\rho V g$

## ***Principle of floatation***:

A needle made of steel sinks in water whereas huge cargo ships made of steel do not sink, instead they float. The reason for this observation is that a needle displaces a small volume of liquid and the weight of liquid displaced is quite small as compared to weight of the needle. Hence the upthrust acting on the needle is not enough to support it, therefore, it sinks. A huge cargo ship because of its shape and because of large empty spaces inside the ship displaces large volume of liquid. Weight of this large volume of liquid displaced is enough to support a loaded cargo ship and prevents it from sinking.

In other words:

" ***A floating object displaces weight of fluid equal to its own weight***".

Ships and submarines are designed on this principle; principle of floatation is based on Archimede's principle.

***In gasses (eg air)***:

***"An object (e.g. a balloon) surrounded by air experiences a force equal to weight of air displaced"***.

Airships and hot air balloons etc. displace large quantity of air, such that if weight of air displaced is ***more*** than balloon's own weight than balloon rises up (because there will be more upthrust). But if weight of air displaced is ***less*** than balloon's own weight than balloon descends (because there will be less upthrust). If weight of air displaced is ***equal*** to balloon's own weight than balloon hovers at a constant altitude. (because there will be equal upthrust)

## ***Surface tension***:

Surface tension of a liquid may be defined as:

"***The force per unit length acting on either side of an imaginary line drawn on the liquid surface at rest. Direction of this force is along tangent to the surface and is perpendicular to the line***"

$$\text{Surface tension} = \frac{F}{L} \qquad \text{(SI unit N/m)}$$

Due to surface tension, surface of a liquid acts as a **membrane**. It is the surface tension due to which surface of a liquid tends to have **minimum surface area**. Due to surface tension, a liquid **supports** a needle, liquid surface assumes a **concave shape**, liquid **rises in a capillary tube** and a liquid **bubble is spherical** in shape and so on.

## ***Section–A Multiple-Choice Questions MCQs:***

***Q.No.1*:** A completely submerged object always displaces its own ………
(a) weight of fluid. (b) volume of a fluid
(c) density of a fluid. (d) Area of a fluid.

***Q.No.2*:** The pressure exerted on the ground by a man is greater when:
(a) he stands with both feet flat on the ground.
(b) he stands flat on one foot.
(c) he stands on the toes of one foot.
(d) he lies down on the ground.

***Q.No.3*:** In a stationary homogeneous liquid:
(a) pressure is the same at all points.
(b) pressure depends on the direction.
(c) pressure is independent of any atmospheric pressure on the upper surface of the liquid.
(d) pressure is the same at all points at the same level.

***Q.No.4*:** one piston in a hydraulic lift has an area that is twice the area of all others. When the pressure at the smaller piston is increased by $\Delta p$ the pressure at the larger piston:
(a) increases by 2 $\Delta p$ (b) increases by $\Delta p/2$
(c) increases by $\Delta p$. (d) increases by 4 $\Delta p$

***Q.No.5*:** In vacuum, an object has:
(a) No buoyant force. (b) no mass.
(c) no weight. (d) none of these.

***Q.No.6*:** The pressure at the bottom of a pond does NOT depend on:
(a) Water density. (b) depth of the pond.
(c) surface area of the pond. (d) None of these

***Q.No.7*:** A rock suspended by a weighing scale weighs 5 N out of water and 3 N when submerged in water. What is the buoyant force on the rock?
(a) 2 N. (b) 5 N. (c) 8 N. (d) 15 N.

***Q.No.8*:** "An object completely submerged in a fluid displaces its own volume of fluid". This is:
(a) Pascal's paradox. (b) Archimede's principle
(c) Pascal's principle. (d) true, but none of the above

***Q.No.9*:** Salt water has greater density than freshwater. A boat floats in both freshwater and salt water. The buoyant force on the boat in saltwater is …………. that in freshwater.
(a) equal to (b) smaller than
(c) larger than (d) same as

***Q.No.10*:** You fill a tall glass with ice and then add water to level the glass's rim, so some fraction of the ice floats above rim. When ice melts, what happens to water level?
(a) water overflows the rim.
(b) water level drops.
(c) water level stays at the rim.
(d) it depends on the difference in density between water and ice.

***Q.No.11*:** The S.I unit of surface tension is: (***2025 Karachi Board***)
• N. • $\frac{N}{m}$ • $\frac{N}{m^3}$ • $\frac{m}{N}$

***Answers*:**

(1) Weight of fluid.
(2) he stands on the toes of one foot.
(3) pressure is the same at all points at the same level.
(4) increases by $\Delta p$.
(5) No buoyant force.
(6) surface area of the pond.
(7) 2 N.
(8) Archimede's principle
(9) Larger than
(10) Water level stays at the rim.
(11) $\frac{N}{m}$

# Numericals:

**Q.No.1: In a hydraulic press a force of 20 N is applied to a piston of area 0.20 m². The area of the other piston is 2.0 m². What is:**

**(a) Pressure transmitted through the fluid**

**(b) The force on the piston.**

**Data:**

Force on smaller piston $F_1 = 20$ N

Area of smaller piston $A_1 = 0.20$ m²

Areas of larger pistons $A_2 = 2.0$ m²

(a) Pressure transmitted P =?

(b) Force on the piston (Larger) $F_2$ =?

**Solution:**

(a) Pressure is equally transmitted everywhere (Pascal's law), Hence pressure on smaller piston:

$P = \frac{F_1}{A_1} = \frac{20}{0.20}$ = **100 N/m² or 100 Pa.**

(b) Force on the larger piston is given by:

$\therefore$ $F_2 = P A_2 = 100 \times 2.0$ **$F_2$ = 200 N.**

**Q.No.2: The pressure in a water pipe on the ground floor of a building is 4×10⁵ Pa but three floors up it is only 2×10⁵ Pa. What is height between the ground floor and the third floor? Water in the pipe may be assumed to be stationary, density of water =1×10³ kg/m³.**

**Data:**

Pressure of water at ground floor $P_1 = 4\times10^5$ Pa

Pressure of water at third floor $P_2 = 2\times10^5$ Pa

Density of water $\rho = 1\times10^3$ kg/m³.

Height between the floors h = ?

**Solution:**

Difference of pressures = $P_1 - P_2$

But $P_1 - P_2 = \rho g h$

$\therefore$ $4\times10^5 - 2\times10^5 = 1\times10^3 \times 9.8\ h$

$2\times10^5 = 9.8 \times10^3\ h$

$\therefore$ $h = \frac{2\times10^5}{9.8\times10^3}$ **h = 20.4 m**

**Q.No.3: The small piston of hydraulic press has an area of 10.0 cm². If the applied force is 50.0 N, what must the area of the large piston be to exert a pressing force of 4800 N ?**

**Data:**

Area of small piston $A_1$=10.0 cm²= 0.001m²

Applied force $F_1 = 50.0$ N

Output force $F_2 = 4800$ N

Area of large piston $A_2$ =?

**Solution:**

According to Pascale's law:

$P_1 = P_2$ OR $F_1/A_1 = F_2/A_2$

$\therefore$ $A_2 = \frac{F_2 A_1}{F_1} = = \frac{4800 \times 0.001}{50.0}$

**$A_2$ = 0.096 m³ = 960 cm³**

**Q.No.4: Mechanical advantage of a hydraulic jack is 420. Find weight of the heaviest automobile that can be lifted by an applied force of 55 N.**

**Data:**

Mechanical advantage M.A = 420

Applied force F = 55 N.

Heaviest weight lifted W =?

**Solution:**

By definition: $M.A = \frac{W}{F} = \frac{\text{Output force (Load lifted)}}{\text{Input force (applied force)}}$

W = M.A × Input or applied force F

W = 420 × 55 **W = 23100 N.**

**Q.No.5: A flat-bottom river barge is 30 ft wide, 85 ft long and 15 ft deep.**

**(a) how many ft³ of water will displace while the top stays 1 m. above the water.**

**(b) What load in tons will the barge contain under these conditions if the empty barge weighs 160 tons in dry dock.**

**Data:**

Length L = 85 ft. Width (breadth) b = 30 ft.

Depth h = 15 ft

(a) Volume of water displaced V=?

(b) Load in the barge = ?

Weight of barge W =160 tons.

**Solution:**

(a) Volume displaced V= L b (depth immersed)

Volume displaced V = 85 × 30 × (15–3.28)

(1m above water =3.28ft) = **29886 ft³.**

But 1 m³ = (3.28)³ ft³.

$\therefore$ Volume displaced $V = \frac{29886}{(3.28)^3}$ m³= **847 m³**.

(b) Weight of water displaced =Bouncy = $\rho V g$

= 1 × 10³ × 847 × 9.8 = 8300.6 N

(Density of water is taken as $\rho = 1 \times 10^3$ kg/m³)

But 1 tone = 9807 N

$\therefore$ Bouncy = $\frac{8300.6}{9807}$ tones = **846.4 tones.**

Load in the barge = Bouncy – weight of the barge

Load in the barge = 846.4 –160 = **686.4 tones**

***Q.No.6*: A canal lock gate is 20 m wide and 10 m deep. Calculate the thrust acting on it assuming that the water in the canal is in level with the top of the gate. Density of water is 1000 kg/m$^3$.**

***Data*:**

Width W = 20 m, Depth h = 10 m

Density of water $\rho$ = 1000 kg/m$^3$.

Thrust F = ?

***Solution*:**

Average pressure on the gate P= $\rho$ g (average depth)

$= 1000 \times 9.8 \times \frac{10}{2} = 49000$ N/m$^2$.

Thrust F = P A = 49000 (20 × 10)

Thrust F = ***9800000 N*** = ***9.8 ×10$^6$ N***.

***Q.No.7*: A tank 4 m long, 3 m wide and 2 m deep is filled to the brim with paraffin (density 800 kg/m$^3$). Calculate the pressure on the base. What is the thrust on the base?**

***Data*:**

Length L = 4 m. width (or breadth)W = 3 m

Depth h = 2 m density $\rho$ = 800 kg/m$^3$

Pressure P = ? Thrust -?

***Solution*:**

Since $P = \frac{\text{Force}}{\text{Area}} = \frac{mg}{L\,b}$

Density $\rho = \frac{\text{mass}}{\text{Volume}} = \frac{m}{L\,b\,h}$ $m = \rho L b h$

$\therefore \quad P = \frac{\rho L b h}{L b} g = \rho g h$

Pressure on the base P = 800 × 9.8 × 2

= **15680 N/m$^2$.(or Pa)**

Thrust on the base = Pressure × Area

Thrust = 15680 × L b =15680×4×3 = **188160 N**.

***Q.No.8*: A rectangular boat is 4.0 m wide, 8.0 m long and 3.0 m deep.**

**(a) How much water will it displace if the top stays 1 m above the water?**

**(b) What load will the boat contain under these conditions if the empty boat weighs 8.60×10$^4$ N in dry dock?**

***Data*:**

Length L = 8.0 m. Width W = 4.0 m

Depth of boat inside water h = 3.0 – 1.0 = 2.0 m.

(a) Volume of water displaced V =?

(b) Load in the boat W =?

***Solution*:**

(a) Water displaced = volume of boat submerged

= L W h = 8.0×4.0×2.0 = **64.0 m$^3$**.

(b) Bouncy = Weight of boat + Load in the boat

But Bouncy = weight of liquid displaced = $\rho$ g V

$\therefore$ $\rho$ g V = Weight of boat + Load in the boat

$\therefore$ Load in the boat = $\rho$ g V – Weight of boat

Load in the boat=1000×9.8×64 – 8.60×10$^4$

Load in the boat = 627200 – 8.60×10$^4$

$\therefore$ Load in the boat = **541200 N**.

***Q.No.9*: A hot air ballon has a volume of 2200 m$^3$. The density of air at temperature of 20ºC is 1.205 kg/m$^3$. The density of the hot air inside the ballon at a temperature of 100ºC is 0.946 kg/m$^3$.**

**How much weight can the hot air balloon lift?**

***Data*:**

Volume of balloon volume of displaced air V=2200 m$^3$

Density of air inside the ballon $\rho_1$= 0.946 kg/m$^3$.

Density of air outside the ballon $\rho_2$= 1.205 kg/m$^3$.

Weight lifted W = ?

***Solution*:**

Weight of balloon = $\rho_1$ g V = 0.946 × 9.8 × 2200

= 20395.76 N

Upthrust = $\rho_2$ g V = 1.205 × 9.8 × 2200

= 25979.8

But Upthrust = Weight of balloon + Weight lifted

$\therefore$ Weight lifted = Upthrust – Weight of balloon

$\therefore$ Weight lifted = 25979.8 – 20395.76

$\therefore$ Weight lifted **= 5584.04 N.**

(This gives us maximum load that can be lifted)

***Q.No.10*: A spherical balloon has a radius of 7.15 m and is filled with helium. How large a cargo can it lift, assuming that the skin and structure of the balloon have a mass of 930 kg? Neglect the buoyant force on the cargo volume itself.**

***Data*:**

Radius of balloon r = 7.15 m

Volume of balloon $V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} (7.15)^3$

Volume of balloon V = 1531.73 m$^3$.

Mass of balloon m = 930 kg

Cargo lifted W = ?

Density of Helium $\rho_1$= 0.179 kg/m$^3$.

Density of air $\rho_2$ = 1.29 kg/m$^3$.

***Solution*:**

Upthrust = Weight of balloon + Weight of helium + Cargo lifted

$\therefore$ Cargo lifted = Upthrust – Weight of balloon – Weight of helium

$\therefore$ Cargo lifted = $\rho_2 g V - m g - \rho_1 g V$

(volume of air displaced=volume of helium or of balloon)

$\therefore$ Cargo lifted = $1.29\times9.8\times1531.73 - 930\times9.8 - 0.179\times9.8\times1531.73$

$\therefore$ Cargo lifted = $19364.13 - 9114 - 2686.96$

$\therefore$ Cargo lifted = $19364.13 - 11800.96$

$\therefore$ Cargo that can be lifted **= <u>7563.17 N</u>.**

***Q.No.11*: A tank 4 m long, 3 m wide and 2 m deep is filled to the brim with paraffin of density 800 kg/m$^3$.**

**Calculate the pressure on the base.**

**(*2025 Karachi Board*)**

***Data*:**

Length L = 4 m. width (or breadth)W = 3 m

Depth h = 2 m density $\rho$ = 800 kg/m$^3$

Pressure P = ?

***Solution*:**

Since $P = \frac{Force}{Area} = \frac{mg}{L\,b}$

Density $\rho = \frac{mass}{Volume} = \frac{m}{L\,b\,h}$ $\quad m = \rho L b h$

$\therefore \quad P = \frac{\rho L b h}{L b} g = \rho g h$

Pressure on the base P = $800 \times 9.8 \times 2$

= **<u>15680 N/m$^2$.(or Pa)</u>**

***Unit. 7*****.....**

# Fluid dynamics

## ***Fluids*:**

There are three states of matter. States of matter that can flow are called ***fluids***. Since liquids and gasses can flow, therefore, broadly speaking they are called **fluids**.

### ***Fluid dynamics*:**

**Fluid dynamics** is a branch of physics in which we study properties of a flowing fluid.

### ***Fluid friction or viscosity*:**

Imagin a number of layers of a flowing liquid. The lowest layer of a flowing liquid which is in contact with a solid surface will be at ***rest***, It retards the flow of layer immediately above it, which in turn retards the flow of next layer above it, in this manner each layer opposes the motion of layer immediately above it. Hence there is an internal force of friction between adjacent layers of a liquid, this force of friction acts parallel to the surface of layers and is known as ***fluid friction*** or ***viscous force*** and this property of fluids is called ***VISCOSITY***.

Liquids have higher viscosity than gasses.

Viscosity slows down the flow of a fluid. Since honey has a very high viscosity as compared with water, therefore, honey flows slowly whereas water can flow quickly. Similarly, a gas can flow easily as compared a liquid because gasses have low viscosity.

Honey, syrup, motor oil, cooking oil, paints etc. are liquids of high viscosity, hence they flow slowly. Whereas water, sprit, kerosene oil etc. are liquids of low viscosity, therefore they can flow quickly. In general gasses have even lower viscosity as compared to liquids.

Viscous force "F" between layers of a liquid is directly proportional to the rate of change of velocity of liquid layers with respect to distance "$\Delta v/l$" perpendicular to the direction of flow of layers. Viscous force is also directly proportional to area "A" of contact of layers.

Hence:

$$F \propto -A\frac{\Delta v}{l}$$

OR

$$F = -\eta A\frac{\Delta v}{l}$$

Where "$\eta$" is the constant of proportionality known as **"*coefficient of viscosity*"**.

Highly viscous fluids have high value of coefficient of viscosity "$\eta$". ***In other words,*** "$\eta$" is a measure of viscosity of a fluid. Negative sign is used just to show that the viscous force opposes internally the relative motion between different layers. Hence if there is velocity gradient between different layers of a fluid there will be viscous force between them due to which it will have some viscosity.

For Honey the value of "$\eta$"is high and for water the value of "$\eta$" is low, which shows that honey is a highly viscous and water is less viscous, as a result of which honey flows slowly and water flows quickly.

Viscosity of a fluid also depends upon its ***temperature***. As the temperature of a liquid rises its viscosity ***decreases***. But viscosity of a gas ***increases*** with a rise in its temperature.

S.I unit of $\eta$ is **N . s/m$^2$ = Pa . s** (N/m$^2$ = Pascal Pa)

C.G.S unit of $\eta$ is **dyne . s/cm$^2$**.

But C.G.S unit of $\eta$ is commonly known as "Poise" (i.e. **dyne . s/cm$^2$ = poise**)

**1 N . s/m$^2$ = 10 poise.**

## *Velocity gradient*:

Velocity gradient may be defined as:

"***The rate of change of velocity with respect to distance normal (perpendicular) to the direction of flow of liquid layers"***.

Higher the value of velocity gradient more viscous is a fluid i.e. the fluid will offer more force to resist the liquid flow or to the motion of a body through it.

## *Terminal velocity*:

When a small spherical body of radius "r" is dropped slowly in a viscous fluid of coefficient of viscosity "$\eta$" then at first it falls under its weight and has some acceleration. As soon as it enters the fluid the fluid exerts a force opposite to its weight. This force exerted by the fluid because of Its viscosity is called "***Drag force***". As soon as the drag force balances weight of the spherical body, net force acting on the falling body will be zero, hence acceleration of the body also becomes **zero**. It then on moves down with a **uniform velocity**. This constant velocity with which the spherical body moves through the fluid is called its ***"terminal velocity."***

**Viscous or drag force** exerted by a fluid on a body moving through it depends upon viscosity of the fluid, size, shape and terminal velocity of the body.

According to Stocke's law viscous or drag force "F" acting on a small **spherical** body of radius "r" moving with a uniform velocity "v" through a viscous fluid of coefficient of viscosity "$\eta$" is given by:

$$F = 6 \pi \eta r v$$

**Note that** this formula is valid only for a **spherical body** moving with a uniform velocity through a viscous fluid. Formula for drag force for any other shape will be different.

When a spherical body moves with a constant terminal velocity through a viscous medium, its weight is balanced by viscous drag force, hence:

$$6 \pi \eta r v = m g$$

If "$\rho$" is the density of the viscous medium and $(4/3) \pi r^3$ is volume of the spherical body, then is mass "m" is given by:

$$m = \rho \times (4/3) \pi r^3$$

$$\therefore \quad 6 \pi \eta r v = \rho \times (4/3) \pi r^3 g$$

$$\therefore \quad v = \frac{\rho \times (4/3) \pi r^3 g}{6 \pi \eta r}$$

Simplifying this equation we get:

$$\boxed{v = \frac{2 r^2 \rho g}{9 \eta}}$$

This formula gives us terminal velocity "v" of a spherical body of density "$\rho$" moving through a viscous medium of coefficient of viscosity "$\eta$" at a given location (constant g). It shows that the terminal velocity "v" is directly proportional to square of radius ($r^2$) of the spherical body.

## *Types of fluid flow*:

Liquid flow is two basic types **steady** and **unsteady** flow.

## *Steady (Streamline or Laminar) flow*:

In steady flow of a fluid properties of the fluid **do not change with time**.

For example, velocity "v" of particles of the fluid, viscosity of the fluid "$\eta$" and pressure "P" of the fluid etc. **do not change** with time.

**It means that** velocity "v" of particles of the fluid will be the same everywhere and in the same direction at different instants. **In other words,** particles of the fluid will travel parallel to each other with the same velocity at different instants, and will **not cross** each other's path.

- Streamline is a line which shows path of a particle in a flowing fluid, such that a tangent to the line at any particular instant shows the direction of velocity of a particle at that instant.

In steady or streamline or laminar flow of a fluid streamlines showing the path of moving particles of the fluid are **parallel** to each other, in other words all the particles of the fluid move in the **same direction** with the **same velocity**.

Steady, streamline or laminar flow of fluids is usually observed when a fluid of high viscosity is flowing slowly.

## ***Unsteady (irregular or turbulent) flow*:**

In unsteady flow of a fluid properties of the fluid **change with time**.

For example, velocity "v" of particles of the fluid, viscosity of the fluid "η" and pressure "P" of the fluid etc. **change** with time.

**It means that** velocity "v" of particles of the fluid will be different everywhere. **In other words,** particles of the fluid will travel with different velocities in different directions, and will **cross** each other's path.

In unsteady, irregular or turbulent flow of a fluid streamlines showing the path of moving particles of the fluid are **not parallel** to each other, in other words particles of the fluid move in **different directions** with **different velocities**, different streamlines cross each other.

## ***Incompressible flow*:**

Incompressible flow of a fluid is that in which during the flow density **does not change**, although pressure may change. Mostly liquids are incompressible whereas, gasses are compressible.
(In other words, under normal conditions liquids may be considered incompressible, whether they are at rest or in motion)

## ***Non-viscous flow*:**

It is the flow of an incompressible and non-viscous fluid in which a fluid can flow without any dissipation of energy for doing work against viscous force.

All real fluids have some viscosity; some have **low** and some have **high** viscosity. Hence depending upon their viscosity there is some dissipation of energy during their flow. **It means that** a totally incompressible fluid having zero viscosity will be an **ideal fluid**.

## ***Raynold number*:**

Raynold number is ***the ratio of inertial force to the viscous force.***
Since it is the ratio of two similar quantities therefore, it is a ***dimensionless*** quantity.

Raynold number is given by:

$$\text{Raynold number} = \frac{\text{fluid velosity * internal diameter}}{\text{Kinetic viscosity}}$$

Where

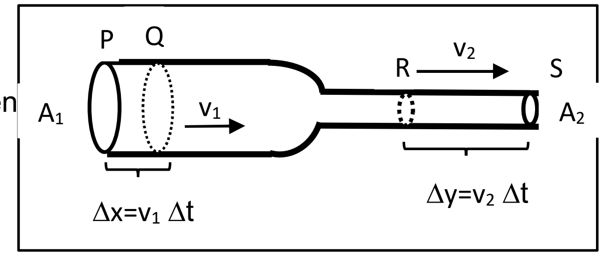
$$\text{Kinetic viscosity} = \frac{\text{Dynamic viscosity}}{\text{Fluid density}}$$

- If Raynold number "$R_e$" is less than 2300 ($R_e < 2300$) then the flow of fluid is Laminar or regular.
- If Raynold number is greater than 2300 but less than 4000 ($2300 > R_e < 4000$) then the flow of fluid is transient, this type of flow of a fluid occurs just before the turbulent conditions.
- If Raynold number is greater than 4000 ($R_e > 4000$) then the flow of fluid is turbulent.

## ***Equation of continuity*:**

For an incompressible fluid the mass flow rate $\Delta m/\Delta t$ (rate of flow of its mass) at different points through the fluid is constant. This is in accordance with **law of conservation of mass**.

Let a liquid flow from point P to Q in time $\Delta t$ with velocity $v_1$ through wider section of a pipe of area $A_1$,




then the volume of liquid flowing from P and Q will be "$A_1 \Delta x$". But velocity "$v_1$" with which liquid is flowing is given by: velocity $v_1 = \frac{\text{Distance covered } \Delta x}{\text{time taken } \Delta t}$ OR $\Delta x = v_1 \Delta t$

$\therefore$ volume of liquid flowing from P to Q will be "$A_1 v_1 \Delta t$".

$$\text{Density of liquid } \rho = \frac{\text{mass of liquid } \Delta m}{\text{Volume of liquid}}$$

$$\rho = \frac{\Delta m}{A_1 v_1 \Delta t}$$

OR
$$\frac{\Delta m}{\Delta t} = \rho A_1 v_1 \ldots\ldots\ldots\ldots (i)$$

This equation gives us **mass flow rate** of the liquid between P and Q.
Similarly mass flow rate of the liquid between R and S is given by:

$$\frac{\Delta m}{\Delta t} = \rho A_2 v_2 \ldots\ldots\ldots\ldots (ii)$$

Mass flow rate $\Delta m/\Delta t$ (rate of flow of mass of liquid) at different points in a flowing liquid is constant. Hence left hand side of both equations will be equal.

$$\rho A_1 v_1 = \rho A_2 v_2$$

Since the liquid is **incompressible** therefore, its density "$\rho$" throughout the pipe will be the same.

$$\therefore \quad \boxed{A_1 v_1 = A_2 v_2}$$

This equation is known as **equation of continuity**.
From this equation we can see that when a liquid flows through wide pipe of area of cross section "$A_1$" its velocity "$v_1$" will be **small** i.e. the liquid will flow **slowly** whereas while flowing through narrow section of the pipe of **small** area of cross-section "$A_2$" its velocity "$v_1$" will be **high** i.e. the liquid will flow **with high speed.** In other words, **large** "$A_1$" **small** "$v_1$" and **small** "$A_2$" **large** "$v_2$". This shows that speed of a fluid changes with diameter of the pipe.

The product "A v" gives us "**volume rate of flow**".
Its S.I unit is $m^3/s$.

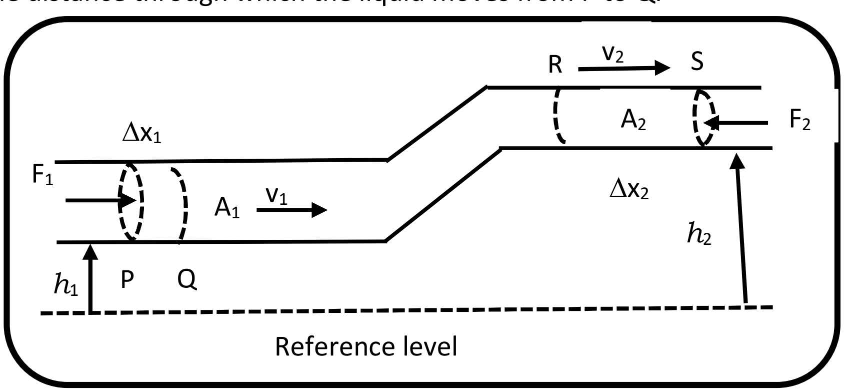
# Bernoulli's principle:

Bernoulli's principle (or Bernoulli's theorem or Bernoulli's equation) is based upon law of conservation of energy. It states that:

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}}$$

**Derivation:**

Consider an incompressible fluid (such as water) flowing with velocity "$v_1$" through a pipe of area of cross- section "$A_1$". Force "$F_1$" is exerted from left side moves the liquid through "$\Delta x_1$". This force will do some **work "$F_1 \Delta x_1$" on** the liquid, where $\Delta x_1$ is the distance through which the liquid moves from P to Q.




Similarly force "$F_2$" exerted to oppos its Flow, this force acts over distance "$\Delta x_2$". Hence **work done by** this force will be "$F_2 \Delta x_2$". Net **work done on** the liquid Is given by:
Net work done **on** = $F_1 \Delta x_1 - F_2 \Delta x_2$
This work done on the liquid will change Its kinetic and potential energies, as the liquid flows with different velocities "$v_1$" and "$v_2$" between points P Q and R S.

at the same time, it is lifted from height "$h_1$" to "$h_2$" ("$h_1$" and "$h_2$" are measured from some reference level). Hence:

$$\text{Change in kinetic energy} = \tfrac{1}{2}\, m_1 v_1^2 - \tfrac{1}{2}\, m_2 v_2^2$$

$$\text{Change in potential energy} = m_1 g h_1 - m_2 g h_2$$

Net **work done on** the liquid must be equal to the sum of change in Kinetic and potential energies, Hence

$$F_1\, \Delta x_1 - F_2\, \Delta x_2 = (\tfrac{1}{2}\, m_1 v_1^2 - \tfrac{1}{2}\, m_2 v_2^2) + (m_1 g h_1 - m_2 g h_2)$$

OR

$$F_1\, \Delta x_1 + \tfrac{1}{2}\, m_1 v_1^2 + m_1 g h_1 = F_2\, \Delta x_2 + \tfrac{1}{2}\, m_2 v_2^2 + m_2 g h_2$$

For incompressible liquid mass "$m_1$" flowing from P to Q will be equal to "$m_2$" flowing from R to S,

$$\therefore \quad m_1 = m_2 = m$$

$$\therefore \quad F_1\, \Delta x_1 + \tfrac{1}{2}\, m v_1^2 + m g h_1 = F_2\, \Delta x_2 + \tfrac{1}{2}\, m v_2^2 + m g h_2$$

But $F_1 = P_1 A_1$ ($P_1$ is the pressure of fluid at the left-hand side),

and $F_2 = P_2 A_2$ ($P_2$ Is the pressure of fluid at the right hand side)

$$\therefore \quad P_1 A_1\, \Delta x_1 + \tfrac{1}{2}\, m v_1^2 + m g h_1 = P_2 A_2\, \Delta x_2 + \tfrac{1}{2}\, m v_2^2 + m g h_2$$

But $A_1\, \Delta x_1 = V_1$ ($V_1$ = volume of liquid between P and Q)

and $A_2\, \Delta x_2 = V_2$ ($V_2$ = volume of liquid between R and S)

$$\therefore \quad P_1 V_1 + \tfrac{1}{2}\, m v_1^2 + m g h_1 = P_2 V_2 + \tfrac{1}{2}\, m v_2^2 + m g h_2$$

Since the liquid is incompressible (cannot be compressed) its volume "$V_1$" flowing from P to Q will be equal to its volume "$V_2$" flowing during the same time from R to S, i.e. $V_1 = V_2 = V$

$$\therefore \quad P_1 V + \tfrac{1}{2}\, m v_1^2 + m g h_1 = P_2 V + \tfrac{1}{2}\, m v_2^2 + m g h_2$$

Dividing this equation throughout by "V" we get:

$$\therefore \quad P_1 + \tfrac{1}{2}\, \frac{m}{V}\, v_1^2 + \frac{m}{V}\, g h_1 = P_2 + \tfrac{1}{2}\, \frac{m}{V}\, v_2^2 + \frac{m}{V}\, g h_2$$

But density $\rho = \dfrac{\text{mass } m}{\text{Volume } V}$ ( $\rho$ is the density of fluid, which is constant everywhere for an incompressible fluid)

$$\therefore \quad P_1 + \tfrac{1}{2}\, \rho v_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\, \rho v_2^2 + \rho g h_2$$

**In other words**: $\mathbf{P + \tfrac{1}{2}\, \rho v^2 + \rho g h = Constant}$

This is Bernoulli's equation.

**Note that** while flowing through a pipe of variable area of cross section, in wider section of the pipe **pressure** of the liquid is **high,** but **speed** of flow is **low**, whereas, in narrow section of the pipe **pressure** is **low** but **speed** of flow is **high**. These observations are in accordance with Bernoulli's theorem, equation of continuity is also satisfied.

## ***Applications of Bernoulli's principle*:**

There are many devices, machines etc. whose working is based upon Bernoulli's principle. For example working of "***Venturi meter***" which is used to measure rate of flow of a fluid through a pipe, just by measuring the pressure difference at two points of different area of cross section, "***Bunsen burner***" (used for heating purpose in laboratory), ordinary spray pump, also called an "***Atomizer***" (used for spraying insecticide, mosquito sprayer), shape of wings of airplane acts as "***an airfoil***" that moves through air with such a speed that a pressure difference above and below the airplane is produced. This difference in pressure provides lift to the airplane. This pressure difference takes place in accordance with "Bernoulli's principle".

## Section–A Multiple-Choice Questions MCQs:

**Q.No.1**: For an incompressible fluid the flow rate is:

(a) equal for all surfaces.
(b) constant throughout the pipe.
(c) greater for the larger parts of the pipe
(d) None of the above.

**Q.No.2**: Bernoulli's principle states that for horizontal flow of a fluid through a tube, the sum of the pressure and energy of motion per unit volume is:

(a) increasing with time.
(b) decreasing with time.
(c) constant. (d) varying with time.

**Q.No.3**: Which of the following is associated with the law of conservation of energy in fluids?

(a) Archimede's principle.
(b) Bernoulli's principle.
(c) Pascal's principle.
(d) Equation of continuity

**Q.No.4**: As the speed of a moving fluid increases, the pressure in the fluid:

(a) increases. (b) remains constant.
(c) decreases,
(d) may increase or decrease, depending on the viscosity.

**Q.No.5**: If the cross- sectional area of a pipe decreases, what happens to the fluid viscosity?

(a) Increases. (b) decrease.
(c) remains the same.
(d) depends on the fluid density

**Q.No.6**: A sky diver falls through the air at terminal velocity. The force of air resistance on him is:

(a) half his weight.
(b) equal to his weight.
(c) twice his weight.
(d) cannot be determined from the information given.

**Q.No.7**: Wind speeding up as it blows over the top of a hill:

(a) Increases atmospheric pressure.
(b) decrease atmospheric pressure.
(c) Doesn't affect atmospheric pressure there.
(d) equal's atmospheric pressure.

**Q.No.8**: A fluid is undergoing "incompressible" flow. This means that:

(a) the pressure at a given point cannot change with time.
(b) The velocity at a given point cannot change with time.
(c) The velocity must be the same everywhere.
(d) The pressure must be the same everywhere.
(e) the density cannot change with time or location.

**Q.No.9**: A fluid is undergoing steady flow. Therefore:

(a) the velocity of any given molecule of fluid does not change.
(b) the pressure does not vary from point to point.
(c) the velocity of any given point does not vary with time.
(d) the density does not vary from point to point.

**Q.No.10**: The equation of continuity for fluid flow can be derived from the conservation of:

(a) energy. (b) mass.
(c) volume. (d) pressure.

**Q.No.11**: This is associated with law of conservation of energy in fluids:
(***2025 Karachi Board***)

- Archimede's principle.
- Pascal's principle.
- Bernoulli's principle.
- Equation of continuity.

***Answers*:**

(1) constant throughout the pipe.
(2) constant.
(3) Bernoulli's principle.
(4) decreases,
(5) remains the same.
(6) equal to his weight.
(7) decrease atmospheric pressure.
(8) the density cannot change with time or location.
(9) the density does not vary from point to point.
(10) mass.
(11) Bernoulli's principle.

# Numericals:

***Q.No.1*: Two spherical raindrops of equal size are falling through air at a velocity of 0.08 m/s. if the drops join together forming a large spherical drop, what will be the new terminal velocity?**

***Data*:**

Initial terminal velocity of rain drops (**before** combining) $v_1 = 0.08$ m/s

Newl terminal velocity of rain drop (**after** combining) $v_2 = ?$

***Solution*:**

Let "r" be the radius of each drop **before** combining and "R" be the radius of lager drop **after** combining. Then since terminal velocity of a spherical body moving through a viscous liquid is directly proportional to square of its radius, therefore:

$v_1 \propto r^2$ and $v_2 \propto R^2$

$\frac{v_1}{v_2} = \frac{r^2}{R^2}$ OR $\frac{0.08}{v_2} = \frac{r^2}{R^2}$

$v_2 = \frac{0.08}{(r/R)^2}$ ………………..($i$)

But volume of drops before and after are given by: $\frac{4}{3}\pi r^3$ $\frac{4}{3}\pi R^3$

But volume of combined drop will be **twice** the volume of each combining drop.

$\frac{4}{3}\pi R^3 = 2\left(\frac{4}{3}\pi r^3\right)$

$R^3 = 2r^3$

$\therefore \frac{r^3}{R^3} = \frac{1}{2}$

Taking cube root of both sides we get:

$\frac{r}{R} = \sqrt[3]{\left(\frac{1}{2}\right)} = 0.7937$

Put this value of (r/R) in equation ($i$).

$v_2 = \frac{0.08}{(r/R)^2}$ ………($i$)

$v_2 = \frac{0.08}{(0.7937)^2} =$ **0.127 m/s.**

🗣New terminal velocity is **0.127 m/s**.

***Q.No.2*: Calculate the viscous drag on a drop of oil of 0.1 mm radius falling through air at its terminal velocity.**

**(Viscosity of air = $1.8\times10^{-5}$ Pa-s.**
**Density of oil = 850 kg/m$^3$.)**

***Data*:**

Viscous drag $F = ?$

Radius of oil drop $r = 0.1$ mm $= 1\times10^{-4}$m

Viscosity of air $\eta = 1.8\times10^{-5}$ Pa-s.

Density of oil $\rho = 850$ kg/m$^3$.

***Solution*:**

According to Stocke's law viscous or drag force on a small **spherical** body is given by:

$F = 6\pi\eta r v$

But (first we will have to find terminal velocity) terminal velocity of the spherical oil drop is given by: $v = \frac{2r^2\rho g}{9\eta}$

$v = \frac{2\times(1\times10^{-4})^2\times850\times9.8}{9\times1.8\times10^{-5}}$

$v =$ **1.028 m/s.**

Drag force $F = 6\pi\eta r v$

$F = 6\pi\times1.8\times10^{-5}\times1\times10^{-4}\times1.028$

**$F = 3.4879\times10^{-8}$ N.**

🗣Drag force on the oil drop is **$3.4879\times10^{-8}$ N**.

***Q.No.3*: What area must a heating duct have if air moving 3.0 m/s along it can replenish the air every 15 minutes in a room of volume 300 m$^3$. Assume air density remains constant.**

***Data*:**

Velocity $v = 3.0$ m/s.

Time taken t =15 min.=15×60 s.= 900 s

Volume of the room $V = 300$ m$^3$.

Area of heating duct $A = ?$

***Solution*:**

Volume of a fluid passing a given point per second, called **volume rate of flow** is given by: $\frac{V}{t} = Av$

$\therefore \frac{300}{900} = A\times3$

$$A = \frac{300}{900 \times 3} = \mathbf{0.111\ m^2.}$$

Area of heating duct must be **<u>0.111 m²</u>**.

***Q.No.4*: Water circulates throughout a house in a hot -water heating system. If water is pumped at a speed of 0.50 m/s through a 4.0 cm diameter pipe in the basement under a pressure of 3.0 atm.**

**What will be the flow speed and pressure in a 2.6 cm diameter pipe on the second floor 5.0 m above? Assume the pipes do not divide into branches.**

***Data*:**

Speed of water in the basement $v_1 = 0.5$ m/s

Diameter of pipe in the basement r = 4 cm=0.04 m

Radius of pipe in the basement $r_1 = 0.02$ m.

Pressure in the basement

$P_1 = 3$ atm.$=3 \times 1.01 \times 10^5$ N/m$^2$$=3.03 \times 10^5$ N/m$^2$

Flow speed at second floor $v_2$ =?

Pressure at second floor $P_2$ =?

Diameter of second floor pipe d=2.6 cm

Radius of second floor pipe $r_2 = 0.013$m

Height of second floor $h_2 = 5$ m.

Density of water $\rho = 1000$ kg./m$^3$

Basement is taken as reference hence, $h_1 = 0$

***Solution*:**

Since $A_1 v_1 = A_2 v_2$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi r_1^2 v_1}{\pi r_2^2} = \frac{\pi (0.02)^2 \times 0.5}{\pi (0.013)^2}$$

$v_2 =$ **1.183 m/s.**

According to Bernoulli's theorem:

$$P_1 + \tfrac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2} \rho v_2^2 + \rho g h_2$$

$$3.03 \times 10^5 + \tfrac{1}{2}\, 1000 \times (0.5)^2 + 0$$

$$= P^2 + \tfrac{1}{2} \times 1000 \times (1.183)^2 + 1000 \times 9.8 \times 5$$

$$P_2 = 3.03 \times 10^5 + \tfrac{1}{2}\, 1000 \times (0.5)^2 - \tfrac{1}{2} \times 1000 \times (1.183)^2 - 1000 \times 9.8 \times 5$$

$$P_2 = 3.03 \times 10^5 + 125 - 699.745 - 49000$$

$$P_2 = 253425.255 \text{ N/m}^2 = \frac{253425.255}{1.01 \times 10^5}$$

$P_2 =$ **2.51 atm.**

Flow speed on the second floor is **<u>1.183 m/s</u>** and pressure is **<u>2.51 atm</u>.**

***Q.No.5*: What is the volume rate of flow of water from a 1.85 cm diameter faucet if the pressure head is 12 m?**

***Data*:**

Volume rate of flow $\frac{V}{t}$ =?

Diameter of faucet d =1.85 cm.

= 0.0185 m.

Radius of faucet r = d/2 = 0.0185/ 2 m

r = 0.00925 m

Height of pressure head h = 12 m.

***Solution*:**

According to law of conservation of energy:

P.E lost while falling down = K.E gained

$$mgh = \tfrac{1}{2} m v^2$$

$$v^2 = 2 g h \qquad v = \sqrt{2 g h}$$

$v = \sqrt{2 \times 9.8 \times 12} =$ **15.336 m/s**

("v" is the velocity with which water flows from the faucet).

According to equation of continuity:

$VA = A v$ But $A = \pi r^2$

$\therefore$ $A = \pi r^2 = \pi (0.00925)^2 =$ $\mathbf{2.688 \times 10^{-4}\ m^2}$

$A v = \frac{V}{t}$ = volume rate of flow of water

$\therefore$ $A v = 2.688 \times 10^{-4} \times 15.336$

$A v = \frac{V}{t} =$ $\mathbf{4.122 \times 10^{-3}\ m^3/s.}$

Volume rate of flow of water will be **<u>$4.122 \times 10^{-3}$ m³/s</u>.**

***Q.No.6*: The stream of water emerging from a faucet 'neck down' as it falls. The cross-sectional area is 1.2 cm² and 0.35 cm². The two levels are separated by a vertical distance of 45 mm as shown in figure. At what rate does water flow from the tape.**

***Data*:**

Cross sectional area at upper level

$A_1 = 1.2$ cm$^2$

Cross sectional area at lower level

$A_2 = 0.35$ cm$^2$

Vertical distance h = 45 mm = 4.5 cm

Rate of flow v/t = ?

***Solution*:**

According to equation of continuity:

Rate of flow of water at upper level $v/t = A_1 v_1$

Rate of flow of water at lower level $v/t = A_2 v_2$

Hence: $A_1 v_1 = A_2 v_2$ Squaring both sides

$A_1^2 v_1^2 = A_2^2 v_2^2$ ................ ($i$)

Acceleration with which water falls is "g": hence applying equation of motion,

$v_2^2 - v_1^2 = 2\,g\,h$ We get:

$v_2^2 = v_1^2 + 2\,g\,h$

Putting this value of $v_2^2$ in equation ($i$) we get:

$A_1^2 v_1^2 = A_2^2 (v_1^2 + 2\,g\,h)$

$A_1^2 v_1^2 = A_2^2 v_1^2 + 2\,A_2^2 g\,h$

$A_1^2 v_1^2 - A_2^2 v_1^2 = 2\,A_2^2 g\,h$

$(A_1^2 - A_2^2)\, v_1^2 = 2\,A_2^2 g\,h$

$$v_1^2 = \frac{2\,A_2^2 g\,h}{(A_1^2 - A_2^2)}$$

Substituting the values of known quantities we get: (**Note that** in this numerical all quantities are in C.G.S units.)

$$v_1^2 = \frac{2 \times (0.35)^2 \times 980 \times 4.5}{(1.2)^2 - (0.35)^2}$$

$$v_1^2 = \frac{1080.45}{1.44 - 0.1225} = \frac{1080.45}{1.3175} = 820.076$$

$v_1 = \sqrt{820.076} =$ **28.637 m/s.**

Rate of flow of water at upper level = $A_1 v_1$

Rate of flow of water at upper level

$= 1.2 \times 28.637 =$ **<u>34.36 m³/s</u>.**

Rate of flow of water at upper level is **<u>34.36 m³/s</u>.**

***<u>Q.No.7</u>*: Water leaves the jet of a horizontal hose at 10 m/s. If velocity of water within the hose is 0.40 m/s. Calculate the pressure within the hose. Density of water is 1000 kg/m³ and the atmospheric pressure is 100000 Pa.**

***<u>Data</u>*:**

Velocity of water inside the hose $v_1 = 0.4$ m/s

Pressure within the hose $P_1 =$?

Velocity of water at the hose $v_2 = 10$ m/s.

Atmospheric pressure $P_2 = 100000$ Pa.

Density of water $\rho = 1000$ kg/m³

***<u>Solution</u>*:**

According to Bernoulli's theorem:

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

Since the hose is horizontal, therefore, $h_1 = h_2$ (which means that there is no change in P.E)

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$P_1 + \frac{1}{2} \times 1000 (0.4)^2 = 100000 + \frac{1}{2} \times 1000 (10)^2$

$P_1 + 80 = 100000 + 50000$

$P_1 =$ **149920 Pa = 1.499×10⁵ Pa.**

Pressure of water with the hose is **<u>1.499×10⁵ Pa</u>.**

***<u>Q.No.8</u>*: What is the maximum weight of an aircraft with a wing area of 50 m² flying horizontally. Velocity of air over the upper surface of the wing is 150 m/s and that at lower surface is 140 m/s. density of air is 1.29 kg/m³.**

***<u>Data</u>*:**

Maximum weight an aircraft W =?

Wing area $A = 50$ m².

Velocity of air over the upper surface of the wing $v_2 = 150$ m/s.

Velocity of air over lower surface $V_1 = 140$ m/s.

Density of air $\rho = 1.29$ kg/m³.

***<u>Solution</u>*:**

According to Bernaulli's equation:

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

(Since the plane if flying horizontally therefore there is no change in potential energy involved $\therefore \rho g h_1 = \rho g h_2$)

$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$

$P = \frac{1}{2} \times 1.29 \times (150)^2 - \frac{1}{2} \times 1.29 \times (140)^2$

$P = 14512.5 - 12642 = 1870.5$ N/m

But $P = F/A$

$F = W = P\,A = 1870.5 \times 50 =$ **93525 N.**

Maximum weight of the aircraft is **<u>93525 N</u>.**

***<u>Q.No.9</u>*: A liquid flows through a pipe with a diameter of 0.50 m at a speed of 4.20 m/s. What is the rate of flow in L/min.?**

***<u>Data</u>*:**

Diameter of pipe $d = 0.50$ m.

Radius of pipe $r = 0.50/2$ m $= 0.25$ m

Speed of liquid $v = 4.20$ m/s.

Rate of flow in L/min. $V/t =$ ?

***<u>Solution</u>*:**

Rate of flow of a liquid $V/t = A\,v = \pi r^2 v$

Rate of flow $= \pi (0.25)^2 \times 4.20 =$ **0.824668 m³/s.**

1 litre = $10^3$ m³ and 1 min. = 60 s

$$\therefore \text{Rate of flow of a liquid} = \frac{0.824668 \times 60}{10^3} \text{ L/min}$$

$\therefore$ Rate of flow of a liquid = **49480 L/min.**

Rate of flow of a liquid through the pipe is **49480 L/min.**

***Unit. 8*****.....**

# Electric field

## ***Electrostatics***:

Electrostatics is a branch of physics which deals with the behavior of charges at rest,

***Like charges*** (both positive or both negative) ***repel*** and ***unlike charges*** (one positive other negative) ***attract*** each other.

***Unit of charge***:

S.I unit of charge is ***coulomb***.

### ***Comparison between electric and gravitational force***:

- Gravitational force is ***attractive*** but electric force between charges may be ***attractive*** or ***repulsive*** in nature.
- Gravitational force is a ***long-range force*** but electric force is a ***short-range force.*** It means that the gravitational force acts over long distances but electric force acts over a short distances.
- Gravitational force is ***weaker*** than electric force.
- Formation of our solar system is due to the gravitational force between sun and other planets.
- Formation of atoms, molecules etc. is due to electric force between oppositely charged electrons and protons or between oppositely charged ions.

## ***Coulomb's law***:

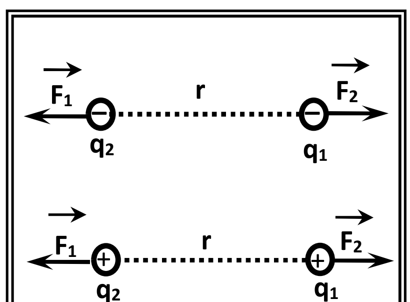
***Q.No.1: State and explain Coulomb's law.***

***Statement***:

***"The force of attraction or of repulsion between two point charges is directly proportional to the product of charges and inversely proportional to the square of distance between them".***

***Formula***:

$$\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$




***Like charges repel***

## ***Derivation***:

Since like charges repel and unlike charges attract each other, therefore, according to Coulomb's law, the magnitude of force "$F_1$" with which "$+q_1$" repels "$+q_2$" when the distance between them is "r", is given by:

$F_1 \propto q_1 q_2$ $\qquad$ $F_1 \propto \frac{1}{r^2}$

On combining both the proportionalities we get:

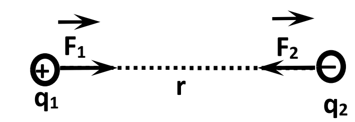
$$F_1 \propto \frac{q_1 q_2}{r^2}$$

$$F_1 = K\frac{q_1 q_2}{r^2}$$

Where "K" is the constant of proportionality, it's value depends upon ***nature of medium present between the charges*** and upon the ***system of units*** in which "F", "q" and "r" are measured.

The value of "K" for vacuum in S.I or M.K.S system is:

$$K = \frac{1}{4\pi\varepsilon_o}$$




***Unlike charges attract***

***Note that*** force exerted by charges on each other always act along the ***line joining their centers*** and are always ***equal*** in magnitude but ***opposite*** in direction.

Where "$\varepsilon_o$" is a universal constant, it represents the permittivity of free space. Its S.I value is $8.85\times10^{-12}$ $C^2/N\text{-}m^2$. Hence.

$$K = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\ N\text{-}m^2/C^2.$$

***In other words***,

$$F_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$$

OR

$$F_1 = 9\times10^9\frac{q_1 q_2}{r^2}$$

***Note that*** when two charges exert force on a third charge, the total or net force on that charge is the ***vector sum*** of forces that the two charges would exert individually. Similarly, if there are more charges then net force on any charge will be the vector sum of forces exerted by all the other charges.

These equations give us the magnitude of force "$F_1$" with which $+q_1$ repels $+q_2$ (These equations help to solve numericals). Since "$F_1$" is a vector quantity, therefore, its direction must be specified.

To include its direction in the above formula, multiply the right side by a unit vector $\hat{r}_{12}$, directed from $+q_1$ to $+q_2$.

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Since action and reaction are equal and opposite (Newton's third law), therefore, force "$\vec{F}_2$" exerted by "$+q_2$" on "$+q_1$" will be $\vec{F}_2 = -\vec{F}_1$. Hence it is given by:

$$\vec{F}_2 = -\frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

## ***Effect of medium upon force between charges***:

***Q.No.2: What happens to force between charges when a medium is present between them?***

***Ans***: As compared to vacuum, force between charges decreases by "$\varepsilon_r$" times when a medium is present between them.

$$\vec{F}_1 = \frac{\frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r}_{12}}{\varepsilon_r}$$

Constant "$\varepsilon_r$" is known as ***"relative permittivity of the medium"***, it is taken relative to the vacuum. Its value for a given medium is constant and depends upon nature of the medium. Force between two point charges with a medium of relative permittivity "$\varepsilon_r$" between them is given by:

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1q_2}{r^2}\hat{r}_{12}$$

$$\boxed{\vec{F}_1 = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}\hat{r}_{12}}$$

Where $\varepsilon = \varepsilon_o\varepsilon_r$ is the (Total) permittivity of the medium.

- For air $\varepsilon_r$=1.0006, hence force between two point charges in air and in vacuum will be practically equal and can be calculated by above equations.
- For water "$\varepsilon_r$" is 80, ***it means that*** force between charges, as compare to vacuum, ***decreases*** by eighty times if they are put in water.

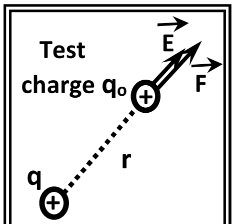
# ***Electric field:***

***Q.No.1: What is electric field?***

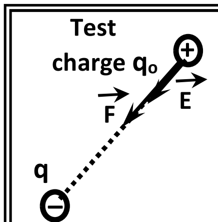
***Ans: Electric field is said to exist at a point at which a positive test charge experiences a force of electrical nature.***

OR

***"The space around a charged body in which it attracts or repels other charges is called electric field".***




***Electric field due to a positive charge q***



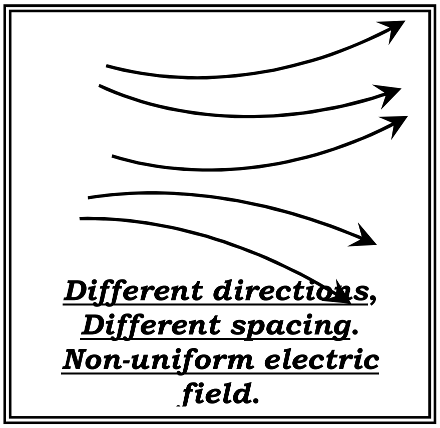

***Electric field due to A negative charge q***

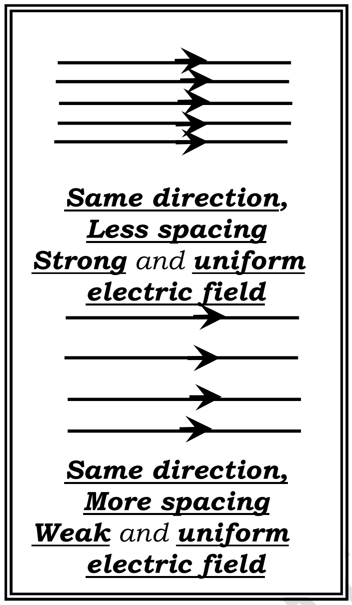
An electric field modifies the space around a charge such that other charges experience a force when they are placed in this space (attraction when they are unlike, repulsion when they are like). This modified space around a charged body in which it attracts or repels other charges is called electric field of the charge.

***Q.No.2: What are electric lines of force?***

***Ans***: An electric line of force is an imaginary line drawn in such a manner that tangent to this line at a given point gives the direction of electric field at that point.

1. Electric lines of force show the direction of $\vec{E}$ at each point.
2. These lines are continuous, they originate from **positive** and terminate at the **negative** charge, and are directed from ***positive*** to ***negative*** charge.
3. Electric lines of force are radial and ***do not intersect*** each other.
4. Spacing of these lines give a general idea about the ***strength*** of the field, hence where lines of force are bunched together (closely spaced) $\vec{E}$ is ***strong*** and where they are farther apart $\vec{E}$ is ***weak***.
5. Parallel electric lines of force, equally spaced represent a ***uniform*** electric field.
6. Hence for oppositely charged plates the electric field or ***in other words*** lines of electric force are directed from a ***positively charged*** to a ***negatively charged*** plate.




***Note that*** the test charge must be positive and of a very small magnitude.
It must be very small so that it does not alter the already existing electric field whose intensity it is to measure.

## *Strength of electric field or Electric intensity*:

***Q.No.3*: Define electric intensity at a point in an electric field. Also give its S.I unit.**

***Ans*:** ***"Electric intensity or strength of an electric field at a point in an electric field is the force experienced by the test charge divided by the magnitude of test charge".***

OR

***"Electric intensity at a point is the force per unit charge'.***

$$\vec{E} = \frac{\vec{F}}{q_o}$$

"$q_o$" is the magnitude of test charge.

Electric intensity is a ***vector quantity;*** its direction is **same** as the direction of ***electric force*** experienced by a ***positive*** test charge.

- Electric intensity due to a ***positively charged*** body is directed ***away*** from the body. (Because a positively charged body repels positive test charge).
- Electric intensity due to a ***negatively charged*** body is directed ***towards*** the body. (Because a negatively charged body attracts positive test charge).

## *S.I unit of electric intensity*:

The S.I unit of electric intensity is N/c (newton per coulomb) or v/m (volt per meter).

***Q.No.4*: Derive an expression for electric intensity at a point near an isolated point charge.**

***Ans*:** Consider an isolated point charge "+q". If a positive test charge "+$q_o$" is placed at distance "r" from it in its electric field, the test charge will experience a repulsive force, according to the strength of the field at that point.

According to Coulomb's law the force $\vec{F}$ with which "+q" repels the test charge "$+q_o$" is given by:

$$\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{q\,q_o}{r^2}\hat{r}$$

"$\hat{r}$" is a unit vector directed away from the field producing charge (i.e. away from point charge +q). Hence electric intensity $\vec{E}$ at a distance "r" from "+q" is given by:

$$\vec{E} = \frac{\vec{F}}{q_o}$$

Hence,
$$\vec{E} = \frac{\frac{1}{4\pi\varepsilon_o}\frac{q\,q_o}{r^2}\hat{r}}{q_o}$$

$$\therefore \quad \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}}$$

For a negative point charge "– q", electric intensity at a point is given by:

$$\therefore \quad \boxed{\vec{E} = -\frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}}$$

In this case the direction of $\vec{E}$ will be opposite to the direction of $\hat{r}$ (i.e. towards the charge).

## ***Effect of medium upon electric intensity:***

***Q.No.5: What is the effect of the presence of a medium upon electric intensity at a point?***

***Ans:*** Electric intensity at a point ***decreases*** by $\varepsilon_r$ times if a medium is present between the field producing charge and the given point ($\varepsilon_r$ is the relative permittivity of the medium). ***In other words***, a test charge placed at such a point will experience a weaker force as the electric field has become weak.

***Q.No.6: How is net (or resultant) electric intensity at a point due to a distribution of several isolated point charges found?***

***Ans:*** If there are two or more isolated point charges distributed randomly, then the net (or resultant) electric intensity at a point due to such a charge distribution will be equal to the ***vector sum*** of intensity due to individual point charges.
In such a case, first of all using the above formula, find the intensity at the given point due to each point charge separately. Then considering their directions find their vector sum.

If $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3$,……. represent strength of electric fields (or electric intensities) at a point due to charges $q_1$, $q_2$, $q_3$, …… etc. then the net electric intensity $\vec{E}$ at the point is given by:

$$\boxed{\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3, \ldots\ldots}$$

## ***Electric dipole*:**

An ***electric dipole is a pair of two equal and opposite charges "+q" and "– q" separated by a very small distance "d"***.

Electric field produced by an electric dipole at any point is directed from positive to the negative charge and it has maximum strength along a straight line joining the two charges, called **axis of the dipole**.

***Electric dipole moment*** $\vec{p}$ is a ***vector*** quantity. Its magnitude p is equal to the product "q d" the magnitude of charge "q" and the distance "d" between the charges, it represents the strength of the dipole and is directed from the **negative charge** "– q" **to positive charge** "+q".

## ***Electric field due to a dipole along its axis*:**

Let the small distance between two equal and opposite charges "+q" and "– q", forming a dipole be "d". Along the axis of the dipole at point "P" at distance "r" from "+q" electric intensity due to "+q" will be:

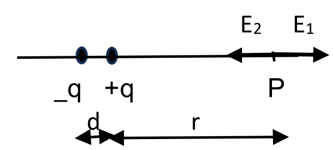
$$E_1 = K\frac{q}{r^2} \qquad \text{(where } K = \frac{1}{4\pi\varepsilon_o} \text{ it is called Coulomb's constant)}$$

"$E_1$" is directed **away** from charge "+q". Similarly at the same point "P" electric intensity due to "– q" will be:

$$E_2 = K\frac{q}{(r+d)^2}$$

"$E_2$" is directed **towards** charge "– q".

Since "$E_1$" and "$E_2$" are in opposite directions therefore, net electric intensity at "P" will be:




***Electric field due to a dipole along its axis.***

$$E = E_1 - E_2$$

$$E = K\frac{q}{r^2} - K\frac{q}{(r+d)^2}$$

$$E = Kq\left\{\frac{1}{r^2} - \frac{1}{(r+d)^2}\right\}$$

$$E = Kq\left\{\frac{(r+d)^2 - r^2}{r^2(r+d)^2}\right\}$$

$$E = Kq\left\{\frac{r^2+2rd+d^2-r^2}{r^2(r^2+2rd+d^2)}\right\}$$

Since distance "d" between the charges is **very small** (Imagine the distance between hydrogen atom and oxygen atom forming a dipole in water molecule) as compared with "r", therefore, neglecting the negligible terms we get:

$$E = Kq\left\{\frac{2rd}{r^4}\right\}$$

OR $$E = \frac{2Kqd}{r^3}$$

But (product of a charge and distance between charges) "q d = P", it is known as "**dipole moment**":

$$E = \frac{2KP}{r^3}$$

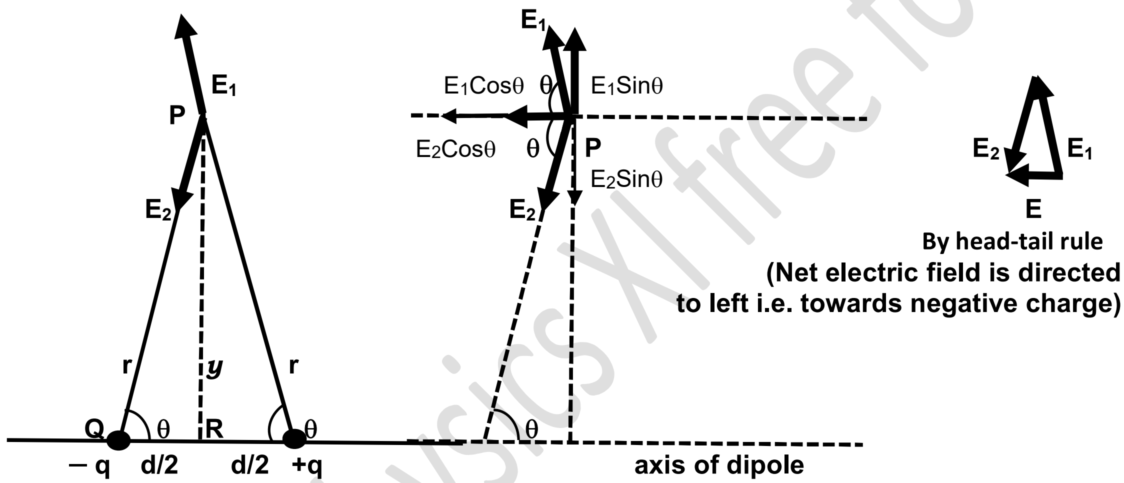
But $K = \frac{1}{4\pi\varepsilon_o}$

$$\boxed{E = 2 \times \frac{1}{4\pi\varepsilon_o}\frac{P}{r^3}} \quad \ldots\ldots\ldots\ldots\ldots.. (i)$$

This formula gives us strength of electric field (or electric intensity) at a point along the axis of a dipole. It shows that the strength of electric field of a dipole along its axis depends upon its dipole moment "P" which is the product of charge "q" and small distance "d" between the charges and not only upon the magnitude of charge, it also shows that the strength of field **decreases rapidly** with "r". (inversely proportional to $r^3$).

## ***Electric field due to a dipole at a point perpendicular to its axis:***

Horizontal line joining a dipole "+q" and "–q" shows its axis. Strength of electric field at a point "P" at an equal distance "r" from each charge and perpendicular to axis of the dipole will be equal to "$E_1$" and "$E_2$", where "$E_1$" and "$E_2$" are strength of electric field at "P" due to "+q" and "–q" respectively.




***Electric field due to a dipole at "P"***

Electric intensity "$E_1$" due to **positive** charge is **away** and "$E_2$" due to **negative** charge is **towards** the charge. Resolving $E_1$ and $E_2$ into horizontal and vertical components we see that vertical components $E_1 \sin\theta$ and $E_2 \sin\theta$ cancel each other (Since they are equal and opposite to each other, because "q" and "r" for each charge is equal). But horizontal components $E_1 \cos\theta$ and $E_2 \cos\theta$ are in the **same** direction (**towards** the negative charge) hence they will add up, net strength of electric field at point P is given by:

$$E = E_1 \cos\theta + E_2 \cos\theta \qquad E_1 = E_2 = \frac{Kq}{r^2}$$

$$E = 2\frac{Kq}{r^2}\cos\theta$$

In triangle PQR: $\cos\theta = \frac{d/2}{r}$

$$E = 2\frac{Kq}{r^2}\frac{d/2}{r} = 2\frac{Kqd}{2r^3} = \frac{1}{4\pi\varepsilon_o}\frac{qd}{r^3}$$

But $qd = P$ (called **electric dipole moment**)

$$E = \frac{1}{4\pi\varepsilon_o}\frac{P}{r^3}$$

In triangle PQR: $r^2 = (d/2)^2 + y^2$ therefore, $r = \sqrt{(d/2)^2 + y^2}$

Hence
$$E = \frac{1}{4\pi\varepsilon_o}\frac{P}{\left\{(d/2)^2 + y^2\right\}^{3/2}}$$

Distance "d" between the two charges, forming a dipole is **very small** as compared to "$y$" hence it's higher power terms are negligibly small. Neglecting $(d/2)^2$ we get:
$$E = \frac{1}{4\pi\varepsilon_o}\frac{P}{y^3}$$

"$y$" can be taken as equal to "r" since d/2 is negligible.
$$E = \frac{1}{4\pi\varepsilon_o}\frac{P}{r^3} \quad \ldots\ldots\ldots\ldots\ldots\ (ii)$$

Comparing equations (*i*) and (*ii*) we conclude that the electric field is stronger at a point along the axis of a dipole than at an equal distance from charges perpendicular to the axis.

Type of electric field due to a dipole is used in microwave oven. Food to be heated is subjected to an alternating electric field.

- Due to high electro-negativity shared pair of electrons shifts slightly towards oxygen atom in a water molecule, due to which a **small negative charge** appears at Oxygen end and a **small positive charge** at Hydrogen ends. As a result of which water molecules behave as a **dipole**. When electric field is applied to a water molecule it turns to align with the direction of applied field. If an alternating field is applied, water molecules start vibrating/rotating (as it tends to align itself with the direction of electric field), thereby rubbing with their neighboring molecules. As a result of which its temperature rises.
- In a Microwave oven food to be heated is subjected to an alternating field, due to the presence of water molecules in the food, it is heated by the mechanism described above.

# Electric flux:

***Q.No.1*: Define electric flux passing through a surface.**

***Ans*: "Electric flux "Δϕ" passing through a surface is the scalar or dot product of electric intensity "$\vec{E}$" and vector area "$\Delta\vec{A}$" of the surface".**

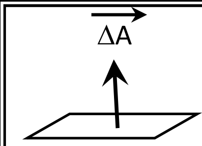
$$\Delta\phi = \vec{E} \cdot \Delta\vec{A}$$

Electric flux is a ***scalar quantity***. The magnitude of electric flux is given by:
$$\Delta\phi = E\,\Delta A \cos\theta$$
Where "θ" is the angle between the electric intensity $\vec{E}$ and vector area "$\Delta\vec{A}$" (When tails of $\vec{E}$ and $\Delta\vec{A}$ are joined together).

- Physically the electric flux is a measure of ***number of lines of electric force passing normally*** through a surface in an electric field.
- S.I unit of electric flux is N m$^2$/C.

***Q.No.2*: What is vector area "$\overrightarrow{\Delta A}$" ?**

***Ans*:** Area normally is a scalar quantity, but in this case area of a surface has a ***directional effect*** upon the number of lines of force passing through the surface. Because if the surface makes different angles with the electric field then the flux passing through it (i.e. the number of lines passing through it) is different for different angles. Hence area of the surface is taken as a ***vector***. It is called ***vector area*** of the surface. Its magnitude is equal to the ***geometrical area of the surface*** and its direction is taken ***along the outward drawn normal*** to the surface.

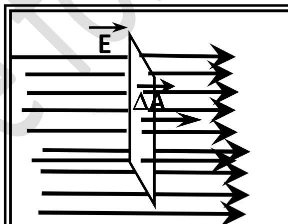



***Vector area***

***Q.No.3*: Under what condition is flux passing through a surface: (*i*) maximum (*ii*) minimum?**

***Ans*:** (*i*) ***Condition for maximum flux*:**

If a surface is perpendicular to the electric field, then "$\vec{E}$" will be parallel to the vector area "$\overrightarrow{\Delta A}$", so that angle between them is "$\theta=0°$". Under this condition maximum number of lines of force will pass through it, ***in other words*** flux passing through it will be ***maximum***.




***Maximum flux***

Magnitude of maximum flux passing through a surface is given by:

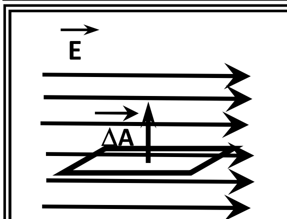
$$\Delta\phi = E\,\Delta A \cos 0°$$

$$\Delta\phi = E\,\Delta A \qquad (\text{Con } 0° = 1)$$

(*ii*) If the surface is ***parallel*** to the electric field, then "$\vec{E}$" will be perpendicular to the vector area "$\overrightarrow{\Delta A}$", so that "$\theta = 90°$". Under this condition no line of force will pass through the surface, ***in other words***, flux passing though the surface will ***zero***. This is the condition for ***minimum*** flux.

$$\Delta\phi = E\,\Delta A \cos 90°$$

$$\Delta\phi = 0 \qquad (\text{Con } 90° = 0)$$




***Minimum flux***

***Electric flux through surface of any shape enclosing a charge*:**

By following these necessary steps electric flux passing through a closed surface of any geometrically regular or irregular shape can be determined:

***Step 1*:** Divide the given surface into a large number of patches. The process of dividing the surface into patches should be continued until each patch obtained must be ***plane*** in shape.

***Step 2*:** Determine the electric flux through each patch.

Let

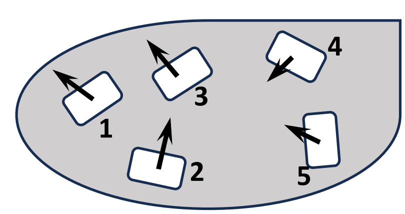
$$\Delta\phi_1 = (\vec{E}\cdot\overrightarrow{\Delta A})_1$$

$$\Delta\phi_2 = (\vec{E}\cdot\overrightarrow{\Delta A})_2$$

$$\Delta\phi_3 = (\vec{E}\cdot\overrightarrow{\Delta A})_3$$

$$\vdots$$

$\Delta\phi_n = (\vec{E}\cdot\overrightarrow{\Delta A})_n$ be the flux through 1st, 2nd, 3rd, …., $n$th patch.




***Flux through a closed surface***

***Step 3*:** Since electric flux is a **scalar** quantity, therefore, total flux through the given closed surface will be equal to the sum of flux through all the patches.

Hence, total flux "$\phi$" through the surface having $n$ patches is given by:

$$\phi = \sum_{i=n}^{i=1} \Delta\phi_n$$

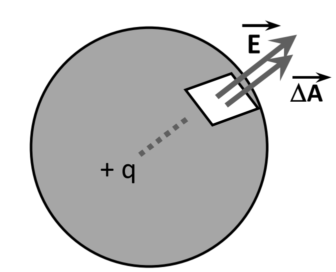
OR

$$\phi = \sum_{i=n}^{n=1} (\vec{E} \cdot \vec{\Delta A})_n$$

$\sum_{i=n}^{n=1}$ is read as sigma $i$=1 to $i$=$n$. ***It means that*** the closed surface has been divided into "$n$" patches and to find the total flux "$\phi$" add flux through patches number 1 to patch number $n$.

***Q.No.4*: *Derive a formula for electric flux through the surface of a sphere due to a point charge "+q" at its center.***

***Ans*:** Imagine a small positive point charge "+q" located at the center of a sphere of radius "r". To find the electric flux through the surface of a sphere, divide the surface into large number of small patches each of area "$\vec{\Delta A}$". Each patch must be so small that it is practically plane in shape, so that the direction of "$\vec{\Delta A}$" comes out to be the ***same*** at all points on a patch. Since the point charge is located at the center, lines of force will originate from the center of the sphere. ***In other words***, each patch will be perpendicular to the electric field (i.e. to $\vec{E}$). Hence electric flux through each patch will be ***maximum*** and is given by:




***Flux through a sphere***

$\Delta\phi = E\,\Delta A \cos 0°$ ($\vec{E}$ and $\vec{\Delta A}$ are parallel to each other)

$\Delta\phi = E\,\Delta A$

Since flux is a ***scalar quantity***, therefore, the total flux through the sphere will be equal to the sum of flux through individual patches.

Total flux through the sphere $\phi = \Sigma\, E\,\Delta A$

But the magnitude of electric intensity at the surface of the sphere due to a point charge "+q" at its center is given by:

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

$\therefore$ Total flux through the sphere $\phi = \Sigma \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\,\Delta A$

Since each patch is at equal distance from the point charge "+q" therefore, $\vec{E}$ is same for all the patches and can be taken as common in the above summation.

$\therefore$ Total flux through the sphere $\phi = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\,\Sigma.\,\Delta A$

But $\Sigma.\,\Delta A = 4\pi r^2$ = Total surface area of the sphere.

$\therefore$ Total flux through the sphere $\phi = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\,4\pi r^2$

$\therefore \quad \boxed{\phi = \dfrac{q}{\varepsilon_o}}$

This formula shows that the electric flux through the surface of a sphere is independent of position of the point charge inside the sphere.

## ***Gauss's law***:

According to Gauss's law:

***"Electric flux passing through any closed surface (of any shape) is $1/\varepsilon_o$ times the total charge enclosed by that closed surface".***

Hence: $\phi = \dfrac{1}{\varepsilon_o} \times$ Total charge enclosed

$\therefore \quad \boxed{\phi = \dfrac{q}{\varepsilon_o}}$

# ***Electric potential:***

***Q.No.1: Define potential difference between two points in an electric field.***

***Ans.*** Potential difference between two points in an electric field may be defined as:

***"The amount of work done to move a positive test charge from one point to another divided by the magnitude of the test charge".***

> ***Note that*** when work is done **on** a charge it **gains** electric P.E but when work is done **by** the charge it **looses** electric P.E.

$$\text{Potential difference} = \frac{\text{Work done}}{\text{Test charge}}$$

$$\Delta V = \frac{\Delta U}{q_o}$$

OR ***"It is the change in potential energy per unit charge".***

Potential difference is a ***scalar*** quantity.

***Q.No.2: Define S.I unit of potential difference.***

***Ans.*** The S.I unit of potential difference is ***volt***.

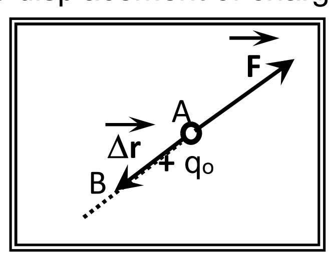
Potential difference between two points in an electric field is said to be one volt *(*1V*)* if one joule *(*1J*)* of work is done to move one coulomb (1 c) charge from one point to another.

$$1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

## ***Potential difference between two points***:

***Q.No.3: Derive an expression for potential difference between two points in an electric field.***

***Ans.*** If a small charge "$+q_o$" is moved from point "A" to another point "B" against the direction of electric field then work has to be done **on it**. Let "$\vec{\Delta r}$" be the displacement of charge "q" from point A to point B,




and $\vec{F}$ be the force on charge 'q' in the electric field, then the amount of work done (work is the dot product of $\vec{F}$ and $\vec{\Delta r}$) is given by:

$$\Delta W = \vec{F} \cdot \vec{\Delta r}$$

But $\vec{F} = q_o \vec{E}$ where $\vec{E}$ is the electric intensity (or strength of electric field) throughout displacement $\vec{\Delta r}$.

$$\Delta W = q_o \vec{E} \cdot \vec{\Delta r}$$

This work done on '$q_o$' is ***independent of the path followed***, it changes electrical potential energy of the charge. The change in electric potential energy "$\Delta U$" (increase in P.E) of charge "$+q_o$" is equal to the amount of **work done on** it.

$$\Delta U = \Delta W$$

But $\Delta W = q_o \vec{E} \cdot \vec{\Delta r}$ $\therefore$ $\Delta U = q_o \vec{E} \cdot \vec{\Delta r}$

$\therefore$ $$\frac{\Delta U}{q_o} = \vec{E} \cdot \vec{\Delta r}$$

$\Delta U/q_o$ represents the **change in potential energy per unit charge** or the ***work done per unit charge***. But work done per unit charge is called **potential difference $\Delta V$** between the two points A and B. $\therefore$

$$\Delta V = \vec{E} \cdot \vec{\Delta r}$$

Potential difference between two points is also the ***dot*** or ***scalar product*** of ***electric intensity*** $\vec{E}$ and the ***displacement*** $\vec{\Delta r}$.

***Note that***: This formula for calculating potential difference between two points ***is valid only when the electric field is uniform throughout the displacement "$\vec{\Delta r}$"***. Electric intensity between two given points may be uniform only through small displacements, hence this formula can be applied when the displacement $\vec{\Delta r}$ is ***small***.

## ***Absolute potential***:

***Q.No.4: Define absolute potential at a point in an electric field.***

***Ans.*** Absolute potential at a point in an electric field may be defined as:

***"The amount of work done per unit charge when the charge is moved from infinity to that point".***

- The point at infinity is outside the electric field. Hence, it will be at zero potential (no work is done to move a test charge).

Absolute potential at a point in electric field due charge "q" (at distance "r" from the field producing charge "q") is given by::

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

## ***Potential difference between any two points***:

Potential difference between any two points in an electric field can also be determined by finding the difference in absolute potential at these points.

To find the potential difference between two points 1 and 2 in electric field produced by a charge "q" let the distance of each point from charge "q" be "$r_1$" and "$r_2$" respectively. Potential (absolute potential) at each point will be:

$$V_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{r_1} \quad \text{and} \quad V_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{r_2}$$

Hence the potential difference between these points is given by:

$$\Delta V = V_2 - V_1 = \frac{q}{4\pi\varepsilon_o}\left\{\frac{1}{r_2} - \frac{1}{r_1}\right\}$$

This is a general formula for finding potential difference between any two points in an electric field produced by a charge "q".

***Q.No.5*: Derive a relation between electric intensity E and potential difference ΔV. (2015 Karachi Board)**

***Ans.*** Potential difference between two nearby points in an electric field is given by:

$$\Delta V = \vec{E} \cdot \vec{\Delta r}$$

$$\Delta V = E\,\Delta r \cos\theta$$

Where "θ" is the angle between the direction of electric field $\vec{E}$ and the direction of displacement $\vec{\Delta r}$. In this case θ = 180°, because the displacement $\vec{\Delta r}$ is opposite to the direction of electric field $\vec{E}$.

∴ $\Delta V = E\,\Delta r \cos 180°$

∴ $\Delta V = -E\,\Delta r$

∴ $$E = -\frac{\Delta V}{\Delta r}$$

- ΔV/Δr is called the ***space rate of variation of potential*** between the points. The space rate of variation or the ***rate of change of potential with respect to distance*** is called "***potential gradient***". Hence,

  "***Electric intensity at a point is equal to the negative potential gradient at that point***".

This is also the relation between electric intensity E, potential difference ΔV and the distance Δr between the points.

- The above formula also gives "***volt/meter***" as the unit of electric intensity.
- ***Note that*** when a charge is moved ***against*** an electric field work has to be done ***on*** it and it ***gains*** potential energy but if it moves ***in the direction*** of a field work is done ***by*** it and therefore, it ***loses*** potential energy.

If a charge is free to move, then the positive charge moves from a point at ***higher*** potential to a point at ***lower*** potential, whereas a negative charge moves from a point at ***lower*** potential to a point at ***higher*** potential.

***Q.No.6*: Give a relation between kinetic energy and potential difference.**

***Ans.*** Since ,

$$\text{Potential difference } \Delta V = \frac{\Delta W}{q}$$

$$\Delta W = q\,\Delta V$$

Where ΔW is the work done, it is also equal to the change in potential energy ΔU. When a charge looses electrical potential energy it gains an equal amount of kinetic energy and vice versa. Hence if a charge is allowed to fall through (or accelerated by) a potential difference ΔV then it loses potential energy given by:

$$\Delta U = q\,\Delta V$$

At the same time, it will gain an equal amount of kinetic energy, given by:

$$\Delta U = \tfrac{1}{2} m v^2$$

***In other words***,

$$\tfrac{1}{2} m v^2 = q\,\Delta V$$

## ***Electron volt*:**

***Q.No.7*: What is electron volt?**

***Ans*. "Electron volt"** is the unit of energy commonly used in atomic and nuclear physics.

***"One electron volt (1ev.) is the amount of energy gained by an electron when it is accelerated through a potential difference of 1 volt".***

Hence,

$\Delta V = 1$ volt, $q = e = 1.6021\times10^{-19}$ cool.

The kinetic energy gained by the electron is given by:

$$\tfrac{1}{2}mv^2 = q\,\Delta V$$

$$\tfrac{1}{2}mv^2 = 1.6021\times10^{-19}\times 1$$

$$\tfrac{1}{2}mv^2 = 1.6021\times10^{-19}\ J$$

$$\therefore \quad \boxed{1\ ev. = 1.6021\times10^{-19}\ J}$$

Similarly:

1 Mev = $10^{6}$ ev

1 Bev = $10^{12}$ ev

## Section–A Multiple-Choice Questions MCQs:

**Q.No.1**: A 2 μC point charge is located at a distance "d" away from 6 μC point charge, what is the ratio of $F_{12}/F_{21}$?
(a) 1/3 (b) 3 (c) 1 (d) 12

**Q.No.2**: The minimum charge on an object cannot be less than:
(a) $1.6\times10^{-19}$ C. (b) $3.2\times10^{-19}$ C.
(c) $9.1\times10^{9}$ C.
(d) No definite value exists

**Q.No.3**: Two charges are placed at a certain distance. If the magnitude of each charge is doubled the force will become:
(a) ¼th of its original value.
(b) 4 times its original value.
(c) 1/8th of its original value.
(d) 8 times its original value.

**Q.No.4**: Which of the following can be deflected while moving in the electric field?
(a) neutron. (b) photon.
(c) electron. (d) (a) and (b)

**Q.No.5**: The flux through a flat surface of area "**A**" in a uniform electric field "**E**" is maximum when the surface area is:
(a) Parallel to **E** (b) perpendicular to E
(c) placed 45° to **E** (d) placed 60° to **E**.

**Q.No.6**: The product of charge "q" and small separation "d" between two charges of same magnitude and opposite in nature is known as:
(a) Electric dipole. (b) Moment arm.
(c) Electric dipole moment.
(d) Flux of electric field.

**Q.No.7**: 12 J of work is to be done against an electric field to take a charge of 0.01 C from one point A to another point B. The potential difference between B and A is:
( a) 120 V. (b) 1200 V.
(c) 1.2 V. (d) 12 V.

**Q.No.8**: The force between two charges placed in air is F. If air is replaced by a medium of relative permittivity $\varepsilon_r$ then force is reduced to:
(a) F $\varepsilon_r$ (b) F / $\varepsilon_r$ (c) $\varepsilon_r$ / F (d) $\varepsilon$ $\varepsilon_r$

**Q.No.9**: The negative gradient of potential is:
(a) potential energy. (b) voltage.
(c) electric field intensity.
(d) electric flux.

**Q.No.10**: The electric flux through a plane area will be half of its maximum value when area is held at an angle of ……. With electric field.
(a) 30° (b) 60° (c) 45° (d) 90°

**Q.No.11**: The concept of electric lines of force was introduced by a famous scientist called: ***(2006 Karachi Board)***
• Newton. • Einstein.
• Coulomb. • Faraday.

**Q.No.12**: Which of the following cannot be the unit of electric intensity?
***(2007 Karachi Board)***
• N/coulomb. • Volt/meter.
• Joule./ coul.-meter. • Joule/coulomb.

**Q.No.13**: Two positive point charges repel each other with a force of $4\times10^{-4}$ N when placed at a distance of 1 meter. If the distance between them is increased to 2 m, the force of repulsion will be: ***(2008 Karachi Board)***
• $1\times10^{-4}$ N. • $8\times10^{-4}$ N.
• $2\times10^{-4}$ N. • $4\times10^{-4}$ N.

**Q.No.14**: The change in potential energy per unit charge between the two points in an electric field is called: ***(2009 Karachi Board)***
• Intensity. • Flux.
• Potential difference. • Permittivity.

**Q.No.15**: This is not a scalar quantity:
***(2018, 2014 Karachi board)***
• Electric flux. • Electric intensity.
• E.M.F. • Electric potential.

**Q.No.16**: The scalar product of electric intensity ($\vec{E}$) and vector area ($\Delta\vec{A}$) is called:
***(2019 Karachi Board)***
• electric flux. • electric force.
• electric potential.
• electric flux density

**Q.No.17**: Joule per coulomb is equal to:
***(2021, 2014, 2012 Karachi board)***
• Farad • Henry • Ampere • Volt

**Q.No.18**: When a dielectric is placed in an electric field, it becomes.
***(2021 Karachi Board)***
• Negatively charged only.

- Positively charged.
- Polarized
- Conductive

***Q.No.19*:** If the separation of two positive charges is halved then electrostatic force of repulsion becomes:

(***2021,2015, 2007 Karachi Board***)

- $\frac{1}{4}$ times
- $\frac{1}{2}$ times
- 2 times
- 4 times

***Q.No.20*:** The unit of potential gradient is:

(***2023 Karachi Board***)

(A) Volt/meter.
(B) Coulomb/meter. (C)Volt/Ampere
(D) Volt × Ampere.

***Q.No.21*:** The flux through an area "$\vec{A}$" in a uniform electric field "$\vec{E}$" is maximum when the angle between $\vec{E}$ and $\vec{A}$ is:

(***2025 Karachi Board***)

- 0°
- 30°
- 60°
- 90°

***Answers*:**

(1) 1.
(2) $1.6\times10^{-19}$ C
(3) 4 times its original value.
(4) electron.
(5) Parallel to **E**
(6) Electric dipole moment.
(7) 1200 V
(8) F / $\varepsilon_r$
(9) electric field intensity.
(10) 60°
(11) Faraday
(12) Joule/coulomb.
(13) $1\times10^{-4}$ N.
(14) Potential difference.
(15) Electric intensity.
(16) electric flux.
(17) Volt
(18) Polarized
(19) 4 times
(20) Volt/meter
(21) 0°.

# <u>Numericals</u>:

***Q.No.1***: **(a) Calculate the value of two equal charges if they repel one another with a force of 0.1 N when situated 50 cm apart in vacuum.**

**(b) What would be the size of charges if they were situated in an insulating liquid whose permittivity is ten times that of vacuum?**

***Data***:

(a) Charges in vacuum $q_1 = q_2 = ?$
Force between charges $F = 0.1$ N
Distance between charges $r = 50$cm. $= 0.5$m
(b) Charges in insulating medium $q_1 = q_2 = ?$
Permittivity of medium $\varepsilon_r = 10\ \varepsilon_o$

***Solution***:

(a) Force between charges in vacuum is given by:

$F = k\dfrac{q_1 q_2}{r^2}$ $\left(k = \dfrac{1}{4\pi\varepsilon_o} = 9\times10^9 Nm^2/C^2\right)(q_1 = q_2 = q)$

$q^2 = \dfrac{F\,r^2}{k} = \dfrac{0.1\times(0.5)^2}{9\times10^9} = 2.778\times10^{-12}\ C^2$

$q = \sqrt{2.778\times10^{-12}} = \mathbf{1.667\times10^{-6}\ C.}$

**OR <u>1.667 μC</u>.**

🗣Value of each charge in vacuum is **<u>1.67×10⁻⁶ C</u>**. or **<u>1.667 μC</u>.**

(b) For an insulating medium (coulomb constant)

$K = \dfrac{k}{\varepsilon_r} = \dfrac{9\times10^9}{10} = 9\times10^8\ Nm^2/C^2$

Force between charges in the medium:

$F = K\dfrac{q^2}{r^2}$

$\therefore\quad q^2 = \dfrac{F\,r^2}{K} = \dfrac{0.1\times(0.5)^2}{9\times10^8} = 2.778\times10^{-11}\ C^2$

$q = \sqrt{2.778\times10^{-11}} = \mathbf{5.271\times10^{-6}\ C.}$

**OR <u>5.271 μC</u>.**

🗣Value of each charge in an insulating medium is **<u>5.271×10⁻⁶ C</u>**. or **<u>5.271 μC</u>.**

***Q.No.2***: **How far apart must two protons be if the magnitude of electrostatic force acting on either one due to the other is equal to the magnitude of gravitational force on a proton at Earth's surface?**

**(Mass of proton $= 1.67\times10^{-27}$ Kg.
charge on proton $= 1.6\times10^{-19}$ C)**

***Data***:

Distance between proton $r = ?$
Mass of proton $m = 1.67\times10^{-27}$ Kg.
Charge on proton $e = 1.6\times10^{-19}$ C
Electrostatic force on either = Gravitational force on a proton at earth surface is equal to its weight mg.

***Solution***:

Electrostatic force between protons $F = \dfrac{K\,e^2}{r^2}$

$F = \dfrac{9\times10^9\,(1.6\times10^{-19})^2}{r^2} = \dfrac{2.304\times10^{-28}}{r^2}$

Since: $F = m\,g$ (gravitational force)

$\therefore\quad \dfrac{2.304\times10^{-28}}{r^2} = 1.67\times10^{-27}\times9.8$

$r^2 = \dfrac{2.304\times10^{-28}}{1.6366\times10^{-26}} = 0.014$

$\therefore\quad r = \sqrt{0.014} = \mathbf{0.119\ m.}$

🗣Distance between protons is **<u>0.119 m</u>**

***Q.No.3***: **An electron of charge $1.6\times10^{-19}$ C is situated in a uniform electric field of intensity 1200 volt/cm. Find the force on it, it's acceleration and the time it takes to travel 2 cm. from rest.**

**(electronic mass $= 9.1\times10^{-31}$ kg)**

***Data***:

Charge of electron $q = e = 1.6\times10^{-19}$ C.
Mass of electron $m = 9.1\times10^{-31}$ kg.
Electric intensity $E = 1200$ volt/cm.
$= 1200\times100$ V/m. $= 12\times10^4$ volt/m.
Force on electron $F = ?$
Acceleration of electron $a = ?$
Distance moved $S = 2$ cm. $= 0.02$ m
Initial velocity $v_i = 0$
Time taken $t = ?$

***Solution***:

Force on electron is given by:

$F = q\,E = 1.6\times10^{-19}\times12\times10^4$

$\mathbf{F = 1.92\times10^{-14}\ N.}$

Since: $F = m\,a$ (Newton's second law).

$\therefore \quad 1.92\times10^{-14} = 9.1\times10^{-31} \times a$

$$a = \frac{1.92\times10^{-14}}{9.1\times10^{-31}} = \mathbf{2.11\times10^{16}\ m/s^2}.$$

Since: $\quad S = v_i t + \frac{1}{2} a t^2$

$\therefore \quad 0.02 = 0 + \frac{1}{2} \times 2.11\times10^{16} t^2$

$$t^2 = \frac{0.02}{1.055\times10^{16}} = 1.8957\times10^{-18}$$

$$t = \sqrt{1.8957\times10^{-18}} = \mathbf{1.3769 \times10^{-9}\ s}.$$

***Q.No.4*: An alpha particle (nucleus of helium atom) has a mass of $6.64\times10^{-27}$ kg and a charge of 2e. What are the (a) magnitude and (b) direction of the electric field that will balance the gravitational force on the particle.**

***Data*:**

Mass of alpha particle m= $6.64\times10^{-27}$ kg
charge of alpha particle q =2e
$= 2\times1.6\times10^{-19}C = 3.2\times10^{-19}C$
Magnitude and direction of electric field =?
Electric force = gravitational force.

***Solution*:**

Since: Electric force = gravitational force.

$\therefore \quad qE = mg$

$$E = \frac{mg}{q} = \frac{6.64\times10^{-27} \times 9.8}{3.2\times10^{-19}} = \mathbf{2.0335\times10^{-7} N/C}$$

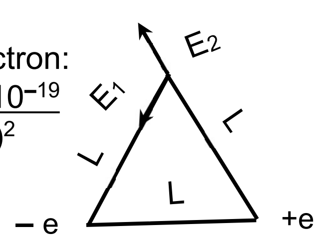
Gravitational force is downward therefore, electric field must be **upward** so that it exerts an upward force on the alpha particle to balance the gravitational force acting it.

***Q.No.5*: A proton and an electron form two corners of an equilateral triangle of side length $2.0\times10^{-6}$ m. What is the magnitude of net electric field these two particles produce at the third corner?**

***Data*:**

Charge of proton = Charge of electron = e
Length of each side of equilateral triangle
$L = 2.0\times10^{-6}$ m.
Magnitude of net electric field at third corner E =?

***Solution*:**




Electric field due to electron:

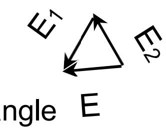
$$E_1 = k\frac{e}{L^2} = \frac{9\times10^9\times1.6\times10^{-19}}{(2.0\times10^{-6})^2}$$

$E_1$ = **360 N/C.**
(Directed towards Electron, as shown)

Electric field due to proton:

$$E_2 = k\frac{e}{L^2} = \frac{9\times10^9\times1.6\times10^{-19}}{(2.0\times10^{-6})^2} = \mathbf{360\ N/C.}$$

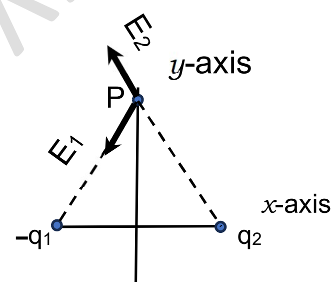
(Directed away from proton as shown)




Since the charges of equal magnitude are at the corners of an equilateral tringle (triangle of equal sides), therefore, all electric fields will also be equal in magnitude (as shown in vector diagram).

$\therefore$ Magnitude of net electric field at third side will also be **360 N/C**.

***Q.No.6*:** Figure shows two charged particles on x-axis $-q = -3.20\times10^{-19}$ C at x = – 3.00 m and $q = 3.20\times10^{-19}$ C at x = 3.00 m. What are the (a) magnitude and (b) direction (relative to the +direction of x-axis) of the net electric field produced at point P at y = 4.00 m?




***Data*:**

Charge $q_1 = -3.20\times10^{-19}$ C.
Distance $x_1 = -3.00$ m.(along x-axis)
Charge $q_2 = 3.20\times10^{-19}$ C.
Distance $x_2 = 3.00$ m. (along x-axis)
Net electric field at point P E =?
Distance $y = 4.00$ m. (along y-axis)

***Solution*:**

Distance of $q_1$ from P (Pythagoras theorem)

$$=\sqrt{(x_1)^2 + (y)^2} = \sqrt{(-3)^2 + (4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = \mathbf{5\ m}.$$

Similarly distance of $q_2$ from P = **5 m.**

(a) Electric field due to $q_1$:

$$E_1 = k\frac{q_1}{5^2} = \frac{9\times10^9\times3.20\times10^{-19}}{25} = \mathbf{1.152\times10^{-10}\ N/C}$$

($E_1$ is directed towards $q_1$ as shown)

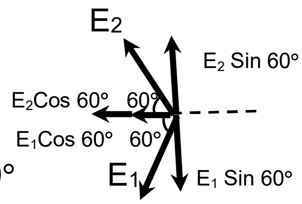
Electric field due to $q_2$:

$$E_2 = k\frac{q_2}{5^2} = \frac{9\times10^9\times3.20\times10^{-19}}{25} = \mathbf{1.152\times10^{-10}\ N/C}$$

($E_2$ is directed **away** from $q_2$ as shown)

Net electric field (Electric intensity) at point P will be equal to the resultant of $\mathbf{E_1}$ and $\mathbf{E_2}$.

Resolving $\mathbf{E_1}$ and $\mathbf{E_2}$ into components we see that $E_1$ Sin 60° is cancelled by $E_2$ Sin 60°. ($E_1$ Sin 60° and $E_2$ Sin 60° are equal and opposite)




Hence the net electric field at point P will be:

$$E = E_1 \text{ Cos } 60° + E_2 \text{ Cos } 60°$$

( $E_1$ Cos 60° and $E_2$ Cos 60° are in the same direction)

$$E = 1.152\times10^{-10}\times0.5+1.152\times10^{-10}\times0.5$$

$$\mathbf{E = \underline{1.152\times10^{-10}\ N/C}.}$$

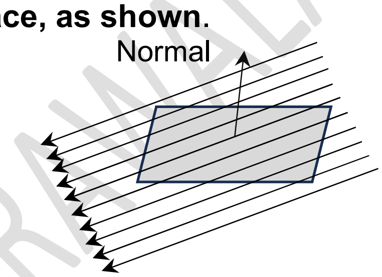
Net electric field **E** is directed along **– x-axis**.

***Q.No.7*: A proton and an electron form two corners of an equilateral triangle of side length 5 μm. What is the magnitude of net electric field these two particles produce at the third corner?**

***Solution*:**

See solution of Q.No.5.

***Q.No.8*: The square surface shown in fig, measures 3.2 mm on each side. It is immersed in a uniform electric field with magnitude E = 1800 N/C and with field lines at an angle of θ = 35° with a normal to the surface, as shown.**




Take that normal to be directed "outward" as though the surface were one face of a box. Calculate the electric flux through the surface.

***Data*:**

Length of each side L = 3.2 mm
$= 3.2\times10^{-3}$ m

Area of square A $= 3.2\times10^{-3}\times3.2\times10^{-3}$ m$^2$
$= 1.024\times10^{-5}$ m$^2$

(Vector area **A** is taken along the outward drawn normal to the surface)

Magnitude of electric field E = 1800 N/C

Angle with the normal = 35°

Angle between **E** and the normal (or **A**)

$\theta = 180–35 = 145°$

Electric flux $\phi$ =?

***Solution*:**

Electric flux through the surface is given by:

$$\phi = E A \text{ Cos } \theta$$

$$\phi = 1800 \times 1.024\times10^{-5} \text{ Cos } 145°$$

$$\phi = -\ \mathbf{\underline{1.51\times10^{-2}\ N\,m^2/C}.}$$

***Q.No.9*: An electron is liberated from the lower of two large parallel metal plates separated by a distance h = 2 cm. The upper plate has a potential of 2400 V relative to the lower. How long does the electron take to reach it?**

***Data*:**

Charge on electron e $= 1.6\times10^{-19}$ C

Mass of electron m $= 9.1\times10^{-31}$ kg.

Separation of plates S = 2 cm.= 0.02 m

Potential difference V = 2400 volt.

Time taken t =?

Initial velocity of electron $v_i$ =0

***Solution*:**

F = q E and E = V/d

$$F = \frac{qV}{d} = \frac{1.6\times10^{-19}\times2400}{0.02} = 1.92\times10^{-14}\text{ N}$$

But F = ma (second law of motion)

$$1.92\times10^{-14} = 9.1\times10^{-31}\times a$$

$$a = \frac{1.92\times10^{-14}}{9.1\times10^{-31}} = 2.11\times10^{16}\text{ m/s}^2.$$

Therefore, $S = v_i t + \frac{1}{2} a t^2$

$$0.02 = 0 + \tfrac{1}{2}\, 2.11\times10^{16}\, t^2$$

$$t^2 = \frac{0.02}{1.055\times10^{16}} = 1.896\times10^{-18}.$$

$$\mathbf{\underline{t = 1.377\times10^{-9}\ s}}$$

Electron will take **1.377×10⁻⁹ s** to reach the upper plate.

***Q.No.10*: Two large parallel plates are 1.5 cm apart and have charges of equal magnitudes but opposite signs on their facing surfaces. Take the potential of the negative plate to be zero. If the potential half way between the plates is 5.0 V, What is the electric field in the region between the plates?**

***Data*:**

Separation of plates d=1.5 cm =0.015m

Potential of negative plate $V_1$ =0

Potential half way $V_2$ = 5.0 volts.

Electric field E =?

***Solution*:**

Potential difference $\Delta V = V_2 - V_1 = 5 - 0 = 5$ volt

Distance of the point half between the plates where electric field is required

$$d = \frac{\text{Total separation of plates}}{2} = \frac{0.015}{2} = 0.0075\ m$$

Electric field in this region between the plates can be found by:

$\Delta V = E\,d$ OR $5 = E \times 0.0075$

$$E = \frac{5}{0.0075} = \mathbf{666.67\ V/m.}$$

***Q.No.11*: Two unequal point charges repel each other by a force of 10 newtons when they are 10 cm. apart. Find the force which they exert on each other when they are 1 cm apart. If the magnitude of one point charge is $-4.25\times10^{-6}$ C, find the magnitude of the other. (*2018 Karachi Board*)**

***Data*:**

$F_{10}$ = 10 N, d = 10 cm = 0.1 m

$F_1$= ? when d = 1 cm = 0.01 m

$q_1 = -4.25\times10^{-6}$ C $q_2$ = ?

***Solution*:**

$$F_{10} = 9\times10^9 \frac{q_1 q_2}{(0.1)^2}$$

$$10 = 9\times10^9 \frac{q_1 q_2}{(0.1)^2}$$

$$q_1 q_2 = \frac{10 \times (0.1)^2}{9\times10^9} = 1.1111111\times10^{-11}\ C^2$$

$$F_1 = 9\times10^9 \frac{q_1 q_2}{(0.01)^2} \quad (q_1q_2 = 1.111111\times10^{-11}\ C^2)$$

$$F_1 = 9\times10^9 \frac{1.11111111\times10^{-11}}{(0.01)^2} = \mathbf{1000\ N.}$$

Force between charges when they are 1 cm apart will be ***1000 N***.

(Distance between charges decreases by 10 times, therefore, force increases by $10^2$ times Coulomb's law)

$q_1 q_2 = 1.1111111\times10^{-11}\ C^2$

But $q_1 = -4.25\times10^{-6}$ C

$\therefore\ -4.25\times10^{-6}\times q_2 = 1.1111111\times10^{-11}$

$$q_2 = \frac{1.1111111\times10^{-11}}{-4.25\times10^{-6}} = \mathbf{-2.614\times10^{-6}C}$$

Magnitude of the other charge is ***$-2.614\times10^{-6}$C***.

***Q.No.12*: A charged particle of charge $2\times10^{-9}$ C, in an electric field between two parallel metal plates 4 cm. apart, is acted upon by a force of $10^{-4}$N.**

**(a) What is the intensity of electric field?**

**(b) What is potential difference between the plates? (*2022 Karachi Board*)**

***Data*:**

$q = 2\times10^{-9}$C, d = 4 cm.= 0.04 m $F = 10^{-4}$ N.

E =? V =?

***Solution*:**

(a) $E = \frac{F}{q} = \frac{10^{-4}}{2\times10^{-9}}$ $\mathbf{E = 5\times10^4\ N/C}$

(b) $V = E\,d = 5\times10^4\times0.04$ $\mathbf{V = 2000\ volt.}$

***Q.No.13*: A particle of charge $3.2\times10^{-19}$ C and mass $2.48\times10^{-27}$ kg is held motionless between two horizontal parallel plates separated by 5 cm. Find the potential difference between the plates. (*2023, 16, 09 Karachi Board*)**

***Data*:**

Mass of particle $m = 2.48\times10^{-27}$kg

Charge of α-particle $q = 3.2\times10^{-19}$ C

Separation of plates d = 5 cm = 0.05 m

Potential difference between plates V = ?

***Solution*:**

Since particle is in equilibrium, therefore, upward force F = down ward force mg

But $F = qE$ and $E = \frac{V}{d}$

$$\therefore\ q\frac{V}{d} = m g$$

$$3.2\times10^{-19}\frac{V}{0.05} = 2.48\times10^{-27}\times 9.8$$

$$V = \frac{2.48\times10^{-27}\times 9.8\times 0.05}{3.2\times10^{-19}}$$

$$V = \frac{1.2152\times10^{-27}}{3.2\times10^{-19}}$$

$$\mathbf{V = 3.8\times10^{-7}\ volt}$$

Potential difference between the plates to make the particle motionless is ***$3.8\times10^{-7}$ volt.***

***Q.No. 14***: **How many electrons should be removed from each of the two similar spheres each of 10 gm. so that the e.s. repulsion be balanced by gravitational force. (*2015,2010, 2006 Karachi Board*)**

***Data***:

Mass of each sphere m = 10 gm = 0.010 kg.
Charge on each electron $e = 1.6 \times 10^{-19}$ C.
No. of electrons to be removed from each sphere n =?

***Solution***:

On removing electrons from spheres each of them becomes positively charged and so they repel each other. In this case,

e.s. repulsion = Gravitational force

$$\frac{1}{4\pi\varepsilon_o}\frac{q\,q}{r^2} = G\frac{m\,m}{r^2}$$

$$9\times10^{9} \times q^2 = 6.67\times10^{-11}\times0.010 \times 0.010$$

$$q^2 = 7.41\times10^{-25}$$

$$q = 8.609 \times10^{-13} \text{ Coulomb}$$

'q' is the charge on each sphere for which e.s. repulsion is balanced by gravitational attraction between the two spheres.

But $q = n\,e$

Where 'n' is the number of electrons and 'e' is the charge on each electron.

$$\therefore \quad n = \frac{q}{e} = \frac{8.609 \times10^{-13}}{1.6 \times10^{-19}}$$

$$n = 5.38 \times 10^{6} \text{ electrons}$$

***5.38 × 10⁶ electrons*** must be removed from each sphere so that e.s. repulsion between the spheres is balanced by gravitational force.

## *Microwave oven ...............*

When charged particles are subjected to electric field they experience a force, due to which positive charges move in the direction of force and negative charges move opposite to it, if it is free to do so.

If the electric field is **alternating** then the particles will ***vibrate***, moving in one direction during a half cycle and in opposite direction during the next half cycle. It's frequency of vibration is same as the frequency of the alternating field.

- This is the **basic idea** on which working of a microwave oven is based.
- Because of higher **electronegativity**, shared pair of electrons slightly shifts towards oxygen end. Thus a partial negative charge "$-2\delta$" appears on oxygen and a partial positive charge "$+\delta$" on each hydrogen atom in $H_2O$ molecule. Like charges on hydrogen atoms repel each other, increasing the bond angle to about 104˚.

Water molecule is said to be highly ***polarized*.**

When a water molecule is subjected to an alternating electric field to align itself with the field, it moves in one direction during half cycle then in opposite direction in the remaining half cycle.

Net result is that it starts rotating, While rotating it rubs with its neighboring molecule. Due to friction the temperature rises.

By this mechanism, water contained food can be heated/cooked in a **microwave oven**……..

***Unit. 9*****…..** ✍

# Capacitors

## Capacitors:

***Q.No.1*****: What is a capacitor?**

***Ans*****:** Capacitor is an electrical device used to store ***electric charge*** and ***electrical energy***.

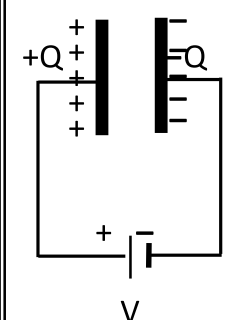
- A simplest capacitor consists of two conducting plates held parallel to each other and separated by small distance. Space between the plates is usually filled with a suitable insulator, called a ***dielectric medium*****.**

The amount of charge stored "Q" on any one plate of a capacitor is directly proportional to the potential difference "V" across it.

$$Q \propto V$$

$$\boxed{Q = C\,V}$$


***Circuit notation***




***Capacitor***

Where "C" is the constant of proportionality, known as ***capacitance of the capacitor***.

## Definition of capacitance C:

***Q.No.2*****: Define capacitance of a capacitor and define its S.I unit.**

***Ans*****:** Capacitance of a capacitor "C" may be defined as:

***"The amount of charge required to produce unit potential difference across its plates"***

- Capacitance of capacitor actually represents its ***charge storing ability*** i.e. it tells us that how much charge a capacitor can store per unit potential difference across its plates.
- Capacitance of capacitor depends upon area "A" of its plates, their separation "d" and properties of the dielectric medium (Which is a suitable insulator) present between its plates **(In other words** capacitance of a capacitor depends upon its ***geometry*** and ***nature of material*** between its plates).

## Unit of capacitance C:

Unit of capacitance is "***farad***".

Capacitance of a capacitor is said to be one farad (1 F) if on giving one coulomb (1 C ) charge to one of its plates the potential difference across it rises by one volt (1 V).

$$1\text{ farad} = \frac{1\text{ coulomb}}{1\text{ volt}}$$

Submultiples of farad are: $1\ \mu F = 10^{-6}\ F$

$1\mu\mu\ F = 1\text{pico farad } (1\ p\ F) = 10^{-12}\ F$

## ***Types of capacitors*: Variable capacitors:**

Capacitance of these capacitors can be changed to any desired value within a suitable range. Usually a variable capacitor has two parallel sets of metallic plates. One set of plates is fixed while the other set can be rotated with its plates rotating between the plates of first without touching it. In this manner distance between plates and the area of a plate facing its nearby plates also changes which results in a change in the overall capacitance of the capacitor.

Variable capacitors are used in tuning circuits of transmitters and receivers of different applications such as radios, televisions etc.

**Ceramic capacitors:**

These capacitors use ceramic as a dielectric medium and are not polarized.
Ceramic capacitors are used in high voltage laser power supplies, power circuit breakers etc.

**Mica capacitors:**

These capacitors use mica as a dielectric medium and are not polarized.
These capacitors can be operated at high voltages and high power but are of low capacitance.

**Film capacitors:**

Usually paper or polyester is used in these capacitors as dielectric medium
Film capacitors are used in high power electronic applications such as X-ray flashes, phase shifters, pulse lasers etc.

**Electrolytic capacitors:**

These capacitors use some electrolyte between their plates as a dielectric medium.
Electrolytic capacitors are polarized hence their terminals are marked with + and - signs.
These are used to reduce the unwanted ripple voltage.
Electrolytic capacitors have high capacitances and are used in almost all power supplies.
These capacitors stabilize output voltage and supply current at a uniform rate.

## ***Some uses of capacitors from daily life electrical appliances*:**

Capacitors are widely used in electrical appliances.

***For example***, capacitors are used in all tuning circuits, such as in channel selector of a TV-set, tuning circuits in radio transmitters and receivers, neon signs, road signals, wireless sets, washing machine, water pump, electric fan, mobile phones, automobile indicators, computers, electronic watches, power supplies used to smooth and regulate current, ………….. etc.

- During charging a capacitor, energy is stored in it in its electric field in the form of electrical potential energy, hence when a camera flash light is switched on, capacitor connected with the flash light discharges quickly and this stored energy produces a strong light flash, some heat is also produced.

| Material | Dielectric constant $\varepsilon_r$ |
|---|---|
| Vacuum | 1 |
| Air | 1.0006 |
| Polyethylene | 2.3 |
| Waxed paper | 2.5 |
| Nylon | 4.1 |
| Mica | 7 |
| Water | 81 |

Mica whose $\varepsilon_r$ is 7 (according to the table given in your text book), is the ***most suitable dielectric medium*** for capacitors, because it can withstand **rapid changes in potential difference** and the resulting **rapid changes in electric field** between plates of a capacitor. Chemically it **does not** easily react.

Although water has the highest value of '$\varepsilon_r$' (81) but it ***cannot be*** used as a dielectric medium in a capacitor. Water **conducts** with a slightest acidic impurity in it, so charge cannot stay on any plate. More over chemically water **reacts** with oxygen of the air and/or plate material of the capacitor forming undesirable compounds (especially ionic compounds which conduct away charges).

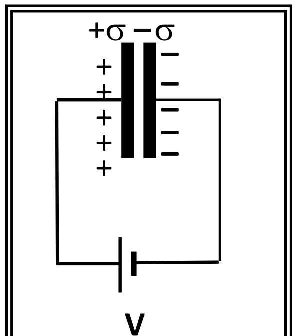
# Capacitance of a parallel plate capacitor

***Q.No.3*: Derive an expression for capacitance of a parallel plate capacitor with air or vacuum between its plates.**

***Ans*:** Consider a parallel plate capacitor whose each plate is of area "A" and are separated by a small distance "d". Let "+ σ" and "– σ" be the ***surface charge density*** (charge per unit area is called surface charge density) on the two plates respectively. Distance between the two plates must be very ***small*** so that the electric field between them is ***uniform***.

The magnitude of electric intensity "E" or strength of the electric field at any point between the two oppositely charged plates is given by:

$$E = \frac{\sigma}{\varepsilon_o}$$



***Parallel plate capacitor with air***

Hence potential difference "V" between the plates will be:

$$V = E\,d$$

$$V = \frac{\sigma}{\varepsilon_o} \times d$$

But $\sigma = \frac{Q}{A}$ $\qquad \therefore \qquad V = \frac{Q\,d}{A\,\varepsilon_o}$

OR $\qquad Q = \frac{A\,\varepsilon_o\,V}{d}$ ………………… ($i$)

Charge "Q" on a plate of any capacitor is given by:

$$Q = C\,V \quad \text{………………………} \quad (ii)$$

Comparing equations ($i$) and ($ii$) we get:

$$C\,V = \frac{A\,\varepsilon_o\,V}{d}$$

$$\therefore \qquad \boxed{C = \frac{A\,\varepsilon_o}{d}}$$

Hence capacitance of a parallel plate capacitor is ***directly proportional to area "A" of the plates*** and ***inversely proportional to separation "d" of plates***.

For high capacitance, plates must be of ***large*** area "A" and their separation "d" must be ***small***.

> ***Note that*:** **Positively charged body is said to be at higher and negatively charged body at a lower potential. Potential of a charged body depends upon the amount of charge on it. If a negatively charged body is brought close to a positively charged body the potential of positive body *lowers* and that of negatively charged body *rises*. *In other words*, when oppositely charged bodies are brought close to each other potential difference between them *decreases* without affecting their charge. This is the idea on which working of a capacitor is based.**

Since plates of a capacitor are oppositely charged, and opposite charges attract each other with a force inversely proportional to squire of distance between them. Therefore, oppositely charged plates must be separated by a small distance, so that plates can hold large quantity of charge. Opposite charges will stay on the plates because of strong attraction between them. Hence for high capacitance (large charge per unit potential difference) distance "d" between the plates must be ***small***.

***Q.No.4*: Derive an expression for capacitance of a parallel plate capacitor with a dielectric medium between its plates.**

***Ans*:** If the space between plates of a capacitor is completely occupied by a dielectric medium of relative permittivity "$\varepsilon_r$", then the strength of electric field (or electric intensity) between the two plates decreases by "$\varepsilon_r$" times and is given by:

$$E = \frac{\sigma}{\varepsilon_o \varepsilon_r}$$

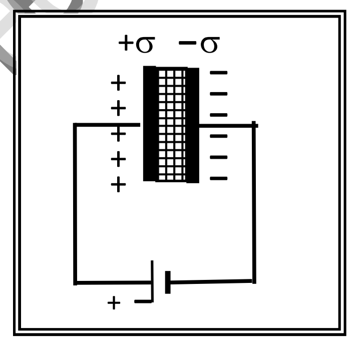
But potential difference between the plates is given by:

$V = E\,d$

$$\therefore \quad V = \frac{\sigma d}{\varepsilon_o \varepsilon_r}$$

$$\therefore \quad V = \frac{Q\,d}{A\,\varepsilon_o \varepsilon_r}$$

$$\therefore \quad Q = \frac{A\,V\,\varepsilon_o \varepsilon_r}{d} \;\ldots\ldots\ldots\ldots (i)$$




But the charge stored by any capacitor of capacitance "C′ " is given by:

$$Q = C'\,V \;\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Comparing equation (*i*) and (*ii*) we get:

$$C'\,V = \frac{A\,V\,\varepsilon_o \varepsilon_r}{d}$$

$$\therefore \quad \boxed{C' = \frac{A\,\varepsilon_o \varepsilon_r}{d}}$$

***Q.No.5*: Show that the capacitance of a parallel plate capacitor (or in general capacitance of any capacitor) with a dielectric medium is <u>more</u> than it's capacitance without a dielectric medium between its plates.**

***Ans*:** Capacitance "C" of a parallel plate capacitor <u>without</u> a dielectric between its plates is given by:

$$C = \frac{A\,\varepsilon_o}{d}$$

But capacitance "C′ " of the same parallel plate capacitor <u>with</u> a dielectric between its plates is given by:

$$C' = \frac{A\,\varepsilon_o \varepsilon_r}{d}$$

Hence,

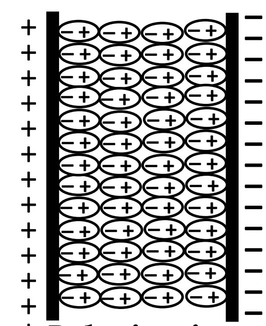
$$\boxed{C' = \varepsilon_r C}$$

It shows that capacitance "C′ " of a capacitor (Or in other words charge storing capacity) with a dielectric medium is $\varepsilon_r$ ***times greater*** than capacitance "C" of the same capacitor without a dielectric medium between its plates (OR we can say that a capacitor will store more charge with a dielectric medium than without any medium). The value of "$\varepsilon_r$" is more than 1. Where "$\varepsilon_r$" is known as ***relative permittivity*** of the medium, it is also known as ***dielectric constant*** of the medium.

# ***Explanation of polarization:***

***Q.No.6: Why does capacitance of a capacitor increase when a dielectric medium is inserted between its plates?***

***Ans:*** When a dielectric medium, which is a suitable ***insulator***, (such as mica, waxed paper etc.) is inserted between plates of a capacitor molecules of the insulator are ***polarized***. Due to the attraction between positively charged plate and the negatively charged electrons of atoms of the medium. The electrons in each atom are shifted slightly towards positively charged plate. This side of the dielectric, therefore, becomes slightly negative whereas the opposite end of each molecule, due this shift, becomes slightly positive. Hence oppositely charged poles appear on ends of each molecule. This phenomenon is called ***polarization***. "$\varepsilon_r$" measures the ability of polarization. Different media have different value of $\varepsilon_r$. Positively charged end of all molecules will be towards the negative and negatively charged end will be towards positive plate of the capacitor. Due to the presence of oppositely charged molecules in the vicinity of charged plates of the capacitor, potential of plates will ***decrease*** without affecting their charge, as a result of which they can hold more charge. ***In other words***, their capacitance ***increases***.



***Polarization***

***Q.No.7: What is relative permittivity or dielectric constant $\varepsilon_r$?***

***Ans:*** Since: $C' = \varepsilon_r C$

$\therefore$ $\varepsilon_r = \dfrac{C'}{C}$

$$\varepsilon_r = \frac{\text{Capacitance of a capacitor with a dielectric medium between its plates}}{\text{Capacitance of the same capacitor without a dielectric medium}}$$

Hence dielectric constant "$\varepsilon_r$" of a medium may be defined as:

***"The ratio of capacitance of a capacitor with the medium to the capacitance of the same capacitor without the medium between its plates".***

Since "$\varepsilon_r$" is a ratio of two similar quantities, therefore, it **does not** have a unit.

***Q.No.8: How can capacitance of a capacitor be increased?***

***Ans:*** Capacitance of a capacitor can be ***increased*** by:

(*i*) ***Increasing*** the area of its plates, because $C \propto A$.

(*ii*) ***Decreasing*** the separation of plates, because $C \propto 1/d$.

(*iii*) Using a suitable ***dielectric medium***, because C increases with $\varepsilon_r$.

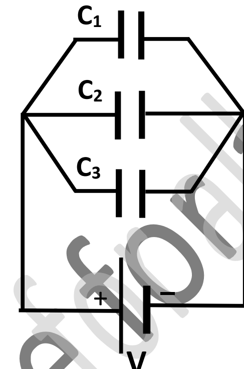
# ***Combinations of capacitors:***

## ***Parallel combination of capacitors:***

***Q.No.9: When capacitors are connected in parallel, derive an expression for net capacitance of the combination.***

***Ans:*** Two or more capacitors are said to be connected ***in parallel*** if each one of them is connected across the ***same two points*** or ***same two wires***. A suitable potential difference "$\Delta V$" is applied across the combination. In a parallel combination of capacitors the potential difference across each capacitor is ***same***, but each capacitor will store ***different charge***, according to its capacitance.

Consider a parallel combination of three capacitors of capacitances "$C_1$", "$C_2$" and "$C_3$", the potential difference across which is "V" Let the charge stored by them be "$Q_1$", "$Q_2$" and "$Q_3$" respectively. Then:



Parallel combination of capacitors

$Q_1 = C_1 V$, $Q_2 = C_2 V$, & $Q_3 = C_3 V$.

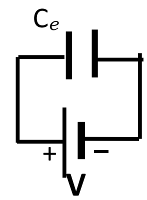
Total charge stored by the combination is given by:

$Q = Q_1 + Q_2 + Q_3$

∴ $Q = C_1 V + C_2 V + C_3 V$

∴ $Q = V (C_1 + C_2 + C_3)$ ……………… (*i*)

This combination of capacitors can be replaced by a single capacitor whose capacitance "$C_e$", called equivalent capacitance, is of such a value that it stores the same charge "Q" stored by the combination when the same potential difference "V" is applied across it. This is the equivalent capacitor for the combination. Hence,



Equivalent capacitor

$Q = C_e V$ ………………………….. (*ii*)

Comparing equations (*i*) and (*ii*) we get:

$C_e V = V (C_1 + C_2 + C_3)$

∴ $$\boxed{C_e = C_1 + C_2 + C_3}$$

***"When two or more capacitors are connected in parallel their equivalent capacitance is equal to the sum of capacitance of individual capacitors"***

***Note that*** in a ***parallel combination*** of capacitors,

- Potential difference across each capacitor is ***same***.
- Amount of charge stored is ***different***, according to their individual capacitance (Large capacitance more charge).
- Net (or equivalent) capacitance is always ***higher than the highest*** individual capacitance.

## ***Series combination of capacitors:***

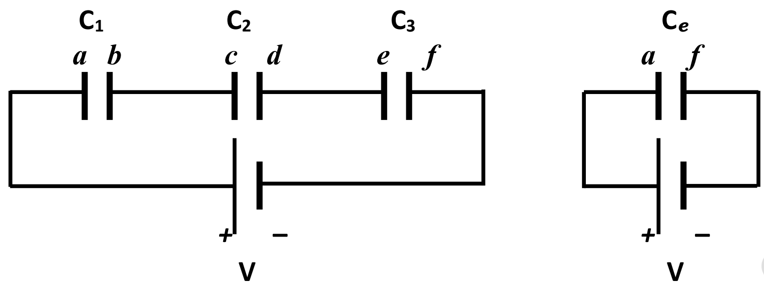
***Q.No.10: When two or more capacitors are connected in series, derive an expression for net or equivalent capacitance of the combination.***

***Ans:*** Two or more capacitors are said to be connected ***in series*** if the second plate of first capacitor is connected with the first plate of second capacitor, second plate of second capacitor is connected with the first plate of third capacitor and so on **……….** A suitable potential difference "ΔV" is applied across the combination.

In a series combination of capacitors, the potential difference across different capacitors is ***different***, but each capacitor will store the ***same amount of charge*** whatever their capacitance.

Let "$V_1$", "$V_2$" and "$V_3$" be the potential difference across capacitors of capacitances "$C_1$", "$C_2$" and "$C_3$" respectively and "Q" be the charge stored on each capacitor. Then:

$Q = C_1 V_1$ ∴ $V_1 = \frac{Q}{C_1}$ Similarly: $V_2 = \frac{Q}{C_2}$ & $V_3 = \frac{Q}{C_3}$




***Series combination of capacitors*** ***Equivalent Capacitor.***

Total potential difference "V" acting across this combination will be equal to the sum of "$V_1$", "$V_2$" and "$V_3$". Hence:

∴ $V = V_1 + V_2 + V_3$

∴ $V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

∴ $V = Q\left\{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right\}$ ………………. ($i$)

This series combination can be replaced by a single capacitor whose capacitance "$C_e$" is of such a value that it stores the same amount of charge "Q" when the same potential difference "V" is applied across it. This single capacitor is called equivalent capacitor, it's capacitance "$C_e$" is called equivalent or net capacitance. Hence,

$V = \frac{Q}{C_e}$ …………….. ($ii$)

Comparing equations ($i$) and ($ii$) we get:

$$\frac{Q}{C_e} = Q\left\{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right\}$$

OR

$$\boxed{\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

***"When two or more capacitors are connected in series reciprocal of their equivalent capacitance is equal to the sum of reciprocals of capacitance of individual capacitors"***

***Note that*** in a ***series combination*** of capacitors,

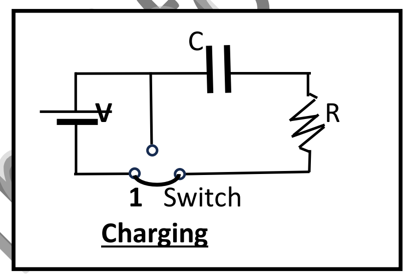
- Charge stored on different capacitors is ***same***.
- Potential difference across different capacitors is ***different***.
- Net (or equivalent) capacitance is always ***less than the least*** individual capacitance.

## ***Charging and discharging of a capacitor through a resistor (RC circuit):***

When a certain potential difference V is applied across a capacitor it charges **exponentially**, similarly a charged capacitor also discharges **exponentially**.***it means that*** the capacitor ***does not*** charge or discharge ***immediately***, but depending upon it's own value and the value of load (resistance) connected with it, it takes ***some time*** to fully charge or fully discharge.

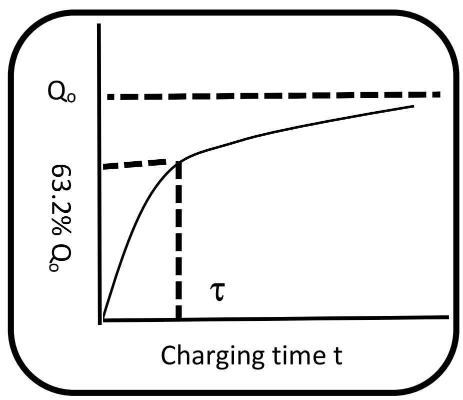
### ***Explanation***: **Charging**:

Consider a capacitor of capacitance "C" connected in series to a resistance "R" and a battery of voltage "V", through a switch. This series combination is called RC- circuit. When the switch is closed in position "1" at that instant since the capacitor initially has no charge, potential difference across it will be zero and the potential difference across the resistor will be "V" (voltage of the battery), as a result of which some current starts flowing through resistor R, this current starts charging the capacitor. As the capacitor charges potential difference across it starts increasing (because Q = CV) whereas potential difference across "R" decreases, so that at any latter instant potential difference of the battery will be equal to the sum of potential difference across "R" and "C" (Series combination). When the capacitor is fully charged the current through "R" stops flowing at that instant potential difference across "R" becomes zero where as potential difference across "C" will be equal to voltage "V" of the battery.




Capacitor ***charges exponentially***, ***it means that***, at first it charges **quickly** and then with the passage of time charging process **slows down**. Hence at any instant if "Q" is the charge stored and "$Q_o$" be the maximum charge that can be stored, making the capacitor fully charged, then:

$$Q = Q_o\,(1 - e^{-t/\tau})$$




Where "e" is the base of natural logarithm, its value is 2.718, " t " is the charging time and "$\tau$ = RC" is known as "**TIME CONSTANT**" for a given RC- circuit. Value of time constant depends upon the value of product "RC".

**Smaller** the value of time constant (or of RC) **shorter** is the time of charging, ***in other words*** capacitor charges ***quickly***.

**Higher** the value of time constant **longer** is the time of charging, ***in other words*** capacitor charges ***slowly***.

When a capacitor is charged for one time constant, ***in other words,*** $t = 1\,\tau = 1$ RC

Then $\quad Q = Q_o\,(1 - e^{-\tau/\tau}) \quad$ OR $\quad Q = Q_o\,(1 - e^{-1}) \quad$ OR $\quad Q = Q_o\,(1 - \frac{1}{e})$

Hence, $\quad Q = Q_o\,(1 - \frac{1}{2.718}) \quad$ OR $\quad Q = Q_o\,(1 - 0.368)$

$\therefore \quad Q = 0.632\,Q_o \quad$ OR $\quad$ **$Q$ = 63.2 % of $Q_o$**

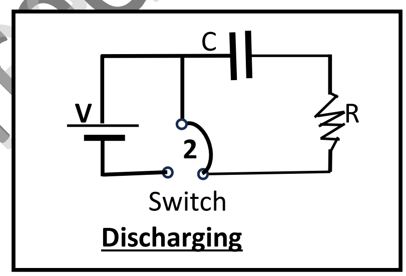
***It means that*** after one time constant ($1\,\tau = 1$ RC) the amount of charge stored on a capacitor will be 63.2% of total amount of charge that a given capacitor can store.
Similarly after 2 time constants the amount of charge stored will be 86.5 % of total charge.

After 3 time constants the amount of charge stored will be 95 % of total charge.
After 4 time constants the amount of charge stored will be 98.2 % of total charge.
After 5 time constants the amount of charge stored will be 99.3 % of total charge.
Hence after five-time constants a capacitor of given value with some load (resistance) connected with it in series may be considered fully charged.

***Note that***, during the charging process charge on the capacitor ***increases***, Current in the circuit ***decreases*** and potential difference (voltage) across the capacitor ***increases***.

## Discharging:

Now if the switch in the circuit is moved to position **2** then the battery will be disconnected ***In other words*** now there is no external source of emf that can supply charge to the capacitor. Hence the capacitor will start **discharging** through the resistor "R" and current will again flow in the circuit through the resistor "R" but this time in opposite direction. Capacitor ***discharges exponentially***, ***it means that***, at first it discharges **quickly** and then with the passage of time discharging process **slows down**. Hence if at the beginning of discharge "$Q_o$" is the charge on capacitor then at any later instant charge "Q" remaining on it is given by:




$$Q = Q_o\, e^{-t/\tau}$$

After one time constant

i.e. $t = 1\,\tau = 1RC$ $\quad Q = Q_o\, e^{-RC/RC}$ OR $Q = Q_o\, e^{-1}$

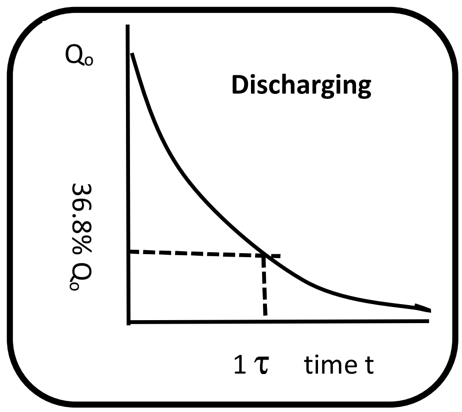
$Q = Q_o \dfrac{1}{2.718}$ OR $Q = 0.368\, Q_o$ OR **Q = 36.8% of $Q_o$**

***It means that*** after 1 time constant **36.8 % of "$Q_o$"** charge will remain on the capacitor whereas 63.2 % of $Q_o$ will discharge through the load (resistance).

After two-time constants $t = 2\,\tau = 2\,RC$

$Q = Q_o\, e^{-2RC/RC}$ OR $Q = Q_o\, e^{-2}$ OR $Q = Q_o \dfrac{1}{(2.718)^2}$

$Q = 0.135\, Q_o$ OR **Q = 13.5% of $Q_o$**




***It means that*** after 2 time constants 13.5 % of "$Q_o$" charge will remain on the capacitor whereas 86.5 % of $Q_o$ will discharge through the load (resistance).

Similarly, after 3 time constants 5% of "$Q_o$" will remain on the capacitor whereas 95% of $Q_o$ will discharge through the load (resistance).

After 4 time constants 1.8 % of "$Q_o$" will remain on the capacitor whereas 98.2 % of $Q_o$ will discharge through the load (resistance).

After 5 time constants 0.7 % of "$Q_o$" will remain on the capacitor whereas 99.3 % of $Q_o$ will discharge through the load (resistance).

***Note that***, during the discharging process charge on the capacitor ***decreases***, Current in the circuit ***increases*** and potential difference (voltage) across the capacitor ***decreases***.

***Time constant***. Time constant may, therefore, be defined as the time during which a fully neutral capacitor stores **63.2 % of $Q_o$** charge during charging process or during discharging process **36.8 % of $Q_o$** charge remains and **63.2 % of $Q_o$** charge flows away from it.

## ***Energy stored in a charged capacitor*:**

- During charging a capacitor, **energy** along with **charge** is also stored in it, in its electric field, in the form of electrical potential energy. Hence when a camera flash light is switched on, capacitor connected with the flash light discharges quickly and this stored energy produces a strong **light** flash, some **heat** is also produced.

Charging of a capacitor takes some time, during this time each plate stores equal and opposite charge. At the beginning of charging process both plates are neutral so they are at ***zero potential***. But as the charging process continues positive plate receives more and more positive and the negative plate receives more and more negative charge. Electrical potential of positively charged plate ***increases*** and that of negative plate ***decreases***, ***in other words***, the potential difference between the plates ***increases***. This happens until the two plates are fully charged. Let the total amount of charge stored at the end of charging process on any one plate is "Q" (+Q on positive, **–** Q on negative plate) and the potential difference across the plates is "V". At the beginning of charging process potential difference across the plates was "**0**" and at the end is "**V**". Hence the average potential difference is "$\frac{1}{2}$ **V**". Since,

$$\text{Potential difference} = \frac{\text{Work or energy}}{\text{Charge}}$$

Therefore, in this case:

$$\frac{1}{2}V = \frac{\text{Energy } E}{Q}$$

$$\therefore \quad \boxed{E = \frac{1}{2}QV} \qquad \text{But } Q = CV$$

$$\therefore \quad \boxed{E = \frac{1}{2}CV^2}$$

Energy stored in a charged capacitor, therefore, depends upon its capacitance "C", charge "Q" stored and the potential difference "V" across its plates.

For a given capacitor (for a given value of "C") energy stored depends upon voltage "V" and charge "Q". Higher the voltage "V" more is the charge "Q" stored and therefore, more is the energy "E". But if a certain limit is crossed charge stored starts leaking. Hence there is a limit to the voltage that can be applied safely. Value of safe maximum operating voltage is written on each capacitor.

## Section–A Multiple-Choice Questions MCQs:

**Q.No.1**: The capacitance of a capacitor is NOT influenced by:
(a) Plate thickness. (b) Plate area.
(c) Plate separation (d) Nature of the dielectric

**Q.No.2**: What is the value of capacitance of a capacitor which has a voltage of 4 V and has 16 C of charge?
(a) 2 F (b) 4F (c) 6 F (d) 8 F

**Q.No.3**: Capacitors are used in electric power supply system to:
(a) Improve power factor. (b) Reduce line current.
(c) Provide voltage stability. (d) Switching.

**Q.No.4**: In a variable capacitor, capacitance can be varied by:
(a) Turning the rotatable plates in or out
(b) Changing the plates.
(c) Colliding rotatable plates.
(d) Changing the material of plates.

**Q.No.5**: Energy stored in a capacitor is:
(a) $E = \frac{1}{4} CV$ (b) $E = \frac{1}{2} CV^2$
(c) $E = CV^2$ (d) $E = \frac{1}{2}CV$

**Q.No.6**: The time constant of a series RC circuit consisting of 100 μF capacitor in series with a 100 Ω resistor is:
(a) 0.1 s (b) 0.1 ms (c) 0.01 s (d) 0.01 ms

**Q.No.7**: The charging of a capacitor through a resistance follows:
(a) Linear law. (b) Square law.
(c) Exponential law. (d) None of the above.

**Q.No.8**: When the total charge on a capacitor is doubled, the energy stored:
(a) Remains the same. (b) is doubled.
(c) is halved. (d) is quadrupled.

**Q.No.9**: The capacitor of capacitance C is charged through a resistor R. The time constant of the charging circuit is given by:
(a) C/R (b)1/ RC (c) RC (d) R/C

**Q.No.10**: Capacitor blocks:
(a) alternating current. (b) direct current
(c) both alternating and direct current.
(d) neither alternating nor direct current.

**Q.No.11**: A dielectric k = 2 is inserted between the plates of a 19.8 μF capacitor. Its capacitance will become. (***2005 Karachi Board***)
• 10 μF • 18 μF • 22 μF • 39 μF

**Q.No.12**: Coulomb /volt is the unit of:
(***2007 Karachi Board***)
• Resistance • Resistivity • Current • Capacity

**Q.No.13**: The capacitance of a capacitor does not depend on the: (***2009 Karachi Board***)
• Area of the plates. • Nature of the plates.
• Distance between the plates.
• medium between the plates.

**Q.No.14**: If the area of the plates of a parallel plate capacitor is doubled, the capacitance:
(***2012 Karachi Board***)
• Remains unchanged. • is half.
• is doubled. • is increased four times.

**Q.No.15**: If two capacitors of capacitances 4 μF and 2 μF are connected in series, the equivalent capacitance is:
(***2021,06 Karachi Board***)
• 6 μF • 2 μF • 1.33 μF • 0.76 μF

**Q.No.16**: When a dielectric is placed in an electric field, it becomes:
(***2021, 09 Karachi Board***)
• Negatively charged only
• Positively charged only • Polarized
• Conductive

**Q.No.17**: If two capacitors 5μF and 7μF are connected in parallel, their equivalent capacitance will be: (***2021 Karachi Board***)
• 0.12μF • 12μF • 0.34μF • 2.9μF

**Q.No.18**: If distance between the plates of a capacitor of capacitance C is doubled and area of plates is halved, the capacitance will become: (***2022,17,16 Karachi Board***)
• C/4 • C • 4C • 2 C

**Q.No. 19**: Coulomb/volt is equal to:
(***2023 Karachi Board***)
(A) Newton (B) Joule (C) Farad (D) Ohm

**Q.No.20**: Charging of a capacitor through a resistance follows: (***2025 Karachi Board***)
• Coulomb's law. • Exponential law.
• Power law. • Inverse square law.

## ***Answers***:

(1) Plate thickness.
(2) 4F
(3) Switching.
(4) Turning the rotatable plates in or out
(5) $E = \frac{1}{2} CV^2$
(6) 0.01 s
(7) Exponential law.
(8) is doubled.
(9) RC
(10) direct current.
(11) 39 $\mu$F
(12) Capacity
(13) Nature of the plates.
(14) is doubled.
(15) 1.33 $\mu$F
(16) Polarized
(17) 12 $\mu$F
(18) C
(19) Farad
(20) Exponential law.

# Numericals:

***Q.No.1***: **The capacitance of air-filled parallel plate capacitor is 1.3 pF. If the separation of plates is doubled and wax is inserted between them. The new capacitance is 2.6 pF.**

**Find the dielectric constant of the wax.**

***Data***:

Capacitance without wax $C = 1.3$ pF

Capacitance with wax $C' = 2.6$ pF

Separation of plates with wax = 2 d

Dielectric constant of wax $\varepsilon_r = ?$

***Solution***:

Capacitance without wax is given by $C = A\varepsilon_o/d$

Capacitance with wax is given by $C' = A\varepsilon_o\varepsilon_r/2d$

$\therefore \quad C' = \varepsilon_r C/2$

$2.6 = \varepsilon_r 1.3/2$ OR $\varepsilon_r = \dfrac{2.6 \times 2}{1.3}$

Dielectric constant of wax is **4.**

***Q.No.2***: **Three capacitors have capacitances 10, 50 and 25 μF respectively as shown in fig. Calculate:**

**(*i*) Charge on each when connected in parallel to a 250 V supply.**

**(*ii*) total capacitance.**

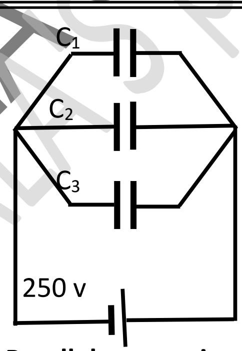
**(*iii*) potential difference across each when connected in series.**

***Data***:

$C_1 = 10\ \mu F$

$C_2 = 50\ \mu F$

$C_3 = 25\ \mu F$




**Parallel connection**

<u>In parallel</u>:

V = 250 v

(*i*) Charge on each.

$Q_1 = ?$, $Q_2 = ?$

$Q_3 = ?$

(*ii*) Total capacitance

C=?

<u>In series</u>:

(*iii*) potential difference across each:

$V_1 = ?$ $V_2 = ?$ $V_3 = ?$

***Solution***:

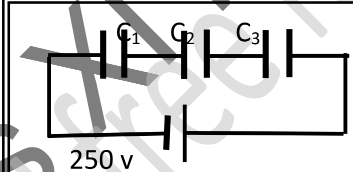
<u>In parallel</u>:

(*i*) In parallel combination potential difference across each capacitor is equal to applied potential difference V, hence

| | | |
|---|---|---|
| $Q_1 = C_1 V$, | $Q_2 = C_2 V$ | $Q_3 = C_3 V$ |
| $Q_1 = 10 \times 250$, | $Q_2 = 50 \times 250$ | $Q_3 = 25 \times 250$ |
| $Q_1 = 2500\ \mu C$ | $Q_2 = 12500\ \mu C$ | $Q_3 = 6250\ \mu C$ |
| $Q_1 = 2.5$ mC | $Q_2 = 12.5$ mC | $Q_3 = 6.25$ mC |

(*ii*) Total capacitance in parallel combination:

$C = C_1 + C_2 + C_3 = 10 + 50 + 25 = \mathbf{85\ \mu F}$




**Series connection**

<u>In series</u>:

In series combination total capacitance is given by:

$\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$ OR $\dfrac{1}{10} + \dfrac{1}{50} + \dfrac{1}{25} = \dfrac{8}{50}$

$C = 50/8 = 6.25\ \mu F$

Charge on each capacitor: $Q = CV = 6.25 \times 250$

$Q = 1562.5\ \mu C$

In series combination of capacitors equal charge stores on each capacitor, hence

$V_1 = \dfrac{Q}{C_1} = \dfrac{1562.5 \times 10^{-6}}{10 \times 10^{-6}}$

$\mathbf{V_1 = 156.25\ v}$

$V_2 = \dfrac{Q}{C_2} = \dfrac{1562.5 \times 10^{-6}}{50 \times 10^{-6}}$

$\mathbf{V_2 = 31.25\ v}$

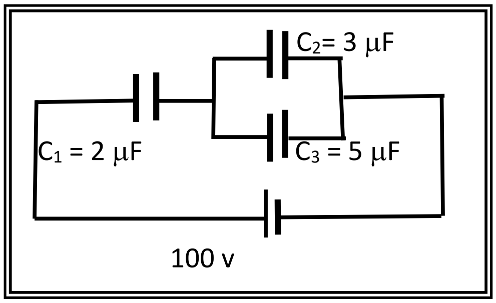
$V_3 = \dfrac{Q}{C_3} = \dfrac{1562.5 \times 10^{-6}}{25 \times 10^{-6}}$

$\mathbf{V_3 = 62.5\ v}$

To check the result: $V = V_1 + V_2 + V_3$

$V = 156.25 + 31.25 + 62.5 =$ **250 v**

***Q.No.3***: **Three capacitors are connected across a potential difference of 100 volts as shown in fig. Find the charge and potential difference across each.**




***Data*:**

$C_1 = 2\ \mu F \quad C_2 = 3\ \mu F \quad C_3 = 5\ \mu F$

$V = 100$ volt

Charge on each $Q_1=?$, $Q_2=?$ and $Q_3=?$

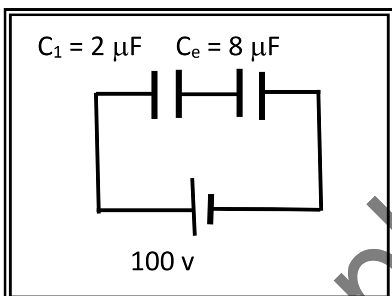
Potential difference across each $V_1=?$ $V_2=?$ $V_3=?$

***Solution*:**

$C_2$ and $C_3$ are connected in parallel,

$\therefore \quad C_e = C_2 + C_3$

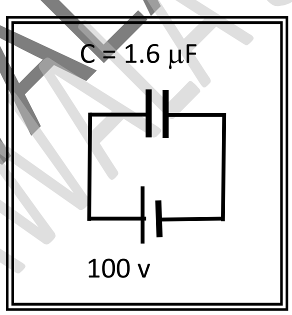
$C_e = 3 + 5 = 8\ \mu F$




$C_e$ and $C_1$ are connected in series.

$$\therefore \quad \frac{1}{C} = \frac{1}{C_e} + \frac{1}{C_1} = \frac{1}{8} + \frac{1}{2} = \frac{5}{8}$$

$$C = \frac{8}{5} = 1.6\ \mu F$$




Charge on C:

$Q = C\,V = 1.6 \times 10^{-6} \times 100 = 1.6 \times 10^{-4}$ C

Since $C_1$ and $C_e$ are in series, therefore charge on each is same ($=1.6 \times 10^{-4}$ C) Hence:

$Q_1 = \mathbf{1.6 \times 10^{-4}\ C = 160\ \mu C}$

Potential difference across $C_1$:

$$V_1 = \frac{Q_1}{C_1} = \frac{1.6 \times 10^{-4}}{2 \times 10^{-6}} = \mathbf{80\ volt}$$

Potential difference across $C_e$:

$$V_e = \frac{Q_e}{C_e} = \frac{1.6 \times 10^{-4}}{8 \times 10^{-6}} = 20 \text{ volt.}$$

Since $C_2$ and $C_3$ are connected in parallel, therefore potential difference across each is 20 V.

$V_2 = V_3 = \mathbf{20\ volts}$

Charge on each:

$Q_2 = C_2 V_2 = 3 \times 10^{-6} \times 20 = \mathbf{60 \times 10^{-6}\ C = 60\ \mu C}$

$Q_3 = C_3 V_3 = 5 \times 10^{-6} \times 20 = \mathbf{100 \times 10^{-6}\ C = 100\ \mu C}$

***Q.No.4*: Capacitor is charged through a large non- reactive resistance by a battery of constant voltage V. For this arrangement, if the capacitor has a capacitance of 10 μF and the resistance is 1 MΩ, calculate the time taken for the capacitor to receive 90% of its final charge.**

***Data*:**

Capacitance $C = 10\ \mu F$

Resistance $R = 1\ M\Omega$

Charge $Q = 90\%$ of $Q_o = 0.9\ Q_o$

Time for 90% total charge $t = ?$

***Solution*:**

Time constant of the circuit:

$\tau = RC = 1\times10^6 \times 10 \times 10^{-6} = 10$ seconds

During charging of a capacitor, at any instant:

$Q = Q_o\,(1 - e^{-t/\tau})$ But $Q = CV$,

$0.90\ V = V\,(1 - e^{-t/\tau})$

$0.90 = (1 - e^{-t/\tau})$ OR $0.9 - 1 = -e^{-t/\tau}$

$0.1 = e^{-t/10}$

Taking natural log (ln) of both sides (use your scientific calculator), we get:

$\ln 0.1 = -t/10$

$t = -10\,(\ln 0.1) = $ **<u>23 sec</u>**

Hence the capacitor will take **<u>23 sec.</u>** to charge up to 90% of total charge.

***Q.No.5*: What capacitance is required to store energy of 10 kWh at a potential difference of 1000 V?**

***Data*:**

$E = 10$ kWh $= 10\times1000\times60\times60$ J $= 36\times10^6$ J

$V = 1000$ volt $\quad C = ?$

***Solution:***

$E = \frac{1}{2} C V^2$

$36\times10^6 = \frac{1}{2} C (1000)^2$ OR $C = \frac{2\times36\times10^6}{(1000)^2}$

**Capacitor must be of C = 72 F**

***Q.No.6*: A 2.0 μF capacitor and a 4 μF capacitor are connected in parallel across a 300 V potential difference. Calculate the total energy in the capacitors.**

***Data:***

$C_1 = 2\ \mu F$ $C_2 = 4\ \mu F$ $V = 300$ volt $E = ?$

***Solution:***

$C_1$ and $C_2$ are in parallel, therefore net capacitance in the circuit:

$C = C_1 + C_2$ $C = 2 + 4 = 6\ \mu F = 6\times10^{-6}$ F

Total energy in the capacitors:

$E = \frac{1}{2} C V^2$

$E = \frac{1}{2}\ 6\times10^{-6} \times (300)^2 = \mathbf{0.27\ J}$

***Q.No.7*: A 12.0 V battery is connected to a capacitor, resulting in 54.0 μC of charge store on the capacitor. How much energy is stored in the capacitor?**

***Data:***

Potential difference V = 12 volt

Charge stored $Q = 54.0\ \mu C = 54\times10^{-6}$ C

Energy stored E = ?

***Solution:***

$E = \frac{1}{2} Q V$

$E = \frac{1}{2}\ 54\times10^{-6} \times 12 = \mathbf{3.24\times10^{-4}\ J.}$

***Q.No.8*: Two capacitors 2 μF and 8 μF are joined in series and a potential difference of 300 volt is applied.**

**Find the charge and potential difference for each capacitor.**

***Data:***

Capacitance $C_1 = 2\ \mu F = 2\times10^{-6}$ F

Capacitance $C_2 = 8\ \mu F = 8\times10^{-6}$ F

Potential difference across the series combination V = 300 volt.

Charge on $C_1$ $Q_1 = ?$

Charge on C $Q_2 = ?$

Potential difference $C_1$, $V_1 = ?$

Potential difference $C_2$, $V_2 = ?$

***Solution:***

$C_1$ and $C_2$ are connected in series, therefore, their net capacitance is given by:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{8} = \frac{4+1}{8} = \frac{5}{8}$$

***C = 1.6 μF***

Total charge stored on the combination = charge stored on C:

$Q = C V = 1.6\times10^{-6} \times 300$

$Q = 4.8\times10^{-4}$ coulomb.

Since $C_1$ and $C_2$ are connected in series, therefore, charge on each capacitor will be same i.e. ***4.8 ×10⁻⁴ coulomb***. Hence,

***$Q_1 = 4.8\times10^{-4}$ coulomb.***

And:

***$Q_2 = 4.8\times10^{-4}$ coulomb.***

Potential difference across $C_1$:

$V_1 = \frac{Q_1}{C_1} = \frac{4.8\times10^{-4}}{2\times10^{-6}}$ ***$V_1$ = 240 volt.***

Potential difference across $C_2$:

$V_2 = \frac{Q_2}{C_2} = \frac{4.8\times10^{-4}}{8\times10^{-6}}$ ***$V_2$ = 60 volt.***

(To check the answer for potential difference, potential difference across the series combination of capacitors, as in this case, must be equal to the total potential difference 'V' across the combination i.e. $V = V_1 + V_2 = 240 + 60 = 300$ volt)

***Q.No.9*: A capacitor of 100 pF is charged to a potential difference of 50 volt. Its plates are then connected in parallel to another capacitor and it is found that the potential difference between its plates falls to 35 volts.**

**What is the capacitance of the second capacitor**? (***1997 Karachi Board***)

***Data:***

Capacitance of first capacitor

$C_1 = 100\ pF = 100\times10^{-12}$ F

Potential difference before connecting $C_1$, $V_1 = 50$ volt

Potential difference before connecting $C_2$, $V_2 = 35$ volt.

Capacitance of second capacitor $C_2$ =?

***Solution:***

***Charge stored on $C_1$:***

Before connecting $C_2$: $Q_1 = C_1 V_1$

$Q_1 = 100 \times 10^{-12} \times 50$

$Q_1 = 5 \times 10^{-9}$ Coulomb.

After connecting $C_2$: $Q_1' = C_1 V_2$

$Q_1' = 100 \times 10^{-12} \times 35$

$Q_1' = 3.5 \times 10^{-9}$ Coulomb.

Charge transferred to $C_2$:

$Q_2 = Q_1 - Q_1'$

$Q_2 = 5 \times 10^{-9} - 3.5 \times 10^{-9}$

$Q_2 = 1.5 \times 10^{-9}$ Coulomb.

Since $C_1$ and $C_2$ are connected in ***parallel***, therefore, potential difference across each will be the same i.e. 35 volts.

∴ $Q_2 = C_2 V_2$

∴ $C_2 = \frac{Q_2}{V_2} = \frac{1.5 \times 10^{-9}}{35}$

$$C_2 = 42.86 \times 10^{-12}\ F = 42.86\ pF$$

Capacitance of second capacitor is ***42.86 × 10⁻¹² F*** or ***42.86 pF***.

***Q.No.10*: A 10 μF capacitor is charged to a potential difference of 220 V. It is then disconnected from the battery. Its plates are then connected in parallel to another capacitor and it is found that the potential difference falls to 100 V.**

**What is the capacitance of the second capacitor?** (***2014,08,1997 Karachi board***)

***Data:***

Capacitance $C_1 = 10\ \mu F = 10 \times 10^{-6} F$

Potential difference $V_1 = 220$ V (Before connecting $C_2$)

Potential difference $V_2 = 100$ V (After connecting $C_2$)

Capacitance of second capacitor $C_2$ = ?

***Solution:***

Charge on $C_1$ before connecting $C_2$:

$Q_1 = C_1 V_1 = 10 \times 10^{-6} \times 220$

$Q_1 = 2.2 \times 10^{-3}$ Coul.

Charge remaining on $C_1$ after connecting $C_2$:

$Q_1' = C_1 V_2 = 10 \times 10^{-6} \times 100$

$Q_1' = 1 \times 10^{-3}$ Coul.

Charge stored by $C_2$ = Charge transferred from $C_1$ to $C_2$, $Q_2 = Q_1 - Q_1'$

$Q_2 = 2.2 \times 10^{-3} - 1 \times 10^{-3}$

$Q_2 = 1.2 \times 10^{-3}$ C

But $Q_2 = C_2 V_2$ Capacitors are connected in parallel. hence the potential difference across them will be same.

∴ $C_2 = \frac{Q_2}{V_2} = \frac{1.2 \times 10^{-3}}{100}$

∴ $C_2 = 1.2 \times 10^{-5}$ **Farad** OR $C_2 = 12\ \mu F$

Capacitance of the second capacitor is ***12 μF***.

***Q.No.11*: Two capacitors of 2 μF and 4 μF are connected in series to a 40 volt battery.**

**Calculate the charge of these capacitors and potential difference across each.**

(***2019 Karachi Board***)

***Data:***

$C_1 = 2\mu F = 2 \times 10^{-6} F$, $C_2 = 4\mu F = 4 \times 10^{-6} F$ C=?

V = 40 volt. $Q_1$ =? $Q_2$ =? $V_1$ =? $V_2$ =?

***Solution:***

$C_1$ and $C_2$ are connected in series

∴ $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ ∴ $\frac{1}{C} = \frac{1}{2} + \frac{1}{4}$

$\frac{1}{C} = \frac{2+1}{4} = \frac{3}{4}$ $C = 1.33\ \mu F$

Total charge stored = charge stored on C:

$Q = C V = 1.33 \times 10^{-6} \times 40 = 5.33 \times 10^{-5}$ coulomb

$C_1$ and $C_2$ are connected in series, therefore charge on each capacitor will be equal

Hence, $Q_1 = Q_2 = 5.33 \times 10^{-5}$ **coulomb**

$V_1 = \frac{Q_1}{C_1} = \frac{5.33 \times 10^{-5}}{2 \times 10^{-6}}$ $V_1 = 26.67$ **volt.**

$V_2 = \frac{Q_2}{C_2} = \frac{5.33 \times 10^{-5}}{4 \times 10^{-6}}$ $V_2 = 13.33$ **volt.**

***Q.No.12*: Four capacitors each of 2 μF are connected in such a way that the total capacitance is also 2 μF. Show what combination gives this value.**

(***2013 Hyderabad Board***)

***Data:***

Capacitances $= C_1 = C_2 = C_3 = C_4 = 2\ \mu F$

Total capacitance $C_e = 2\ \mu F$.

***Solution:***

Let $C_1$ and $C_2$ be connected **in series** to each other, their total capacitance will be

$$\frac{1}{C_{e1}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$$

∴ $C_{e1} = 1\ \mu F$

Similarly, if $C_3$ & $C_4$ are also connected **in series** to each other their total capacitance $C_{e2}$ will also be **1 μF**. Now if these two series combinations are connected **in parallel** to each other their total capacitance is given by:

$$C_e = C_{e1} + C_{e2} = 1 + 1 = \mathbf{2\ \mu F}$$

Therefore, two given capacitors connected in series and the two series combinations are then connected in parallel to each other will give total capacitance of **2 μF**.

***Q.No.12*: A parallel plate capacitor consists of two plates with area of 0.1 m² each, separated by 0.002 m. The capacitor is filled with a dielectric material having a relative permittivity "$\varepsilon_r$" of 4. Calculate the capacitance of the capacitor.**

**($\varepsilon_o = 8.85\times10^{-12}$ C²/N-m²)**

(***2025 Karachi Board***)

***Data*:**

Area of plates $A = 0.1\ m^2$

Separation of plates $d = 0.002$ m.

Relative permittivity $\varepsilon_r = 4$

Permittivity of free space $\varepsilon_o = 8.85\times10^{-12} C^2/Nm^2$

Capacitance $C = ?$

***Solution*:**

Capacitance of a dielectric filled capacitor is given by:

$$C = \frac{A\ \varepsilon_o\ \varepsilon_r}{d} = \frac{0.1\times8.85\times10^{-12}\times 4}{0.002}$$

$$\mathbf{C = 1.77\times10^{-9}\ F}$$

***Unit. 10*** .....

# D.C Circuits

## *Ohm's law*:

***Q.No.1*: State and explain Ohm's law?**

***Ans***: According to Ohm's law:

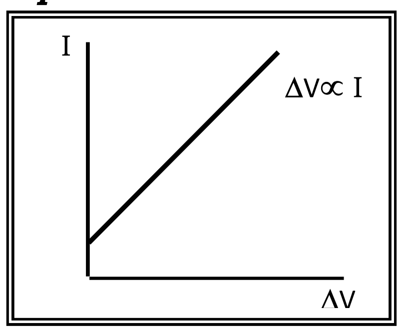
***"Potential difference "ΔV " across the ends of a conductor is directly proportional to Current " I" flowing through it provided temperature and other physical conditions of the conductor do not change".***

$$\Delta V \propto I$$

$$\therefore \quad \Delta V = R\,I$$

Where "R" is the constant of proportionality, it is known as **"Resistance of the conductor"**

Its value depends upon ***dimensions, material*** and ***temperature*** of the conductor.

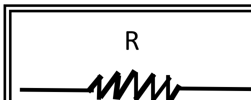



Hence the product of current "I" and resistance "R" of a conductor is equal to the potential difference "ΔV" across its ends, provided ***temperature*** and other physical conditions of the conductor do not change. The graph between potential difference "ΔV" and the resulting current "I" flowing through it, is a **straight line** if Ohm's law is obeyed.

A material that obeys Ohm's law is called ***Ohmic conductor***.

***Q.No.2*: Give an example in which Ohm's law is *not* valid.**

***Ans***: When current is passed through an electric bulb its temperature rises. It converts part of electrical energy into heat and a part into light. In this case since the temperature of filament of the bulb rises, therefore, current flowing through it will not be directly proportional to the potential difference across its ends. ***In other words***, Ohm's law will **not be obeyed**.




***Circuit representation***

***Q.No.3*: What is resistance? How is it offered?**

***Ans***: It is the ***opposition*** offered by a conductor to the flow of electric current through it

When current is passed through a conductor, free electrons move through the conductor in one direction. These free electrons suffer elastic collisions with the vibrating atoms of the conductor. As a result of which their motion is opposed. This opposition to the flow of current (or to the motion of free charges) is called ***resistance*** of the conductor.

***Note that***, with a few exceptions, resistance of materials ***increases*** with a rise in temperature and ***decreases*** with a fall of temperature.

- The change in resistance with temperature, above and below 0°C, over a wide range of temperature is uniform.

***Also note that***, resistance of semiconductors decreases with a rise in temperature.

***Q.No.4*: On what factors does the resistance of a conductor depend?**

***Ans***: Resistance offered by a conductor depends upon:

1. ***Material*** of the conductor.
2. Geometrical ***dimensions*** (length, area of cross section etc.) of the conductor.
3. ***Temperature*** of the conductor.

***Q.No.5*: What is the unit of resistance?**

***Ans***: The S.I unit of resistance is ***"ohm Ω"***.

Resistance of a conductor is said to be one ohm (1 Ω) if it allows one ampere (1 A) current to pass through it when a potential difference of one volt (1V) is maintained across it.

$$1\ \Omega = \frac{1A}{1V}$$

In electronic circuits resistors are used to control the flow of current within desired values.

***Q.No.6*: Why does the resistance of a conductor increase with a rise in its temperature? Why is Ohm's law not obeyed when temperature of the conductor rises?**

***Ans***: When temperature of a conductor is ***raised***, vibrational kinetic energy of its atoms increases. Atoms of the conductor now vibrate **more vigorously** and through **longer distances**, as a result of which the chances of collisions between these rapidly vibrating atoms and the free electrons flowing through it ***increases***. Due to the increased number of collisions, the opposition to the movement of free electrons or ***in other words***, resistance of the conductor ***increases***.

Similarly, when temperature of a conductor is ***lowered***, the atoms of the conductor vibrate slowly with decreased amplitude of vibrations, due to which chances of their collisions with free electrons or ***in other words,*** resistance of the conductor ***decreases***.

**Ohm's law** is valid as long as, along with the other physical properties, mainly temperature of the conductor ***must not change***, because if temperature of the conductor changes it's resistance will also change. Since resistance "R" is a constant of proportionality in Ohm's law, therefore, whenever resistance of a conductor changes while current is flowing through it, ***Ohm's law will not be obeyed***.

***In other words***, when temperature of the conductor changes, potential difference across the conductor ***will not be*** directly proportional to current flowing through it ($V \propto I$).

# Resistor color code:

In electronic circuits carbon resistors are used for introducing high resistances for various purposes. They are ***small***, their value is indicated by a ***color code***.

Color code is in the form of usually four colored coaxial bands in a particular order. Following table having four columns shows how to use the color code.

**First** and **second** bands represent number or significant value, **3rd band** represents multiplier whereas the **fourth band** indicates tolerance (**in other words**, possible variation in resistance, it is expressed in %).

| Color | Value (1st 2nd band) | Multiplier | Tolerance |
|---|---|---|---|
| Black | 0 | $10^0$ | Gold $\pm$ 5% |
| Brown | 1 | $10^1$ | Silver $\pm$ 10% |
| Red | 2 | $10^2$ | No color $\pm$ 20% |
| Orange | 3 | $10^3$ | |
| Yellow | 4 | $10^4$ | |
| Green | 5 | $10^5$ | |
| Blue | 6 | $10^6$ | |
| Violet | 7 | $10^7$ | |
| Grey | 8 | $10^8$ | |
| White | 9 | $10^9$ | |

To understand the color code, let us assume that the first color band is **Orange**, second is **Blue**, third is **Violet** and the fourth is **Gold**. Substituting 3 (for Orange), 6 (for Blue), $10^7$ (for Violet) and $\pm$ 5% (for Gold) we will get the value of resistance as $R = 35\times10^7 \pm 5\%$.

# Resistivity:

***Q.No.1*****:** ***What is resistivity*** (or ***specific resistance*****)** ***of a conductor?***

***Ans*****:** Resistivity or specific resistance of a conductor may be defined as:

***"The resistance offered by a conductor of unit length and unit area of cross section".***

***OR***

***"Resistivity is the resistance offered by unit volume of a conductor".***

***Q.No.2*****:** ***What is the unit of resistivity?***

***Ans*****:** The unit of resistivity is $\Omega$-***m*** (***ohm-meter***).

***Q.No.3*****:** ***On what factors does the resistivity of a conductor depend?***

***Ans*****:** Resistivity of a conductor depends upon its ***material*** and upon it's ***temperature***.

At a given temperature resistivity of a conductor depends only upon it's ***material***.

- Silver is the best conductor of electricity, because its resistivity is the lowest among all the conductors. ***In other words***, its unit volume offers minimum resistance to the flow of electric current among all the conductors.
- Some alloys have high resistivity, some of them find important use under suitable conditions where high resistance is required, e.g. in an electrical room heater usually nichrome is used because it's resistivity is highest.

***Note that***, resistivity of semiconductors is negative ***it means that*** resistance of semiconductors decreases with a rise in temperature.

# Derivation:

At a given temperature resistance of a conductor is directly proportional to its length "L" and inversely proportional to its area of cross section "A":

$$R \propto L \qquad \& \qquad R \propto \frac{1}{A}$$

OR

$$R \propto \frac{L}{A} \qquad \therefore \qquad \boxed{R = \rho \frac{L}{A}} \quad \ldots\ldots\ldots\ldots (i)$$

Where "ρ" is a constant of proportionality, it is known as **resistivity of the conductor**.

At a constant temperature the value of resistivity "ρ" depends upon ***material*** of the conductor. Usually the value of "ρ" for conductors is ***low*** and for insulators (or bad conductors) its value is ***high***.

From the above formula: $\rho = \frac{R\,A}{L}$

Resistivity "ρ" may be defined as ***the resistance offered by a conductor of unit length (L = 1) and unit area of cross section (A = 1). (Or resistance offered by its unit volume)***.

S.I unit of resistivity "ρ" is ohm-m (Ωm).

## Resistance, resistivity and temperature coefficient:

Resistance of a conductor of given material changes with temperature, hence if $R_o$ is its resistance at 0°C and $R_T$ at T°C then:

$$R_T - R_0 \propto R_0\, T$$

$$R_T - R_0 = \alpha\, R_0\, T$$

Where "α" is the constant of proportionality known as **temperature coefficient** of the given material ("α" may be defined as the change in resistance per unit resistance per degree rise in temperature), its value depends upon **material** of the conductor. Its unit is $(°C)^{-1}$ or $K^{-1}$.

$$R_T = R_0 + \alpha\, R_0\, T$$

$$R_T = R_0\,(1+ \alpha\, T)$$

Hence resistivity of a conductor of given material increases with temperature. if "$\rho_0$" and "$\rho_T$" are resistivities of a conductor of given material at 0°C and T°C respectively, then in terms of resistivities the above equation becomes:

$$\rho_T = \rho_0\,(1+ \alpha\Delta T)$$

This equation can be used to find the resistivity of a material at any temperature provided its resistivity at 0°C and its temperature coefficient "α" are known (this equation depends purely on material).

Temperature coefficient of resistance "α" is **positive** for most of the conductors, showing that their **resistance increases** with a rise in their temperature. But in case of semiconductors value of "α" is **negative**, showing that their **resistance decreases** with a rise in their temperature.

In semiconductors number of charge carriers (conduction electrons, holes) increases with a rise in its temperature due to which its "α" is **negative**. ***In other words,*** its resistance **decreases**.

# Conductance and conductivity:

***Q.No.5: What are conductance and conductivity of a conductor?***

***Ans:*** Conductance of a conductor is the reciprocal of its resistance R, similarly it's conductivity "σ" is the reciprocal of its resistivity "ρ" (i.e. $\sigma = 1/\rho$).

Unit of conductance is Siemens "S".

Unit of conductivity is Siemens/meter "S m⁻".

In terms of conductivity "σ" equation (*i*) becomes:

$$\boxed{R = \frac{L}{\sigma A}} \quad \ldots\ldots\ldots\ (ii)$$

Conductance is a measure of how **good** or how **bad** a conductor is. If conductance of a conductor is **low,** its resistance "R" and resistivity "ρ" will be **high** and for a given potential difference it will allow **less** current to pass through it, ***in other words***, it will be a **bad conductor** of electricity. But if conductance of a conductor is **high,** its resistance "R" and resistivity "ρ" will be **low** and for a given potential difference it will allow **more** current to pass through it, ***in other words***, it will be a **good conductor** of electricity.

# ***Electromotive force emf*:**

When current is passed through an electrical circuit, movement of charges (free electrons) is opposed (due to resistance of the conductor). To overcome this opposition charges will have to do work. This work done dissipates energy of the charges and appears as heat. Charges will, therefore, continuously lose energy. To maintain a constant current some source of power is needed. This source must be able to supply power equal to the power dissipated. Strength of such a source is measured by its electromotive force emf.
Hence, the electromotive force "E" of a source of current (Cell, battery, electric generator etc.) may be defined as:

***The potential difference across its terminals when no current is drawn from it.***

# ***Internal resistance*:**

***Q.No.1*: *What is internal resistance?***

***Ans*:** Internal resistance of a source is:

***"The opposition to the flow of charges through a source of emf".***

***Q.No.2*: *What causes internal resistance in a cell, a battery?***

***Ans*:** In the external electrical circuit conventional current flows from positive to negative terminal of a source of emf. To complete the circuit same amount of current also flows through the source from negative to positive terminal. While passing through the source, charges suffer collisions with the atoms of the electrolyte. This opposition to the flow of charges while passing through a source is called ***internal resistance*** of the source. Internal resistance is offered by the electrolyte of a cell or a battery but in case of a generator internal resistance is offered by wire of its coil.

***Q.No.3*: *What is terminal potential difference V and what is its relation with emf?***

***Ans*:** Terminal potential difference is the ***potential difference available across the terminals of a source for the external circuit.***

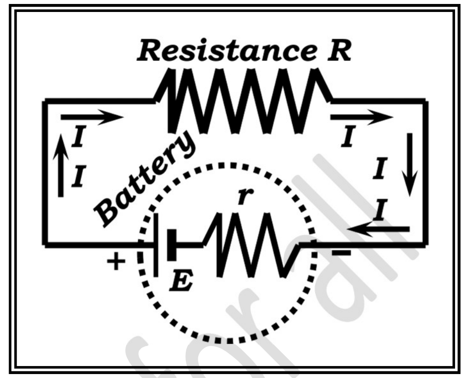
***Explanation and derivation*:**

A battery (or any source) may be considered as a source of pure emf "E" and an internal resistance "r" connected in series to it. If "R" is the total resistance of the circuit connected externally across the battery, then according to Ohm's law current "I" flowing through the circuit (having "r" and "R" connected in series to it) is given by:

$$I = \frac{E}{(R + r)}$$

$$\boxed{E = IR + Ir}$$

"I r" is the ***potential drop*** within the source across its internal resistance "r". Hence the potential difference actually acting across the external resistance "R" is "I R". It is less than emf "E" of the source. Let "I R" be represented by "V", then




$$E = V + I r$$

Hence, $$\boxed{V = E - I r}$$

"V" is known as the ***terminal potential difference***. When current is delivered by a source, terminal potential difference "V" will be less than it's emf "E", but when no current is drawn there will be no potential drop across the internal resistance, therefore, terminal potential difference "V" will be equal to its emf "E". Larger the value of current drawn from a source larger will be the potential drop within the source and smaller will be the terminal potential difference "V" available for the external circuit.

# ***Power dissipation in resistors*:**

When current is passed through a conductor, free electrons flowing through the conductor suffer elastic collisions with the vibrating atoms. During these collisions a part of electric potential energy of these free electrons is transferred to the atoms, as a result of which kinetic energy of atoms increases, resulting in a rise in temperature of the conductor. **Electrical energy** is, therefore, converted into **heat energy**.

Hence power dissipation of any electrical device is the rate at which electrical energy is converted into heat when current passes through it.

***In other words***, we can say that free electrons have to do work to overcome the opposition (resistance of the conductor) to their flow. This work done by the electrons is equal to the amount of heat produced.

If current "I" is passed through the conductor for time "t" under a potential difference "V" t hen by definition:

Work done per unit charge = V

Work done to move charge "Q" from one end to the other = QV

But $I = \frac{Q}{t}$ $\therefore$ $Q = It$

$\therefore$ Work done to move charge "Q" from one end to the other = V I t

Since work done by free electrons to overcome resistance "R" of the conductor is equal to the amount of heat produced, hence:

$$\boxed{\text{Heat produced} = V I t}$$

But power is the rate of doing work or it is the rate at which one form of energy is converted into the other (in this case electrical energy is converted into heat energy).

$$\text{Power} = \frac{\text{Work or energy}}{\text{Time}} \qquad P = \frac{V I \cancel{t}}{\cancel{t}}$$

$$\boxed{P = V I}$$

But according to Ohm's law: $V = I R$

$$\boxed{\begin{array}{l} P = I^2 R \\ P = \dfrac{V^2}{R} \end{array}}$$

S.I unit of power is **watt**. But watt (w) is a small unit of power, it's commonly used multiples are kw and Mw:

1 kilowatt (kw) = $10^3$ w

1 megawatt (Mw) = $10^6$ w

Old unit of power (which is still used in some cases) is horsepower (hp).

1 horsepower (hp) = 746 w = 0.746 kw.

***Q.No.1: What is kilowatt-hour (kwh)?***

***Ans:*** Kilowatt-hour is the ***unit of energy***, it is used to measure electrical energy on commercial scale.

I kilowatt-hour (1kwh) is that amount of energy which is delivered by the electric current by constant power of 1 kilowatt (kw) in 1 hour (h).

Since: $\text{Power} = \dfrac{\text{work (or energy)}}{\text{time}}$

OR $P = \dfrac{E}{t}$ $\therefore$ $E = P \times t$

Hence energy can be expressed in terms of power unit.

If power is 1 kilowatt and time taken is 1hour, then:

Energy E = 1 kw ×1 h = 1 kwh.

But 1kw = $10^3$ w & 1 h = 3600 s

$\therefore$ 1 kwh = $10^3$ w × 3600 s

1 kwh = $10^3$ J/s × 3600 s

1 kwh = $3.6 \times 10^6$ J

1 kwh = 3.6 MJ.

This formula gives the relation between the commercial unit of energy "kwh" and the standard S.I unit of energy "J".

1kwh electrical energy is commonly known as a ***unit*** of electricity.

# Kirchoff's laws:

Kirchoff's laws are useful in analyzing complex networks (a complex network is that electrical circuit which has two or more voltage sources along with different series and parallel combinations of resistances).

## Kirchoff's first law:(OR Kirchoff's Current law KCL)

According to Kirchoff's first law:

***Total current flowing towards a junction is equal to the total current flowing away from it.***

Kirchoff's first law is also known as the ***Kirchoff's current law KCL***, it is the direct result of ***law of conservation of charge***.

Current is the rate of flow of charge (amount of charge flowing per second). Since charge can-not be created nor can it be destroyed, therefore, total amount of charge flowing per second ***towards*** must be equal to the total amount charge per second flowing ***away*** from a junction.

Mathematically we can write:

$$\Sigma I_{towards} = \Sigma I_{away}$$

## ***Kirchoff's second law***: (***OR Kirchoff's Voltage Law KVL***)

According to Kirchoff's second law:

***The sum of electromotive forces in a loop equals the sum of potential drops in the loop.***

***OR***

***The net voltage change in a closed loop is equal to zero.***

Kirchoff's second law is also known as the ***Kirchoff's voltage law (KVL)***, it is the direct result of ***law of conservation of energy***.

When a charge moves from a lower potential to a higher potential it gains energy but when it moves from higher potential to lower potential it looses energy. OR we can say that it gains energy when it moves through a source of e.m.f from its negative to positive terminal but it looses energy when there is a potential drop.

Hence if charge is moved round a closed path starting and ending at the same point (either **clockwise** or **anticlockwise**) then it will have the same final potential energy as its initial potential energy.

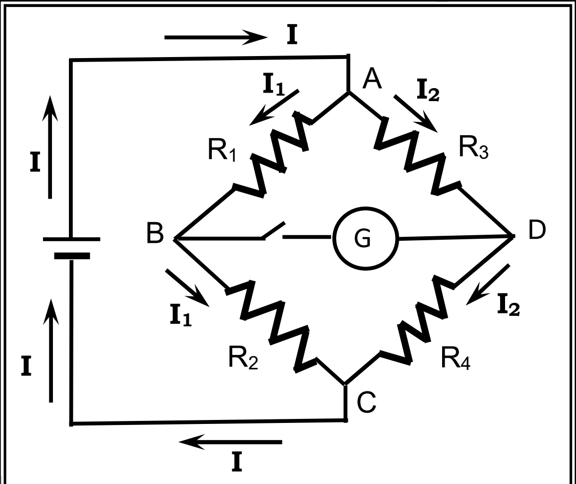
🗣 While solving a complex network (a complex network is that electrical circuit which has two or more voltage sources), by Kirchoff's voltage rule KVL,following steps must be taken:

- Direction of current flowing through all loops is assumed to be either **clockwise** or **anticlockwise**, or you can assume current in a loop in **clockwise** and in other loops **anticlockwise**.
- If the direction of that loop current is such that it flows **through a voltage source** from its **negative** to **positive** terminal, voltage change across that source of emf is taken +E (E is the emf or voltage of the source).
- Similarly, if the direction of that loop current is such that it flows **through a voltage source** from its **positive** to **negative** terminal, voltage change across that source of emf is taken **–** E (E is the emf or voltage of the source).
- Kirchoff's second rule KVL is then applied to each loop separately, finding the potential drop across each resistor in a loop by applying Ohm's law (V = I R).
- Apply Kirchoff's second rule KVL to each loop separately, assuming the current in that particular loop to be **greater** than current in its adjacent loop/s. You will get an equation for each loop. Solve these equations simultaneously. Thus, you will get the value of current in each loop.
- If current in a loop comes out to be **negative**, then its correct direction of flow will be **opposite** to its assumed direction, ***it means that*** if the direction of current assumed is **clockwise** and if it comes out to be **negative** then its correct direction of flow will be **anticlockwise**. With the help of each loop current determine the value of current in each resistor.

# Wheatstone bridge:

***Q.No.1*: What is a Wheatstone bridge? Describe working of a Wheatstone bridge.**

***Ans*:** If four resistances $R_1$, $R_2$, $R_3$, and $R_4$ are connected end to end in such a manner that they form a closed loop. A sensitive galvanometer through a key is connected across two opposite junctions whereas, a cell is connected across the remaining two junctions, then such an electrical circuit is known as ***Wheatstone bridge***



***Wheatstone bridge***

When current is switched on in a Wheatstone bridge, some current "I" is delivered by the battery. It divides into two parts "$I_1$" and "$I_2$" at point "A". These parts will further divide at points "B" and "D". Some current will flow through the galvanometer, due to which it shows deflection. The magnitude and direction of current flowing through the galvanometer and the direction of deflection of its pointer depends upon the potentials at points "B" and "D". If point "B" is at a higher potential then current will flow from "B" to "D" and vice versa.

If points "B" and "D" are at the **same potential** then no current will flow through the galvanometer, hence it's deflection will be **zero** (**null point** is obtained). Under this condition Wheatstone bridge is said to be ***balanced***.

***Q.No.2*: Derive a condition for a balanced Wheatstone bridge.**

***Ans*:** For a balanced Wheatstone bridge potentials at points "B" and "D" must be equal i.e. $V_B = V_D$ or ***in other words***, potential difference across "$R_1$" must be equal to the potential difference across "$R_3$".

But Potential difference across $R_1 = I_1 R_1$

Potential difference across $R_3 = I_2 R_3$

$\therefore \quad I_1 R_1 = I_2 R_3$ …………… (*i*)

Similarly potential difference across $R_2$ and $R_4$ must be equal.

But Potential difference across $R_2 = I_1 R_2$

Potential difference across $R_4 = I_2 R_4$

$\therefore \quad I_1 R_2 = I_2 R_4$ ……………..(*ii*)

Dividing equation (*i*) by equation (*ii*) we get,

$$\frac{I_1 R_1}{I_1 R_2} = \frac{I_2 R_3}{I_2 R_4}$$

$$\therefore \quad \boxed{\frac{R_1}{R_2} = \frac{R_3}{R_4}}$$

This is the condition for a balanced Wheatstone bridge. If this equation is satisfied, the bridge will be balanced i.e. **no current** will flow through its galvanometer and **null point** will be obtained. ***In other words***, if $R_1$ are $R_2$ are in the same ratio as $R_3$ and $R_4$ then the bridge will be balanced i.e. no current will flow through the galvanometer.

- If three resistances are known in the above equation then the fourth one can be determined after balancing the bridge. Many electrical circuits used for measuring unknown resistance, such as meter bridge, post office box etc. are based on the principle of balanced Wheatstone bridge.

## ***Thermoelectric current*:**

When junctions of two wires made of different materials are maintained at different temperatures an electromotive force emf is generated across their ends. This emf is called **thermoelectric emf** and the resulting current is called **thermoelectric current**.

Thermoelectric emf finds many applications in modern day life.

Devices such fire alarms, thermostats etc. work using thermoelectric emf.

## ***Section–A Multiple-Choice Questions MCQs:***

***Q.No.1***: Kirchhoff's laws are useful in determining:

(a) current flowing in a circuit.
(b) emf and voltage drops in a circuit
(c) Power in a circuit.
(d) Only emf in a circuit.

***Q.No.2***:The resistance of a superconductor is

(a) finite. (b) Infinite.
(c) Changes with every conductor.
(d) Zero.

***Q.No.3***: Reciprocal of resistance is called:

(a) Conductance. (b) resistivity.
(c) Resonance. (d) Capacitance.

***Q.No.4***: The graphical representation of Ohm's law is:

(a) Parabola. (b) Hyperbola.
(c) Ellipse. (d) Straight line.

***Q.No.5***: A potential difference is applied across the ends of a wire. If the potential difference is doubled, the drift velocity of free electrons will:

(a) be quadrupled. (b) be doubled.
(c) be halved. (d) remain unchanged.

***Q.No.6***: Internal resistance is the resistance offered by:

(a) Capacitor. (b) Resistor.
(c) Conductor. (d) Source of emf.

***Q.No.7***: Power dissipation in a resistor can be calculated using which formula.

(a) $P= V^2/R$ (b) $P = I^2R$
(c) $P = V I$ (d) $P= R/(V I)$

***Q.No.8***: What is a potentiometer used for:

(a) Measuring electric current.
(b) Measuring electric charge.
(c) Measuring potential difference.
(d) Measuring electric resistance.

***Q.No.9***: A heat sensitive device whose resistivity changes with the change in temperature is called:

(a) Conductor. (b) Resistor.
(c) Thermistor. (d) Thermometer.

***Q.No.10***: A wire of uniform area of cross-section A length L and resistance R is cut into two parts. The resistivity of each part is:

(a) becomes zero. (b) is halved.
(c) is doubled. (d) remains same.

***Q.No.11***: Temperature coefficient of resistance is of a semiconductor is:

• Positive. • Negative. • Zero.
• Can be positive or negative.

***Q.No.12***: The commercial unit of electrical energy is: ***(2005,10 Karachi Board)***

• ohm. • watt. • kwh • ampere

***Q.No.13***: The rate of transfer of charges through a circuit is called:
***(2006 Karachi Board)***

• resistance. • current.
• potential difference. • all of them.

***Q.No.14***: When a resistor carries a current "I", the power dissipation by it is "P". If the same resistor carries the current of '3 I', the power dissipation will be:
***(2007 Karachi Board)***

• P • P/3 • 3P • None of the above

***Q.No.15***: If a wire of uniform area of cross section is cut into two equal parts, the resistivity of each will be:
***(2007 Karachi Board)***

• Halved. • Doubled.
• Remains the same. • none of the above.

***Q.No.16***: A piece of wire of length 'L' and an area of cross section 'A' has a resistance 'R'. Another piece of wire of the same material and the same length but twice the area of cross section is connected end-to-end with the previous wire. The effective resistance is:
***(2008 Karachi Board)***

• R • 2 R • $\frac{1}{3}$ R • $\frac{1}{2}$ R

***Q.No.17***: In a house circuit all the electrical appliances are connected in parallel with the phase and the neutral to get:
***(2008 KARACHI Board)***

• Same current and potential difference.
• Different currents but same potential difference
• Different currents and different potential difference
• Same currents and same potential difference

***Q.No.18***: The electrical energy dissipated as heat in a resistor is:
***(2016,10,9,6,5 Karachi Board)***

• $V^2$ R. • $V^2$ R t. • $I^2$ R t. • $I^2$ R.

***Q.No.19*:** Resistances of 10Ω, 30Ω and 40Ω are connected in series, if the current in 10 Ω is 0.1 A then the current through 40 Ω will be: (***2019, 10 Karachi Board***)

• 0.4A • 0.3A • 0.1A • 0.08A

***Q.No.20*:** Resistivity of a material is ρ, If area of cross section of the material is doubled and length is halved then the resistivity of the conductor will: (***2022, 18 Karachi Board***)

• Remains constant • Increase 4 times
• Decrease 2 times • is one fourth

***Q.No.21*:** Four bulbs of 10Ω, 20Ω, 30Ω and 40Ω are connected in series, the bulb that will shine the most is: (***2022 Karachi Board***)

• 10 Ω • 20 Ω • 30 Ω • 40 Ω

***Q.No.22*:** A current of 9 amp. flows in a circuit of three resistors connected in parallel. If each resistor has magnitude 5 Ω, then current in the 3rd resistor is: (***2022 Karachi Board***)

• 9 amp. • 3 amp. • 1.5 amp. • 0.5 amp.

***Q.No.23*:** A 12 volt battery with internal resistance 0.1 Ω is to be charged at 10 ampere. the charger must be of: (***2022 Karachi Board***)

• 11 volts • 9 volts • 13 volts • 12 volts

***Q.No.24*:** A 2.2 kW electric iron operates at 220 volt, current it draws: (***2022 Karachi Board***)

• 20 amp. • 22 amp. •10 amp. • 5 amp.

***Q.No.25*:** Total potential difference across the combination of three cells becomes maximum when: **(*2003 Karachi Board*)**

• All of the three cells are combined in series.
• All of the three cells are combined in parallel.
• Two cells are connected is parallel and the third cell in series with the combination.
• Two cells are connected in series and the third cell in parallel with the combination.

***Q.No.26*:** If 1A current flows through 2 m long wire, the charge flows through it in 2 hours will be: (***2023 Karachi Board***)

(A) 3600 C (B) 7200 C (C) 1 C (D) 600 C

***Q.No.27*:** Reciprocal of resistance is called: (***2025 Karachi Board***)

• Resistivity. • Conductance.
• Resonance. • Capacitance.

## ***Answers*:**

(1) current flowing in a circuit.
(2) Zero.
(3) Conductance.
(4) Straight line.
(5) Be halved.
(6) Source of emf.
(7) If "V" is constant then $P = V^2/R$, If "I" is constant then $P = I^2R$.
(8) Measuring potential difference.
(9) Thermistor.
(10) remains same.
(11) Negative.
(12) kwh
(13) current
(14) 3P
(15) Remains the same.
(16) $\frac{1}{3}$ R
(17) The different currents but same potential difference.
(18) $I^2$ R t.
(19) 0.1A
(20) Remains constant
(21) 10 $\Omega$
(22) 9 amp.
(23) 13 volts
(24) 10 amp
(25) All of the three cells are combined in series.
(26) 7200 C.
(27) Conductance.

## ***Numericals*:**

***Q.No.1*: The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.5 Ω, What is the maximum current that can be drawn from the battery?**

***Data*:**

EMF E = 12 V Internal resistance r = 0.5 Ω

Max. current I =?

***Solution*:**

Current drown from a battery is given by:

$$I = \frac{E}{(R + r)}$$

Where R is the load resistance.

Maximum current can be drawn from a battery when load resistance is negligible.

$$I = \frac{12}{(0 + 0.5)}$$ **I = 24 amperes.**

***Q.No.2*: A negligibly small current is passed through a wire of length 12 m and uniform cross section $4.0 \times 10^{-7}$ m$^2$ and its resistance is measured to be 6.0 Ω.**

**What is resistivity of the material at temperature of the experiment?**

***Data*:**

Length L = 12 m Resistance R = 6.0 Ω

Area of cross section A = $4.0 \times 10^{-7}$ m$^2$

Resistivity ρ =?

***Solution*:**

Resistance of a conductor of given material is given by:

$$R = \rho \frac{L}{A} \qquad \rho = \frac{RA}{L}$$

$$\rho = \frac{6 \times 4.0 \times 10^{-7}}{12}$$ **ρ = $2.0 \times 10^{-7}$ Ω - m**

***Q.No.3*: In a potentiometer arrangement, a cell of emf 1.20 V gives a balance point at 40.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 74.0 cm, what is the emf of the second cell?**

***Data*:**

$E_1$ = 1.2 V $L_1$ = 40.0 cm.

$E_2$ =? $L_2$ = 74.0 cm.

***Solution*:**

$$\frac{E_1}{E_2} = \frac{L_1}{L_2}$$

$$E_2 = \frac{E_1 L_2}{L_1} = \frac{1.2 \times 74}{40}$$

**$E_2$ = 2.22 volt.**

**EMF of the second cell is <u>2.22 volt.</u>**

***Q.No.4*: (a) Three resistors 1Ω, 2Ω and 3Ω are combined in series. What is the total resistance of the combination?**

**(b) If the combination is connected to a battery of emf 24 V and negligible internal resistance, obtain the potential drop across each resistor.**

***Data*:**

$R_1$ = 1 Ω $R_2$ = 2 Ω $R_3$ = 3 Ω R = ?

(a) V = 24 volt $V_1$ = ? $V_2$ = ? $V_3$ = ?

***Solution*:**

$R = R_1 + R_2 + R_3$ (For series combination)

R = 1 + 2+ 3 = **6** Ω

Total resistance of the combination is **<u>6 Ω</u>**

V = I R (Ohm's law)

$$I = \frac{V}{R} = \frac{24}{6} = 4 \text{ amp.}$$

Hence 4 ampere current flows through the combination. Since in series combination **<u>same current</u>** flows through each resistance, therefore

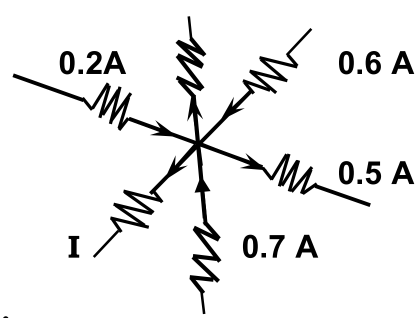
$V_1 = I R_1 = 4 \times 1$ **$V_1$ = 4 volt.**

$V_2 = I R_2 = 4 \times 2$ **$V_2$ = 8 volts.**

$V_3 = I R_3 = 4 \times 3$ **$V_3$ = 12 volts.**

(To check the result, In series combination total voltage $V = V_1+V_2+V_3$)

***Q.No.5*: From the given circuit find the value of "I". 0.4 A**




***Solution*:**

According to **Kirchoff's first law (KCL)**

Total current flowing **<u>towards</u>** a junction = Total current flowing **<u>away</u>** from the junction.

Since 0.2 amp., 0.7 amp. and 0.6 amp. currents are flowing **towards** the junction; therefore, their sum must be equal to the sum of currents "**I**", 0.5 amp. and 0.4 amp. Which are flowing **away** from the junction. Hence,

$\Sigma I_{towards} = \Sigma I_{away}$

$0.2 + 0.7 + 0.6 = I + 0.5 + 0.4$

$1.5 = I + 0.9$ $\boxed{I = 0.6 \text{ ampere.}}$

***Q.No.6*: In a meter bridge with a standard resistance of 15 Ω in the right gap, the ratio of balancing length is 5:3, Find the value of other resistance.**

***Solution*:**

Working of a meter bridge is based on the principle of balanced Wheatstone bridge.

For a balanced meter bridge

Ratio of resistances = ratio of balancing length

Since the balancing lengths are in the ratio of 5:3, therefore, resistances will also be in the ratio of 5:3.

The other resistance will be 5 times the standard resistance.

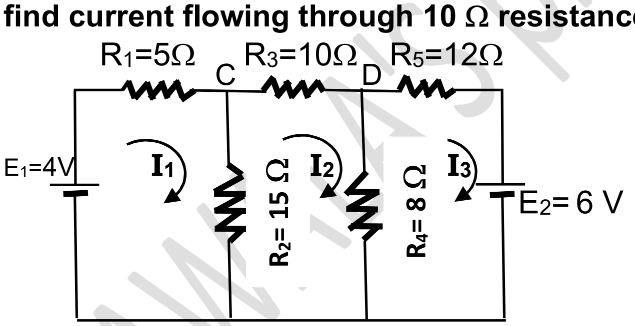
Other resistance = 5 × standard resistance

Standard resistance = 15 Ω

∴ Other resistance = 5 × 15

**Other resistance = 75 Ω**

***Q.No.7*: By KVL (Kirchoff's Voltage Law), find current flowing through 10 Ω resistance.**




***Data*:**

$R_1 = 5\Omega$ $R_2 = 15\Omega$ $R_3 = 10\Omega$ $R_4 = 8\Omega$

$R_5 = 12\Omega$ $E_1 = 4$ V $E_2 = 6$ V

Current flowing through 10Ω $I_2$ =?

***Solution*:**

In this problem direction of currents $I_1$, $I_2$ and $I_3$ in first, second and third loops is given as **clockwise**.

- Applying Kirchoff's voltage law (KVL) to first loop (assuming $I_1 > I_2$) we get:

$E_1 - I_1 R_1 - (I_1 - I_2) R_2 = 0$

$4 - 5 I_1 - (I_1 - I_2)15 = 0$

$4 - 5 I_1 - 15 I_1 + 15 I_2 = 0$

$4 - 20 I_1 + 15 I_2 = 0$ .......... (1)

- Applying Kirchoff's voltage law (KVL) to second loop (assuming $I_2 > I_3$)we get:

$-(I_2 - I_1)R_2 - I_2 R_3 - (I_2 - I_3)R_4 = 0$

$-(I_2 - I_1)15 - 10 I_2 - (I_2 - I_3)8 = 0$

$-15 I_2 + 15 I_1 - 10 I_2 - 8 I_2 + 8 I_3 = 0$

$-33 I_2 + 15 I_1 + 8 I_3 = 0$ .......... (2)

- Applying Kirchoff's voltage law (KVL) to third loop (assuming $I_3 > I_2$) we get:

$-(I_3 - I_2) R_4 - I_3 R_5 - E_2 = 0$

$-(I_3 - I_2) 8 - 12 I_3 - 6 = 0$

$-8 I_3 + 8 I_2 - 12 I_3 - 6 = 0$

$-20 I_3 + 8 I_2 - 6 = 0$

$-10 I_3 + 4 I_2 - 3 = 0$ .......... (3)

To solve these equations simultaneously put $I_1$ in equation (2) From eq. (1) which is

$4 - 20 I_1 + 15 I_2 = 0$ .......... (1)

$I_1 = \dfrac{-15 I_2 - 4}{-20} = \dfrac{15 I_2 + 4}{20}$

Put this value of $I_1$ in eq. (2),

$-33 I_2 + 3\,\cancel{15}\,\dfrac{(15 I_2 + 4)}{\cancel{20}\ 4} + 8 I_3 = 0$

$-132 I_2 + 45 I_2 + 12 + 32 I_3 = 0$

$-87 I_2 + 32 I_3 + 12 = 0$ ......... (A)

To eliminate $I_3$ from eq. (3) and eq. (A) multiply eq. (3) by 32 and eq. (A) by 10 and then add them.

$(\cancel{-10 I_3} + 4 I_2 - 3 = 0\ )\ 32$

$(-87 I_2 + \cancel{32 I_3} + 12 = 0)\ 10$

$128 I_2 - 96 - 870 I_2 + 120 = 0$

$-742 I_2 - 24 = 0$

$I_2 = \dfrac{24}{-742} = \mathbf{-0.03235\ Amp.}$

**Negative** sign shows that $I_2$ is flowing in **anticlockwise** direction.

Current flowing through $R_3$= 10 Ω Is **0.03235 Amp.** In **anticlockwise** direction.

**To check** find the value of current in each resistor, put the value of $I_2$ in eq. (1) :

$4 - 20\,I_1 + 15\,I_2 = 0$ ………. (1)

$I_1 = \dfrac{-15\,I_2 - 4}{-20} = \dfrac{15\,I_2 + 4}{20}$

$I_1 = \dfrac{15\,(-0.03235) + 4}{20} = \dfrac{3.51475}{20}$

$I_1 = \mathbf{0.1757\ amp.}$

Put the value of $I_2$ in eq. (3) :

$-10\,I_3 + 4\,I_2 - 3 = 0$ ………. (3)

$-10\,I_3 + 4\,(-0.03235) - 3 = 0$

$-10\,I_3 - 0.1294 - 3 = 0$

$I_3 = \dfrac{3.1294}{-10} = \mathbf{-0.31294\ amp.}$

Negative sign shows that $I_3$ is **anticlockwise**.

∴ Current through $R_2$:

$(I_1 - I_2) = 0.1757 - -0.03235 = \mathbf{0.20805\ amp}$

∴ Current through $R_4$:

$(I_2 - I_3) = -0.03235 - -0.31294 = \mathbf{0.28059\ amp}$

Apply Kirchoff's current law at **junction C**:

$0.1757 + 0.03235 = 0.20805$

$0.20805 = 0.20805$

**Current flowing towards C= Current flowing away from C**

Apply Kirchoff's current law at **junction D**:

$0.31294 = 0.03235 + 0.28059$

$0.31294 = 0.31294$

**Current flowing towards D= Current flowing away from D**

***Q.No.8*: A silver wire 2 m long is to have a resistance of 0.5 Ω.**
**What should its diameter be?**

***Data*:**

Length of the wire $L = 2$ m.
Resistance of the wire $R = 0.5\ \Omega$.
Resistivity of silver $\rho = 1.52 \times 10^{-8}\ \Omega$-m
Diameter of the wire $d = ?$

***Solution*:**

$R = \rho\,\dfrac{L}{A}$

$0.5 = 1.52 \times 10^{-8}\,\dfrac{2}{A}$

$A = 1.52 \times 10^{-8}\,\dfrac{2}{0.5}$ $\boxed{A = 6.08 \times 10^{-8}\ m^2}$

But $A = \pi r^2$ (For circular wire)

$6.08 \times 10^{-8} = \dfrac{22}{7} \times r^2$

∴ $r^2 = \dfrac{6.08 \times 10^{-8}}{3.142}$

$r^2 = 1.935 \times 10^{-8}$

∴ $r = \sqrt{1.935 \times 10^{-8}}$

∴ $r = 1.39 \times 10^{-4}$ m

But diameter of the wire is given by:

∴ $d = 2\,r = 2 \times 1.39 \times 10^{-4}$

$\boxed{d = 2.78 \times 10^{-4}\ m}$

Diameter of the wire will be ***2.78 × 10⁻⁴ m.***

***Q.No.9*: A 40 Ω resistor is to be wound from platinum wire 0.1 mm in diameter.**
**How much wire is needed?**

***Data*:**

Resistance of the wire $R = 40\Omega$
Diameter of the wire $d = 0.1$ mm
$= \dfrac{0.1}{1000}$ m $= 1 \times 10^{-4}$ m
Resistivity of platinum $\rho = 11 \times 10^{-8}\ \Omega$-m
Length of wire $L = ?$

***Solution*:**

Diameter of the wire $d = 2r$

∴ $r = \dfrac{d}{2} = \dfrac{1 \times 10^{-4}}{2}$ m $= 5 \times 10^{-5}$ m

Area of cross section of the wire:

$A = \pi r^2 = \dfrac{22}{7}(5 \times 10^{-5})^2$

$\mathbf{A = 7.854 \times 10^{-9} m^2}$

Resistance of a wire is given by:

$R = \rho\,\dfrac{L}{A}$

$40 = 11 \times 10^{-8}\,\dfrac{L}{7.854 \times 10^{-9}}$

∴ $L = \dfrac{40 \times 7.854 \times 10^{-9}}{11 \times 10^{-8}}$ $\boxed{L = 2.86\ m}$

***2.86 m*** long wire will be needed.

***Q.No.10*: A resistor is made by using a 50 meter nichrome wire of diameter 0.8 mm at 0°C. Calculate its resistance at 50°C. (Given: resistivity ρ = 1.1×10⁻⁶ Ω-m and α = 0.0002 °C⁻¹ at 0°C.)**
**(*2009 Karachi Board*)**

***Data*:**

Length of the wire $L = 50$ m
Diameter of wire $d = 0.8\text{mm} = 0.810^{-3}$ m

$\therefore \quad r = \frac{d}{2} = \frac{0.8\times10^{-3}}{2}\,m = 0.4\times10^{-3}\,m$

Resistivity of nichrome $\rho = 1.1\times10^{-6}\ \Omega\text{-m}$

Temperature coefficient

$\alpha = 0.0002\ °C^{-1}$ at 0°C

Resistance at 50°C , $R_{50}$ =?

***Solution***:

Resistance of the wire at 0°C is given by:

$R_0 = \rho \frac{L}{A}$

$\therefore \quad R_0 = \frac{1.1\times10^{-6}\times 50}{\pi r^2}$

$\therefore \quad R_0 = \frac{1.1\times10^{-6}\times 50}{\pi (0.4\times10^{-3})^2} = \mathbf{109.42\ \Omega}$

$\therefore$ Resistance at 50°C is given by:

$R_{50} = R_0(1+\alpha t)$

$R_{50} = 109.42\,(1+0.0002\times 50)$

$\therefore \quad \boxed{R_{50} = 110.52\ \Omega}$

**Q.No.11: A 50 ohm resistor is to be wound from a platinum wire 0.1 mm in diameter. How much wire is needed?**

**(Resistivity of the wire $\rho = 11\times10^{-8}\Omega.m$)**

**(*2015,06,03,1999,85 Karachi Board*)**

***Data***:

Resistance $R = 50\ \Omega$

Diameter $d = 0.1\text{ mm} = 0.1\times10^{-3}$ m

Resistivity $\rho = 11\times10^{-8}\ \Omega.m$

Length of wire needed $L$ = ?

***Solution***:

Radius of the wire:

$r = \frac{d}{2} = \frac{0.1\times10^{-3}}{2} = 0.05\times10^{-3}\,m$

Area of cross section of the wire will be:

$A = \pi r^2 = 3.143\times(0.05\times10^{-3})^2$

$= 7.854\times10^{-9}\,m^2$

$\therefore \quad R = \rho\frac{L}{A} \qquad \therefore \quad L = \frac{RA}{\rho}$

$\therefore \quad L = \frac{50\times7.854\times10^{-9}}{11\times10^{-8}} \qquad \boxed{L = 3.57\ m}$

***3.57 m*** long wire will be needed.

**Q.No.12: A 20 m length of cable has a cross-sectional area of 1 mm². and a resistance of 5 ohm. Calculate the resistivity of the cable.**

**(*2025 Karachi Board*)**

***Data***:

Length of cable $L = 20$ m

Area of cross-section $A = 1\ mm^2 = 1\times10^{-6}\ m^2$.

Resistance $R = 5\ \Omega$

Resistivity $\rho$ = ?

***Solution***:

Resistance of a conductor is given by:

$R = \rho\frac{L}{A}$

$\therefore \quad \rho = \frac{RA}{L} = \frac{5\times1\times10^{-6}}{20} = \mathbf{2.5\times10^{-7}\ \Omega\text{–}m}$

Resistivity of the cable is $\mathbf{2.5\times10^{-7}\ \Omega\text{–}m}$.

***Unit.11*****.....**

# Oscillations

## *Vibratory motion*:

**Vibratory** or **oscillatory** motion is that type of motion which repeats itself after a definite interval of time. The vibrating body moves to and fro about a fixed point and completes each vibration in equal interval.

## *Simple harmonic motion*:

***Q*: *What are the characteristics (or conditions) of simple harmonic motion?***

***Ans*:** A body executing simple harmonic motion shows following characteristics:

1. Its motion is **vibratory** or **oscillatory**.
2. Some **restoring force acts on the vibrating system**.
3. **Acceleration** of the body executing simple harmonic motion SHM is **directly proportional to its displacement** "$x$" and **always directed towards its mean position** i.e.

$$a \propto -x$$

The negative sign shows that acceleration "a" of the body execution SHM is always directed towards its mean position.

(Or we can say that direction of acceleration "**a**" is always opposite to the direction of displacement "$x$", displacement is measured from the mean position on either side. When the body is moving **away** from the mean position "$x$" is **increasing** or displacement "$x$" will be **positive** but velocity of the body will **decrease**, **in other words**, its acceleration "**a**" will be **negative** i.e. negative "**a**" is against the direction of motion, in this case it is towards the mean position. These observations show that acceleration "**a**" will always be directed towards the mean position, whether the body moves towards or away from the mean position).

4. Energy of the system oscillates between kinetic energy and potential energy but the total energy remains constant, provided there is no energy loss due to external or internal frictional forces **(**such as those due to roughness of surfaces in contact, cohesive or adhesive force between the surfaces, shape of bodies, air resistance etc.**)**.

# *S.H.M and circular motion*:

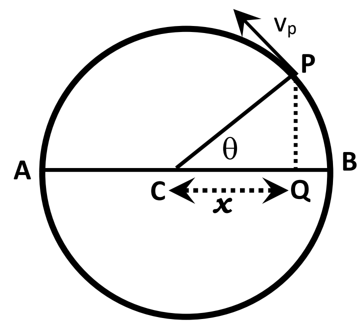
***Q.No.1*: *A point (or a particle) "P" is moving with a constant angular speed along the circumference of a circle of radius "$x_o$".***

***Derive an expression for instantaneous displacement "x" of its projection Q, vibrating along the diameter.***

***Ans.*** Consider a point "P" moving along a circular path of radius "$x_o$" with a constant angular speed "$\omega$". It's projection along the diameter AB of the circular path is point "Q". The circular path may be considered as a part of our **reference circle**. As point "P" moves along the circular path its projection Q moves to and fro along the diameter of circle about point C. Hence motion of point P is **circular** or **angular** and that of Q is **vibratory**.

Point C is the center of the circle for P but since Q vibrates about this point, therefore, it is the **mean position** for Q.

## ***Instantaneous displacement "x".***




Instantaneous displacement of Q is its distance from the mean position C at that instant on either side.

Let $x$ be the instantaneous displacement . Hence in triangle CPQ:

$$\cos\theta = \frac{CQ}{CP}$$

$$\cos\theta = \frac{x}{x_o} \qquad \boxed{x = x_o \cos\theta}$$

"$x_o$" is the maximum displacement of Q on either side of mean position, it is called amplitude of vibration.

This formula shows that when $x = \pm x_o$ (equal to radius of the circle) when the angle is 0° or 180° i.e. when Q is at B or at A. Under this condition its displacement is ***maximum*** and is equal to $x_o$.

- Hence the **amplitude** of the vibrating point Q is $x_o$ (equal to the radius of the circle).

***Q.No.2: For a body, executing S.H.M, at which point is its displacement:***
***(1) Minimum (2) Maximum***

***Ans*:** The displacement of a vibrating body, on either side, at any instant, is measured from its mean or equilibrium position, hence it is:

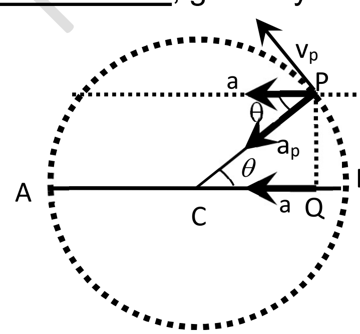
(1) Minimum at its mean position (= 0).

(2) Maximum at its extreme positions, at extreme positions its displacement is called ***amplitude of vibration***.

## ***Acceleration of Q at any instant:***

Acceleration of Q at any instant is the component of acceleration of P at that instant parallel to diameter of the circle. But the acceleration of P is always directed towards the center and is known as **centripetal acceleration**, given by:

$$a_p = \frac{v_p^2}{x_o}$$




Where $v_p$ is the magnitude of linear velocity of P (also called its tangential velocity, because it is along the tangent to the circle at a given instant). But $v_p = x_o\,\omega$.

$$\therefore \quad a_p = \frac{x_o^2\,\omega^2}{x_o} = x_o\,\omega^2$$

Resolving $a_p$ into components we get:

Component of $a_p$ parallel to the diameter $= a_p \cos\theta$

Component of $a_p$ perpendicular to the diameter $= a_p \sin\theta$

But acceleration of Q = component of $a_p$ parallel to diameter.

$\therefore \quad a = -a_p \cos\theta$

Negative sign shows that the acceleration of Q is always directed towards the mean position.

$\therefore \quad a = -x_o\,\omega^2 \cos\theta \qquad$ (since $a_p = x_o\,\omega^2$)

But $x_o \cos\theta = x$

$$\therefore \quad \boxed{a = -\omega^2 x}$$

Since ω represents the constant angular speed of P, therefore, $\omega^2$ is constant, hence:

$$\therefore \quad \boxed{a \propto -x}$$

It shows that at any instant the acceleration of Q is directly proportional to displacement and always directed towards it's mean position, which proves that the motion of the projection point Q is simple harmonic *(*SHM*)*.

From the above formula we can see that the acceleration of Q is maximum when its displacement is maximum. Displacement of Q is maximum when it is at one of its extreme positions. Hence acceleration of Q is maximum at its extreme positions i.e. when it is at A or at B. At these points it's velocity changes rapidly. Maximum acceleration of Q is given by:

$$a_{max.} = -x_o\,\omega^2$$

***Q.No.3*: *If body executes S.H.M, is its acceleration uniform?***

***Ans*:** Acceleration of a body executing S.H.M is ***not uniform***, because the magnitude of its acceleration continuously changes and at any instant depends upon its displacement '$x$'. Acceleration of the body is maximum at extreme positions and zero at the mean position.

During the vibratory motion of the body, at every point its acceleration is directed towards the mean position.

***Q.No.4*: *For a body, executing S.H.M, at which point is its acceleration*:**

**(*1*) *Minimum*** **(*2*) *Maximum***

***Ans*:** The acceleration of a body, executing S.H.M is given by:

$$a \propto -x$$

Hence acceleration of the vibrating body will be **maximum** when "$x$" is maximum. The displacement "$x$" of the body will be maximum at extreme positions, although it's velocity will be minimum but it will have ***maximum acceleration*** (Its velocity will be changing rapidly). Hence:

(1) Minimum acceleration at mean position (= 0). Here its displacement is zero.
(2) Maximum acceleration at its extreme positions. Here its displacement is maximum.

***Note that*** when a body executing S.H.M moves towards it's mean position it's velocity increases; ***it means*** that its acceleration is positive. We know that the direction of a positive acceleration is same as the direction of motion of a body. Since in this case the body is moving towards the mean position, therefore, the direction of acceleration will also be towards the mean position.

Similarly, when the body is moving away from the mean position, it's velocity decreases. ***In other words,*** now it has negative acceleration (called retardation or deceleration). Direction of negative acceleration is opposite to the direction of motion. Since in this case the body is moving <u>away</u> from the <u>mean</u> position, therefore, the direction of acceleration will be <u>towards</u> the <u>mean</u> position.

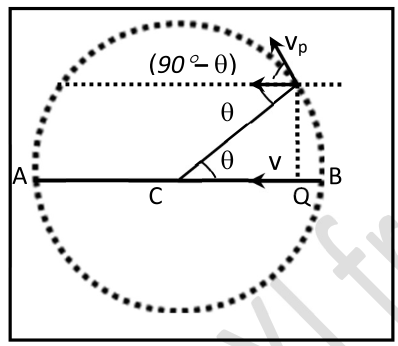
- We have seen that acceleration of a body executing S.H.M is always directed towards it's mean position.

***Q.No.5*: *A point (or a particle) "P" is moving with a uniform speed along the circumference of a circle of radius "$x_o$".***

***Derive expressions for instantaneous and maximum speeds ('$v$ and $v_{max.}$') of its projection Q vibrating along the diameter.***

***Ans.* *Instantaneous speed of Q*:**

Point P has two types of velocities, **angular or circular velocity $\omega$** and **linear velocity**. Linear velocity of P is **along the tangent** to the circle at a given point and its magnitude is $v_p$. At any instant velocity of Q is the component of linear velocity of P parallel to the diameter.




Magnitude of component of $v_p$ parallel to diameter

$$v = v_p \cos(90° - \theta)$$

But speed of Q = component of $v_p$ parallel to diameter.

$\therefore$ speed of Q = $v = v_p \cos(90° - \theta)$

But $\cos(90° - \theta) = \sin\theta$

$\therefore$ $v = v_p \sin\theta$

Or $v = x_o\,\omega \sin\theta$ (Since $v_p = x_o\,\omega$)

But $\sin^2\theta + \cos^2\theta = 1$

$\therefore$ $\sin\theta = \sqrt{1 - \cos^2\theta}$

and $\cos\theta = \dfrac{x}{x_o}$

$$\sin\theta = \sqrt{1 - \frac{x^2}{x_o^2}}$$

$$\therefore \quad \boxed{v = x_o\,\omega\sqrt{1 - \frac{x^2}{x_o^2}}}$$

From trigonometry:
$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
Where $\alpha$ and $\beta$ are any two angles, then
If $\alpha = 90°$ & $\beta = \theta$ Then
$\cos(90° - \theta) = \cos 90° \cos\theta + \sin 90° \sin\theta$
But $\cos 90° = 0$ & $\sin 90° = 1$
$\therefore$ $\cos(90° - \theta) = \sin\theta$

Or $v = x_o\,\omega\sqrt{\dfrac{x_o^2 - x^2}{x_o^2}}$ Or $v = \cancel{x_o}\,\omega\dfrac{\sqrt{x_o^2 - x^2}}{\cancel{x_o}}$

Hence the instantaneous speed of Q is given by:

or $\boxed{v = \omega\sqrt{x_o^2 - x^2}}$

## ***Maximum speed of Q*:**

Speed of Q is <u>maximum</u> when it is passing through its <u>mean position</u> where it's displacement $x = 0$, Hence it's maximum speed $v_{max.}$ is given by:

$$v_{max.} = \omega \sqrt{x_o - 0}$$

$$\boxed{v_{max.} = x_o\,\omega}$$

$x_o\,\omega$ is the speed of P, hence maximum speed $v_{max.}$ of Q is equal to the speed $v_p$ of P.

## Minimum speed of Q:

Speed of Q is minimum (= 0) when it is at one of its extreme positions where $x = \pm x_o$. Hence:

$$v = \omega \sqrt{x_o^2 - x^2} \qquad \text{(but } x = \pm x_o\text{)}$$

$$\therefore \quad v = \omega \sqrt{x_o^2 - (\pm x_o)^2}$$

$$\therefore \quad v = \omega \sqrt{x_o^2 - x_o^2} \qquad \therefore \quad v = 0$$

## Relation between v and $v_{max.}$:

Instantaneous speed of Q is given by:

$$v = x_o\,\omega \sqrt{1 - \frac{x^2}{x_o^2}}$$

But $v_{max.} = x_o\,\omega$

Hence the instantaneous and maximum speeds of Q are related by:

$$\boxed{v = v_{max.}\sqrt{1 - \frac{x^2}{x_o^2}}}$$

## Time period:

***Q.No.6**: A point (or a particle) "P" is moving with a uniform speed along the circumference of a circle of radius "$x_o$".*

***Derive an expression for time period "T" of its projection Q, vibrating along the diameter.***

***Ans.*** Time taken by a vibrating body to complete one vibration is called it's time period T.

Angular speed "ω" of point P at the center of its circular path is given by:

$$\omega = \frac{\theta}{t}$$

Where "θ" is the angular displacement of point P in during time t.

Time in which P completes one revolution Q will complete one vibration.

But the angular displacement of P at the center of its circular path will be $\theta = 2\pi$ radian at the end of one revolution and t = T during this time vibrating point Q would have completed on vibration. Hence T is also the time period for Q:

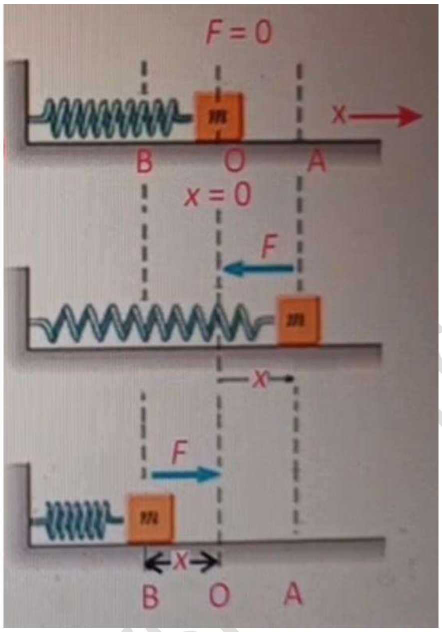
$$\omega = \frac{2\pi}{T} \qquad \text{Hence} \qquad \boxed{T = \frac{2\pi}{\omega}}$$

# ***Motion under elastic restoring force*: (*Horizontally*)**

***Q.No.1*: What is motion under elastic restoring force?**

***Ans*:** When a body placed on a frictionless horizontal surface connected at the end of an elastic and light spring, whose other end is fixed to a rigid support, is pulled to one side and released then the body vibrates about its position of equilibrium.

***Note that*** in this case the spring is parallel to the horizontal surface. **Weight** of the body is balanced by **normal reaction** of the surface, whereas, **friction** between the vibrating body and the horizontal surface on which it vibrates is assumed to be **negligible**. The only unbalanced force acting on the body is the force exerted by the spring due to its elasticity. This force tends to restore the original conditions of the body i.e. it tends to bring the body back to its original position. It is called ***elastic restoring force***.




Vibratory motion of the body is simple harmonic, because the acceleration produced in the body, due to the elastic restoring force of the spring, is directly proportional to its displacement and is always directed towards its equilibrium (or mean) position.

This type of vibratory motion of the body under the influence of elastic restoring force of the spring is called ***motion under elastic restoring force***.

***Q.No.2*: What is *elastic restoring force*?**

***Ans*:** It is a force due to elasticity of the spring that tends to restore the original conditions, i.e. it tends bring the vibrating body back to its original position, hence it is called ***elastic restoring force***.

***Q.No.3*: What is *elastic constant* or *force constant* of a spring?**

***Ans*:** It is the force required to produce unit elongation in the spring.

Spring constant $k = F/x$.

Its value depends upon the elastic properties of the spring. Its value is high for hard springs, as they require large force to elongate and vice versa.

***Q.No.4*: What is the unit of spring constant?**

***Ans*:** Since spring constant is $F/x$, therefore, its S.I unit is N/m or N $m^{-1}$.

***Q.No.5*: A body on a smooth horizontal surface performs vibratory motion under the influence of elastic restoring force.**

**Prove that its motion is simple harmonic.**

***Ans*:** Consider a body of mass 'm' attached to the end of a light and elastic spring, the other end of the spring is attached to a wall. To displace the body from its equilibrium position through some distance '$x$' force required, according to Hook's law is given by:

$$F_{App.} \propto x$$

$$F_{App.} = k\,x$$

Where 'k' is the constant of proportionality and represents the ***spring constant*** (It is the force required to produce unit elongation in the spring, its value depends upon the ***elastic properties*** of the spring). As the body is pulled by applying the above force due to elasticity the spring will also exert an equal and opposite force. Force exerted by the spring tends to restore the original conditions (i.e. it tends to bring the body back to its original position). It is called ***elastic restoring force***, given by:

$$F_{elast.} = -kx$$

If now the applied force is removed, the elastic restoring force becomes unbalanced, body under the action of unbalanced force vibrates about its mean position, with some acceleration.

If 'a' is the acceleration produced in the vibrating body, then according second law of motion:

$$F_{elast.} = ma$$

Therefore,

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

'k/m' is constant for a given spring and a body of given mass, hence:

$$a \propto -x$$

This relation shows that acceleration 'a' of the body vibrating under the influence of elastic restoring force is directly proportional to the displacement and is always directed towards its mean position. Hence motion of the vibrating body is simple harmonic.

**Note that** weight of the body is balanced by upward reaction of the surface on which it is kept.

# *Time period*:

***Q.No.1: Derive a formula for time period of a body executing S.H.M under elastic restoring force?***

***Ans:*** Time period of a vibrating body is the time in which it completes one vibration.

Acceleration of a projection point along the diameter of a reference circle is given by:

$$a = -\omega^2 x$$

But acceleration of a body executing simple harmonic motion under the influence of elastic restoring force is given by:

$$a = -\frac{k}{m}x$$

Comparing the two relations we get:

$$\omega^2 = \frac{k}{m} \quad \text{OR} \quad \omega = \sqrt{\frac{k}{m}}$$

Time period of a projection point executing S.H.M along the diameter of a reference circle is given by:

$$T = \frac{2\pi}{\omega}$$

On substituting $\omega = \sqrt{k/m}$ we get:

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Where "m" is mass of the body and "k" is the elastic constant of material of the spring.

***Q.No.2*: On what factors does the time period of a body executing S.H.M under elastic restoring force of a given spring depend?**

***Ans*:** Time period (time taken to complete one vibration) depends upon mass "m" of the body and upon the spring constant "k" of the spring. If mass of the body is increased its time period also increases, ***it means that*** on increasing mass (taking a heavier body) it slows down and takes a longer time to complete a vibration. But for large value of "k" the value of "T" is small.

Time period of a body executing S.H.M under the influence of elastic restoring force for a given spring is directly proportional to the square root of its mass.

# Energy:

***Q.No.1*: What happens to energy of a body executing S.H.M under the influence of elastic restoring force?**

***Ans*:** Energy of the vibrating system continuously changes its form from K.E to P.E and vice versa. ***In other words***, at any instant the total energy possessed by the vibrating system will be partially in the form of K.E and partially in the form of P.E and the total energy will be equal to the sum of K.E and P.E at that instant. The total energy remains constant if there are no energy losses due to frictional forces.

Potential energy of the system vibrating under elastic restoring force at any instant is given by: $$P.E = \frac{1}{2} k x^2$$

Similarly, its K.E at any instant is given by:

$$K.E = \frac{1}{2} k (x_o^2 - x^2)$$

Hence the total energy of the system executing S.H.M under the influence of elastic restoring force at any instant will be: $$E = P.E + K.E$$

$$E = \frac{1}{2} k x^2 + \frac{1}{2} k (x_o^2 - x^2)$$

$$\boxed{E = \frac{1}{2} k x_o^2}$$

'k' is the spring constant, '$x$' is displacement of the vibrating body at any instant and '$x_o$' is the **amplitude of vibration** (maximum displacement).

Hence energy of a system executing S.H.M under elastic restoring force for a given spring depends upon the amplitude of vibrations '$x_o$'.

***Q.No.2*: Show that energy of a body executing simple harmonic motion under the influence of elastic restoring force is totally in the form of K.E at its mean position.**

***Ans*:** At any position potential energy of a body executing simple harmonic motion under elastic restoring force is given by: $P.E = \frac{1}{2} k x^2$

But at the mean position displacement of the body is zero i.e. $x = 0$

Therefore, at the mean position: $P.E = 0$ ………………… $(i)$

But at any position kinetic energy of a body executing simple harmonic motion under elastic restoring force is given by:

$$K.E = \frac{1}{2} k (x_o^2 - x^2)$$

Therefore, at the mean position ($x = 0$) K.E is given by:

∴ $K.E = \frac{1}{2} k (x_o^2 - 0^2)$

∴ $K.E = \frac{1}{2} k x_o^2$

But '$\frac{1}{2} k x_o^2$' is the total energy of the vibrating body. Therefore,

K.E = Total energy…………….. (*ii*)

From equations (*i*) and (*ii*) we conclude that the energy of the body executing S.H.M is totally in the form of K.E it does not have any P.E at the mean position.

***Q.No.3*: Show that energy of a body executing simple harmonic motion under the influence of elastic restoring force is totally in the form of P.E at extreme positions.**

***Ans*:** At any position kinetic energy of a body executing simple harmonic motion under elastic restoring force is given by: $K.E = \frac{1}{2} k (x_o^2 - x^2)$

But at extreme positions ($x = \pm x_o$) K.E is given by:

∴ $K.E = \frac{1}{2} k \{x_o^2 - (\pm x_o)^2\}$

∴ $K.E = \frac{1}{2} k x_o^2 - \frac{1}{2} k x_o^2$

∴ $K.E = 0$ $(x = x_o)$

Potential energy of a body executing simple harmonic motion under elastic restoring force is given by: $P.E = \frac{1}{2} k x^2$

At extreme positions ($x = \pm x_o$)

Hence, $P.E = \frac{1}{2} K x_o^2$ $(x = x_o)$

But $\frac{1}{2} K x_o^2$ is the total energy, hence P.E of the vibrating system is maximum and is equal to the total energy of the body. Hence energy is totally in the form potential energy (P/E) at the extreme position whereas it's K.E is zero.

## ***Interconversion of K.E and P.E*:**

Energy of a body executing simple harmonic motion continuously changes its form from P.E to K.E and vice versa. Energy of a vibrating body at its mean position is totally kinetic (here its velocity is **maximum**) but at extreme position its energy is totally potential (here its velocity is **zero** for a fraction of a second) but displacement of the body from its mean position will be maximum (potential energy of the vibration body depends upon its displacement from the mean position). At any other position its energy is partially kinetic and partially potential (velocity is between **zero** and **maximum**)).

It the vibrating system is free of frictional forces then at any instant its total energy will be equal to the sum of its K.E and P.E at that instant such that its total energy remains constant.

***Note that*** if the system is frictionless then it's energy will **remain constant** throughout its vibratory motion, although it constantly changes its form from K.E to P.E and vice versa such that it's total energy always **remains constant**.

If there are frictional forces present then a part of energy of the system will be used to do work against friction, as a result of which energy of the system will **decrease** in each vibration, due to which amplitude of vibration will decrease in each vibration. Ultimately the system will come to rest when its energy is totally used up in doing work against friction. This type of oscillations are called ***damped oscillations***.

## ***Motion under elastic restoring force*: (*Vertically*)**

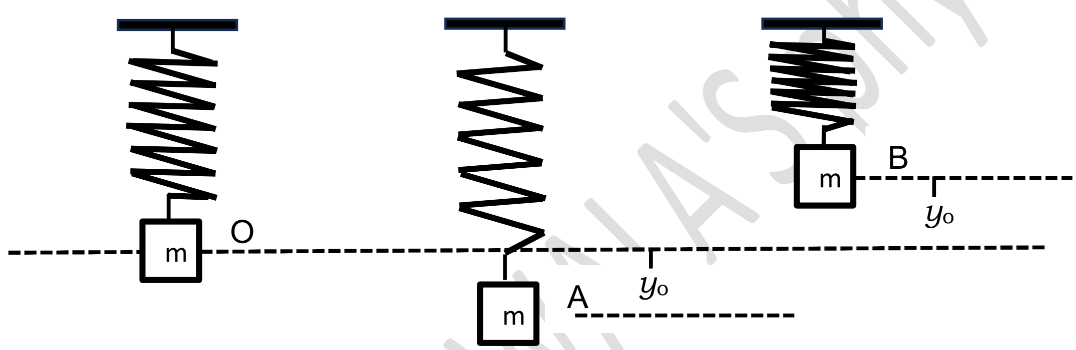
Consider a light body of mass "m" suspended at the end of a light elastic spring whose other end is attached to a rigid support. The suspended body comes to rest at its equilibrium position at point "O" after stretching the spring a little due to its weight.

If the body is now pulled down vertically through distance "$y_o$" to a point "A" then as soon as we start pulling the body downward, due to elasticity spring starts exerting reaction force upward. Force exerted by the spring is called **elastic restoring force**. This force tends to bring the body back to its equilibrium position i.e. it tends to restore original conditions. If the body is now released, it will move upward with some acceleration under the action of this force, its velocity **increases** as it moves up until it crosses its equilibrium position where its velocity becomes **maximum**. The body will continue moving upward but as soon as it crosses the equilibrium position, the spring will start compressing, hence the spring will now exert elastic restoring force against the direction of motion of the body, due to which velocity of the body starts **decreasing**, after covering same distance "$y_o$" above the equilibrium position body comes to rest at point "B". Compressed spring now pushes the body which moves back with increasing velocity. This process repeats itself again and again several times and we will see the body vibrating up and down about its position of equilibrium, between points "A" and "B". Distance "$y_o$" from point A to O or from O to B is **amplitude of vibration**. Point "O" is the **mean position** for the vibrating body. Motion of the body is **vibratory or oscillatory**.

It can be shown that acceleration "a" of the vibrating body at any instant is given by:

$$a \propto -y$$

Where "$y$" is the displacement of the body at any instant. From this equation we conclude that the acceleration "a" of the body at any instant is directly proportional to its vertical displacement "$y$" and is always directed towards its mean position, which shows that motion of the vibrating body is **simple harmonic motion (SHM)**.



# Time period:

***Q.** Give the formula for time period of a body executing S.H.M in vertical direction under elastic restoring force?*

***Ans**:* Time period of a vibrating body is the time in which it completes one vibration.
Time period of a body executing S.H.M in vertical direction is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Where "m" is mass of the suspended body and "k" is the spring constant.

***Q.No.1**: On what factors does the time period of a body executing S.H.M In vertical under elastic restoring force depend?*

***Ans**:* Time period depends upon mass "m" of the body and upon the spring constant "k" of the spring. If mass of the body is **increased** its time period also **increases**, ***it means that*** on increasing mass (taking a heavier body) it slows down and takes a longer time to complete a vibration, actually time period "T" is directly proportional to $\sqrt{m}$. But for large value of "k" the value of "T" is small. Time period T is inversely proportional to $\sqrt{k}$.

# Simple pendulum:

***Q.No.1**: What is an ideal simple pendulum?*

***Ans**:* An ideal simple pendulum consists of a point mass suspended with the help of a weightless, inextensible and flexible string whose other end is attached to a rigid and non-yielding support.

***It means that*** length of the string with which the point mass is suspended must not change as the pendulum vibrates, similarly the support must be rigid so that it should not vibrate along with the pendulum. In the laboratory the center of gravity of a small spherical metallic bob can be used to replace the heavy point mass.

***Q.No.2**: What is the type of motion executed by a simple pendulum?*

***Ans**:* If bob of the pendulum is displaced from its equilibrium position and allowed to move freely then it performs to and fro motion. This vibratory motion of a simple pendulum for small ngular displacement is simple harmonic or ***in other words,*** motion of a simple pendulum is S.H.M when it's amplitude of vibrations is small.

***Q.No.3**: Prove that motion of a simple pendulum for small amplitude is simple harmonic S.H.M?*

***Ans.*** In order to prove that motion of simple pendulum is S.H.M we will find its acceleration. To find it's acceleration first of all we will have to find the unbalanced force acting on it.

At any displaced position, forces acting on the bob are:

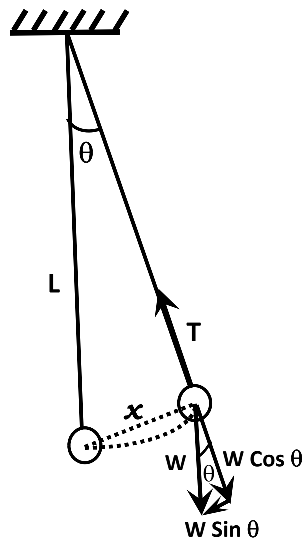
(1) It's weight "W" acting vertically downward.
(2) Tension "T" in string acting along the string.

On resolving "W" into components parallel and perpendicular to the string we get:

Component of "W" parallel to the string = $W \cos\theta$
Component of "W" perpendicular to the string = $W \sin\theta$

Since the bob does not move parallel to the string, therefore, tension "T" in the string is balanced by "W Cos θ". Hence the unbalanced force acting on the bob is "W Sin θ", this force is always **directed towards the mean position** (unbalanced force always produces acceleration in its own direction). Under the action of this force the pendulum performs to and fro motion (This force is also the restoring force, as it tends to bring the bob back to its original mean position).

∴ Unbalanced force = – W Sin θ




The negative sign shows that the unbalanced force is always directed towards the mean position. According to second law of motion:

Unbalanced force = m a

∴ m a = – W Sin θ

m a = – m g Sin θ

∴ a = – g Sin θ

For small values of "θ" measured in radians, or ***in other words***, for small amplitude of vibration of the pendulum

$$\text{Sin}\,\theta = \theta$$

But $$\theta = \frac{x}{L}$$

Where "$x$" is the instantaneous displacement of the bob from its mean position and "L" is length of the pendulum (measured from the point of suspension to center of the bob).

∴ $$a = -g\,\frac{x}{L}$$

For a given pendulum, at a given location g/L is constant,

∴ $$a \propto -x$$

The above relation shows that the acceleration of a simple pendulum at any instant is directly proportional to its displacement at that instant and is always directed towards it's mean position, hence motion of a simple pendulum for small amplitude is ***simple harmonic***.

***Q.No.4*: *When is the tension in a string of a pendulum maximum?***

***Ans*:** Tension in the string of a simple pendulum is maximum when the vibrating pendulum passes through its equilibrium or mean position. At this point the tension in the string will be equal to weight of bob of the pendulum.

The magnitude of tension in the string of a pendulum periodically changes as the pendulum vibrates and "$\theta$" changes according to the following relation:

$$T = W \cos\theta$$

When the angular displacement of the bob "$\theta$" is zero, that happens when the pendulum is passing through its mean position, the tension in the string will be maximum and is equal to weight of the suspended bob $T = W$ $(\cos 0° = 1)$.

***Q.No.5*: *Derive a formula for time period of a simple pendulum, and show that it is independent of mass and material of its bob.***

***Ans*:** Motion of a simple pendulum is S.H.M when its amplitude is small, under this condition its acceleration is given by:

$$a = -g\frac{x}{L}$$

But the equation for acceleration of a point executing S.H.M along the diameter of a reference circle is given by:

$$a = -\omega^2 x$$

Comparing the above two equations, we get:

$$\omega^2 = \frac{g}{L}$$

$$\therefore \quad \omega = \sqrt{\frac{g}{L}}$$

***Time period*** of a body executing S.H.M is the time in which it completes one vibration. Time period of a point executing S.H.M along the diameter of a reference circle can be used to calculate time period of a pendulum by replacing "$\omega$" by $\sqrt{g/L}$ in the following equation;

$$T = \frac{2\pi}{\omega}$$

$$\therefore \quad T = \frac{2\pi}{\sqrt{g/L}}$$

$$\therefore \quad \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

At a given location time period of a simple pendulum is directly proportional to $\sqrt{L}$, hence if length of the pendulum is increased its time period also increases according to the above relation. From the above formula we can also see that the time period of a simple pendulum is independent of ***mass*** and ***material*** of its bob.

***Note that*** the motion of a simple pendulum is S.H.M as long as "θ" is **small** or the amplitude of its vibrations is **small**.

Also $\omega = \sqrt{g/L}$ for a simple pendulum executing S.H.M (or for a mass suspended with an inelastic string executing S.H.M).

$\omega = \sqrt{K/m}$ for a body attached at the end of an elastic spring executing S.H.M.

# Second's pendulum:

***Q.No.6*: What is second's pendulum?**

***Ans***: A pendulum whose time period is ***"2 seconds"*** is called second's pendulum. (This pendulum will take 1 sec. to move from one extreme to the other).

***Q.No.7*: What is the frequency of second's pendulum?**

***Ans***: Frequency of a vibrating body is the number of vibrations completed in one second. Relation between time period "T" and frequency "*f*" is:

$$f = \frac{1}{T}$$

Since time period of a second's pendulum is 2 seconds hence its frequency is 0.5 hertz.

$$\therefore \quad f = \frac{1}{T} = \frac{1}{2} = 0.5 \text{ hertz}$$

and its length "L" where "$g = 9.8\ m/s^2$" is 0.99 m.

***Q.No.8*: What will be difference in the length of a second's pendulum on the surface of the earth and that on the surface of moon?**

***Ans***: Time period 'T' of a simple pendulum executing S.H.M on the surface of any planet is given by:

$$\therefore \quad T = 2\pi\sqrt{\frac{L}{g}}$$

Where 'g' is the acceleration due to gravity on the surface of a planet. For second's pendulum T = 2 second on any planet.

The value of "g" on the surface of moon is less (about $1.63\ m/s^2$) than the average value of "g" on the surface of the earth (about $9.8\ m/s^2$), The length of second's pendulum on earth is 0.99 m and on moon it is about 0.166 m.

***In other words***, length of second's pendulum will be lesser on moon than on the earth.

***Q.No.9*: What will be the length of a second's pendulum on the surface of Jupiter where the value of 'g' is about 2.63 times the value of 'g' on the earth (*i.e.* $g_J = 2.63\ g_e$)?**

***Ans***: Time period 'T' of a simple pendulum executing S.H.M on the surface of any planet is given by:

$$\therefore \quad T = 2\pi\sqrt{\frac{L}{g}}$$

Where 'g' is the acceleration due to gravity on the surface of a planet. For second's pendulum T = 2 second on any planet.

The value of "g" on the surface of Jupiter is about 2.63 times the value of 'g' on the surface of the earth i.e. $g_J = 2.63\ g_e$.

Hence, $$2 = 2\pi\sqrt{\frac{L}{2.63\ g_e}}$$

Hence, $$2 = 2\pi\sqrt{\frac{L}{2.63 \times 9.8}}$$

Therefore, $$\boxed{L = 2.61\ m}$$

Length of second's pendulum on the surface of Jupiter will be ***2.61 m***.

***Q.No.10*: For simple harmonic motion will the time period change or not, by doubling the mass of the bob attached to:**

**(*a*) *Elastic spring.* *b*) *Inelastic string.* *Explain.*** (***2011 Karachi Board***)

***Ans*:** (a) The time period of a bob attached to an elastic spring executing S.H.M (motion under elastic restoring force) is given by:

$$\therefore \qquad \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

This formula shows that the time of the bob attached to an **elastic spring** and executing S.H.M for a given spring ***increases if mass of the bob is increased*** and vice versa.

Hence if mass of a bob is doubled its time period will ***increase by $\sqrt{2}$ times***.

(b) The time period of a bob attached to an inelastic string (a simple pendulum) executing S.H.M is given by:

$$\therefore \qquad \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

This formula shows that the time period of the bob attached to an **inelastic string** (a simple pendulum) and executing S.H.M at a given location is ***independent of mass of the bob***. Hence if mass of the bob is ***increased time period of the pendulum will not change***.

# ***Natural time period and natural frequency*:**

***Q.No.1*: *What is natural time period of a vibrating body?***

***Ans*:** The time period with which a body vibrates in the absence of external forces is known as its ***natural time period***.

Hence a body which is capable to vibrate always vibrates with certain fixed time period. This time period is called ***natural time period*** of the body.

Simple pendulum of a given length (constant L) at a given location (constant g) always vibrates with a fixed time period which is known as its natural time period.

***Q.No.2*: *What is natural frequency of a body?***

***Ans*:** Number of vibrations completed by a vibrating body in one second is known as its frequency. (For waves it is the number of waves passing through a point in one second).

Since frequency is equal to the reciprocal of its time period, hence a body which can vibrate will always vibrate with a fixed frequency and a fixed time period. ***In other words,*** it will always complete the same number of vibrations in one second. This frequency is called ***natural frequency*** of the body.

# Resonance:

***Q.No.3*: What is resonance?**

***Ans*:** When natural frequency of a body capable of vibrating exactly matches with the frequency of an external periodic force then the body starts vibrating with ***increasing amplitude***. This phenomenon is known as ***resonance***.

All receivers, such as T.V, radio set, mobile phones, wireless etc. work on the principle of resonance. These receivers have a tuning circuit. In these tuning circuits the frequency of electrical oscillations is matched with the frequency of incoming signal. When the two frequencies match the receiver starts responding. As a result of which we hear sound and see picture.

## ***Free vibrator/oscillator*:(*OR* *ideal or vibrator*)**

An ideal vibrator/oscillator is that in which frictional forces are not present. If frictional forces are present then a part of energy of the vibrating system will be wasted in doing work against the frictional forces.

**For example**, (1) an ideal simple pendulum is that in which a **point mass** is suspended with the help of an **inextensible**, **weightless** string whose other end is fixed to a rigid support (a support which does vibrate along with the vibrating mass). A real pendulum has a metallic bob suspended by a string attached to a support. Centre of gravity of the bob can be considered as point mass and surface of the bob must be smooth and well-polished to minimize air resistance but viscous air drag remains, which is one of the factors to dissipate energy. Energy dissipated can, therefore be minimized but cannot be made exactly **zero**.

(2) A body executing S.H.M under the action of elastic restoring force must have no friction between the vibrating body and the surface on which it vibrates and also there must be no friction between adjacent turns of the spring. In reality friction between the lower surface of the vibrating body and the surface on which it vibrates can be minimized but cannot be eliminated completely.

In case of an ideal vibrator energy of the vibrating system will remain **constant**. Since energy of a vibrating system depends upon its amplitude of vibrations, therefore for an ideal vibrator its **amplitude** of vibrations will remain **constant**.

## ***Damped vibrations/oscillations*:**

In a real pendulum, or in a body vibrating under the action of an elastic restoring force or in general, in any real vibrating system, dissipating forces are always present. These dissipating forces (frictional forces) can be **minimized** but cannot be completely eliminated. As a result of which part of energy of the vibrating system **decreases**. This lost energy is used to do work against frictional forces. Due to decrease in energy of the system amplitude of the vibrating system **decreases**. Hence vibrations gradually die out and the vibrating system ultimately stops. Vibrations of this type are called ***damped vibrations or damped oscillations***.

Damping of a vibrating system can be eliminated by supplying the energy lost during each vibration.

**For example**, in case of a child taking swings, parent give a little push to keep the amplitude of the swing constant. During each push actually they supply energy lost in doing work to overcome dissipating forces.

***Q- factor*:**

Q-factor may be defined as the ratio of energy stored to energy lost per oscillation.

$$Q = \frac{\text{Energy stored}}{\text{Energy lost}}$$

OR

$$Q = \frac{E_{stored}}{E_{lost}}$$

Q-factor is a **dimensionless quantity**. (since it a ratio of two similar quantities)

## Section–A Multiple-Choice Questions MCQs:

**Q.No.1**: Two simple pendulums A and B with same lengths, and equal amplitude of vibrations but the mass of A is twice the mass of B, their periods are $T_A$ and $T_B$ and energies are $E_A$ and $E_B$ respectively. Choose the correct statement:
(a) $T_A = T_B$ and $E_A > E_B$ (b) $T_A < T_B$ and $E_A > E_B$
(c) $T_A > T_B$ and $E_A < E_B$ (d) $T_A = T_B$ and $E_A < E_B$

**Q.No.2**: In order to double the period of a simple pendulum:
(a) Its length should be doubled.
(b) Its length should be quadrupled.
(c) Its mass should be doubled.
(d) Its mass should be quadrupled.

**Q.No.3**: A simple harmonic oscillator has amplitude A and time period "t". Its maximum speed is:
(a) $\frac{4A}{t}$ (b) $\frac{2A}{t}$ (c) $\frac{4\pi A}{t}$ (d) $\frac{2\pi A}{t}$

**Q.No.4**: A spring attached by a load of weight W is vibrating with a period T. If the spring is divided in four equal parts and the same load is suspended from one of these parts, the new time period is:
(a) $\frac{T}{4}$ (b) 2 T (c) $\frac{T}{2}$ (d) 4 T

**Q.No.5**: The total energy of a particle executing simple harmonic motion is proportional to:
(a) Frequency of oscillation.
(b) Maximum velocity of motion.
(c) Amplitude of motion.
(d) Square of amplitude of motion.

**Q.No.6**: A child swinging on a swing in sitting position, stands up, then the time period of the swing will:
(a) Increase. (b) decrease.
(c) remains the same.
(d) increases if the child is tall and decreases if the child is short.

**Q.No.7**: If a boy oscillates at the angular frequency "$\omega_d$" of the driving force, then the oscillations are called:
(a) Forced oscillations. (b) Coupled oscillations,
(c) Free oscillations. (d) Maintained oscillations

**Q.No.8**: A simple harmonic oscillator with a natural frequency "$\omega_N$" is forced to oscillate with a driving frequency "$\omega_d$". The resonance occurred when:
(a) "$\omega_N < \omega_d$" (b) "$\omega_N > \omega_d$"
(c) "$\omega_N = \omega_d$" (d) "$\omega_N \approx \omega_d$"

**Q.No.9**: In vehicles, shock absorber reduces jerks
(a) The shock absorber is an application of damped oscillations.
(b) Damping effect is due to the fractional loss of energy.
(c) Shock absorbers in vehicles reduce jerk. (d) All of these.

**Q.No.10**: A heavily damped system has a fairly flat resonance curve in:
(a) An acceleration - time graph.
(b) An amplitude - frequency graph.
(c) Velocity - time graph
(d) Distance – time graph.

**Q.No.11**: If the bob of a vibrating simple pendulum is suddenly detached from the string at its mean position, it's path will be: (***2005 Karachi Board***)
- a straight line
- a circle.
- A parabola
- a hyperbola.

**Q.No.12**: which of the following exhibit simple harmonic motion. (***2010 Karachi Board***)
- A hanging spring supporting a weight.
- The balance of a wheel.
- The wheel of an automobile.
- The spring of a violin.

**Q.No.13**: Sin $\theta = \theta$ if $\theta$ is specifically less than: (***2012 Karachi Board***)
- 15°. • 10°. • 5°. • 1 radian.

**Q.No.14**: Time period of a simple pendulum depends upon: (***2013 Karachi Board***)
- mass • length • Acceleration due to gravity
- Both length and acceleration due to gravity

**Q.No.15**: If mass of bob of a simple pendulum is doubled, its time period will: (***2014 Karachi board***)

- be doubled.
- becomes triple
- remains the same.
- halved.

**Q.No.16**: For oscillatory motion of a simple pendulum, restoring force is: (***2022 Karachi Board***)

- mg sin θ
- mg cos θ
- mg tan θ
- mg

**Q.No.17**: A particle is executing simple harmonic motion, particle has maximum velocity when it is: (***2025 Karachi Board***)

- At maximum displacement.
- At equilibrium position.
- At half amplitude.
- At one quarter amplitude.

## Answers:

(1) $T_A = T_B$ and $E_A > E_B$
(2) Its length should be quadrupled.
(3) $2\pi A / t$
(4) T/2
(5) Square of amplitude of motion.
(6) Decrease.
(7) Forced oscillations.
(8) "$\omega_N = \omega_d$"
(9) All of these.
(10) An amplitude - frequency graph.
(11) A parabola
(12) The wheel of an automobile.
(13) 5°.
(14) Both length and acceleration due to gravity
(15) remains the same.
(16) mg sin θ
(17) At equilibrium position.

## Numericals:

***Q.No.1***: **The period of oscillation of an object in an ideal spring and mass system is 0.50 s and the amplitude is 5.0 cm.**

**What is the speed at the equilibrium point? and the acceleration at the point of maximum extension of the spring?**

***Data***:

Time period $T = 0.50$ s
Amplitude $x_o = 5.0$ cm. $= 0.05$m
Speed at equilibrium $v = ?$
Acceleration at maximum extension $a = ?$

***Solution***:

Speed of a body executing SHM is maximum while passing through its equilibrium position and is given by:

$$v = \frac{2\pi x_o}{T} = \frac{2 \times 3.142 \times 0.05}{0.5}$$

$= \mathbf{0.6283\ m/s}$ OR $\mathbf{62.83\ cm./s}$

Acceleration of a body executing SHM is max. at its max. displacement position and is given by:

$a_{max.} = -\omega^2 x_o$ where $\omega = \frac{2\pi}{T}$

Negative sign shows that acceleration of the body is always directed towards its equilibrium position. Magnitude of $a_{max}$ is:

$\therefore$ $a_{max} = (2\pi / T)^2 \times 0.05$

$\mathbf{a_{max} = 7.9\ m/s^2}$.

***Q.No.2***: **A sewing machine needle moves with a rapid vibratory motion, like SHM. As it sews a seam. Suppose the needle moves 8.4 mm from its highest to its lowest position and it makes 24 stitches in 9.0 s. What is the maximum needle speed?**

***Data***:

Distance between highest & lowest points $= 8.4$ mm.
Number of stitches $N = 24$
Time taken for 24 stitches $t = 9.0$ s.
Maximum needle speed $V_{max.} = ?$

***Solution***:

Amplitude "$x_o$" of vibrating needle 'A' is the distance between the highest & lowest points, $x_o = \frac{8.4}{2}$ mm. $= 4.2$ mm $= 0.0042$ m

Time period (in this case time to complete one stitch)

$$T = \frac{\text{time to complete N stitches}}{\text{Total number of stitches N}}$$

$$T = \frac{9}{24} = 0.375 \text{ s.}$$

Speed of any body executing simple harmonic motion is maximum when it is passing through its mean position while coming from extreme position (in this case highest point), it is given by:

$$V_{max.} = \frac{2\pi x_o}{T} = \frac{2\pi \times 0.0042}{0.375}$$

$V_{max.} = \mathbf{0.070\ m/s = 7.0\ cm./s}$

Maximum speed of the needle is **0.07 m/s** or **7.0 cm./s**.

***Q.No.3***: **An ideal spring with spring constant of 15 N/m is suspended vertically. A body of mass 0.60 kg. is attached to unstretched spring and released.**

**(a) What is the extension of the spring when the speed is maximum?**

**(b) What is the maximum speed?**

***Data***:

Spring constant $k = 15$ N/m.
Mass of suspended body $m = 0.6$ kg.
(a) Extension of spring $x_o = ?$
(b) Maximum speed $V_{max.} = ?$

***Solution***:

(a) Extension force $F = k x_o$

$mg = k x_o$ (F = mg weight of body)

$\therefore$ $x_o = \frac{mg}{K} = \frac{0.6 \times 9.8}{15} = \mathbf{0.392\ m.}$

(b) Maximum speed is given by:

$$V_{max.} = \sqrt{\frac{k}{m}}\, x_o$$

$\therefore$ $V_{max.} = \sqrt{\frac{15}{0.6}} \times 0.392$

$V_{max.} = \sqrt{25} \times 0.392 = 5 \times 0.392$

$V_{max.} = \mathbf{1.96\ m/s}$ or $\mathbf{2\ m/s.}$

Extension is **0.392 m** and maximum speed is **1.96 m/s** or **2 m/s.**

***Q.No.4*: A body is suspended vertically from an ideal spring of spring constant 2.5 N/m. The spring is initially in its relaxed position. The body is then released and oscillates about its equilibrium position. The motion is described by**

**y = (4.0 cm) sin [(0.70 rad/s) t ].**

**What is the maximum kinetic energy of the body.**

***Data*:**

Spring constant k = 2.5 N/m.

Maximum kinetic energy of body K.E= ?

Motion of body described by

y = (4.0 cm) sin [(0.70 rad/s) t ].

Amplitude of oscillations $x_o$ = 4.0 cm. = 0.04 m

***Solution*:**

Maximum kinetic energy of the vibrating body is given by:

$K.E_{max.} = \frac{1}{2} k x_o^2$

$\therefore$ $K.E_{max.} = \frac{1}{2} \times 2.5 \times (0.04)^2 =$ **0.002 J**.

Maximum kinetic energy of the vibrating body is **0.002 J** or **2 mJ**. (milli joules).

***Q.No.5*: The period of oscillation of a simple pendulum does not depend on mass of the bob. By contrast the period of a mass-spring system does depend on mass. Explain the apparent contradiction.**

***Solution*:**

Time period of oscillations of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Time period of oscillations of a mass attached to an elastic spring is given by:

$$T = 2\pi\sqrt{\frac{m}{K}}$$

**These equations show that:**

- At a given location (for given "g") time period "T" of a simple pendulum depends upon its length "L"but it is independent of its mass "m".

**But**

- Time period "T" of a mass-spring system depends upon mass "m" of the vibrating body and upon spring constant "K" of the spring.

***Q.No.6*: What is the period of a simple pendulum of a 6.0 kg mass oscillating on a 4.0 m long string?**

***Data*:**

Mass of pendulum m = 6.0 kg.

Length of string L = 4.0 m.

Time period T =?

***Solution*:**

Time period of oscillations of a pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T = 2\pi\sqrt{\frac{4}{9.8}} \quad \text{squaring both sides:}$$

$$T^2 = 4\pi^2 \frac{4}{9.8} = 16.1136$$

$\therefore$ $T = \sqrt{16.1136} =$ **4.01 s.**

Time period of the pendulum is **4.01 s.**

***Q.No.7*: A pendulum of length 75 cm and mass 2.5 kg swings with a mechanical energy of 0.015 J. What is the amplitude?**

***Data*:**

Length of pendulum L =75 cm.= 0.75 m

Mass of pendulum m = 2.5 kg.

Energy of pendulum E = 0.015 J

Amplitude of pendulum $x_o$ = ?

***Solution*:**

Amplitude "$x_o$" of vibrations of the pendulum in terms of its energy can be found using formula for energy of mass-spring system (as both execute SHM). Energy "E" of a vibrating system is given by:

$E = \frac{1}{2} K x_o^2$ ,,,,,,,,,,,,,,,,, (*i*)

But comparing formula for "T" of a simple pendulum with a formula for "T" for mass-spring system we find that:

$$\sqrt{\frac{m}{K}} = \sqrt{\frac{L}{g}}$$

Squaring both sides and re-arranging we get:

$$K = \frac{m\,g}{L} = \frac{2.5 \times 9.8}{0.75} = \mathbf{32.67\ N/m.}$$

Substituting the values of known quantities in equation (*i*) we get:

$0.015 = \frac{1}{2} \times 32.67 \times x_o^2$

$$x_o^2 = \frac{2 \times 0.015}{32.67} = 9.1827\times10^{-4}$$

$x_o = \sqrt{9.1827\times10^{-4}}$ = **0.030 m**. or **3.0 cm**

🗣Amplitude "$x_o$" of vibrating pendulum is **0.030 m**. or **3.0 cm.**

***Q.No.8*: A pendulum of length "$L_1$" has a period of $T_1$ = 0.950 s. length of the pendulum is adjusted to a new value "$L_2$" such that $T_2$ = 1.00 s. What is the ratio $L_2/L_1$**

***Data*:**

Time period of pendulum $T_1$ = 0.95 s (of length $L_1$ before adjustment)

Time period of pendulum $T_2$ = 1.00 s (of length $L_2$ after adjustment)

Ratio of lengths $L_2/L_1$ =?

***Solution*:**

Time period of pendulum before adjustment:

$$T_1 = 2\pi\sqrt{\frac{L_1}{g}}$$

Time period of pendulum after adjustment:

$$T_2 = 2\pi\sqrt{\frac{L_2}{g}}$$

Dividing $T_2$ by $T_1$,

$$\frac{T_2 = 2\pi\sqrt{\frac{L_2}{g}}}{T_1 = 2\pi\sqrt{\frac{L_1}{g}}}$$

* $$\frac{T_2}{T_1} = \sqrt{\frac{L_2}{L_1}}$$

Squaring both sides we get:

$$\frac{L_2}{L_1} = \frac{T_2^2}{T_1^2}$$

$$\frac{L_2}{L_1} = \frac{(1.0)^2}{(0.95)^2} = \frac{1}{0.9025} = \mathbf{1.108}$$

🗣The ratio of lengths ($L_2/L_1$) is **1.108**.

***Q.No.9*: Calculate the length of second's pendulum on the surface of moon where acceleration due to gravity is 0.167 times that on the earth's surface.**(*1995 Karachi Board*)

***Data*:**

Acceleration due to gravity on the surface of moon $g_m$ =0.167 g

Time period of second's pendulum T = 2 sec.

Length of second's pendulum on Moon L=?

***Solution*:**

Time period on the surface of moon is given by

$$T = 2\pi\sqrt{\frac{L}{g_m}}$$

$$2 = 2\pi\sqrt{\frac{L}{0.167\times g}}$$

Squaring both sides we get:

$$4 = 4\pi^2\frac{L}{0.167\times 9.8}$$

$$4 = 4\pi^2\frac{L}{1.6366}$$

$$L = \frac{4\times1.6366}{4\pi^2}$$ ***L = 0.166 m =16.6 cm***

🗣Length of second's pendulum on the surface of moon is ***0.166 m.*** OR ***16.6 cm.***

***Q.No.10*: Calculate the length of second's pendulum at a place where g= 10 m/s².** (*1997 Karachi Board*)

***Data*:**

Time period of second's pendulum T = 2 sec.

Acceleration due to gravity g =10m/s²

Length of second's pendulum L=?

***Solution*:**

Time period of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$2 = 2\pi\sqrt{\frac{L}{10}}$$

Squaring both sides we get:

$$4 = 4\pi^2\frac{L}{10} \quad \text{OR} \quad L = \frac{4\times10}{4\pi^2}$$

***L = 1.01 m.***

🗣Length of second's pendulum is ***1.01 m.*** OR ***101 cm.***

***Q.No.11*: A body of 0.5 kg. attached to a spring is displaced from its equilibrium position and released. If the spring constant is 50 N/m, find: (*i*) Time period. (*ii*) The frequency.**(*1998 Karachi Board*)

***Data*:**

| | |
|---|---|
| Mass of the body | m = 0.5 kg |
| Force constant | K = 50 N/m |
| (*i*) Time period | T =? |
| (*ii*) Frequency | *f* =? |

***Solution*:**

(*i*) Time period of oscillations of a mass attached to an elastic spring is given by:

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$T = 2\pi\sqrt{\frac{0.5}{50}}$$

$$T = 2\pi\sqrt{0.01} = 2\pi \times 0.1$$

$$\boxed{T = 0.63 \text{ sec.}}$$

(*ii*) $f = \frac{1}{T} = \frac{1}{0.63}$

$$\boxed{f = 1.59 \text{ Hz.}}$$

***Q.No.12*: Find the length of second's pendulum on planet Jupiter where the value of 'g' is 2.63 times the value of 'g' on the surface of the earth.**

(***2000 Karachi Board***)

***Data*:**

Length of the pendulum L =?
Time period of second's pendulum T=2s
Acceleration due to gravity $g_J = 2.63g_e$

***Solution*:**

Time period of a pendulum on the surface of Jupiter is given by:

$$T = 2\pi\sqrt{\frac{L}{g_J}}$$

$$2 = 2\pi\sqrt{\frac{L}{2.63 \times g_e}} \quad \text{squaring both sides}$$

$$4 = 4\pi^2 \frac{L}{2.63 \times 9.8}$$

$$L = \frac{4 \times 2.63 \times 9.8}{4\pi^2} \qquad \boxed{L = 2.61 \text{ m.}}$$

Length of second's pendulum on the surface of Jupiter is ***2.61 m***.

***Q.No.13*:A body of mass 0.5 kg. is attached to the end of a spring placed on a smooth horizontal table and is performing SHM.**

**Find the acceleration of the body when it has displacement of 0.6 m. The spring constant of the spring is 150 N/m.**

(***2001 Karachi Board***)

***Data*:**

Mass of the body m = 0.5 kg.
Acceleration of the body a = ?
Displacement $x = 0.6$ m
Spring constant K= 150 N/m.

***Solution*:**

Acceleration of a body attached to a spring executing SHM is given:

$a = -\frac{K}{m}x$ where negative sign shows that 'a' is always towards the mean position. Hence the magnitude of acceleration is given by:

$$a = \frac{K}{m}x = \frac{150 \times 0.6}{0.5} \qquad \boxed{a = 180 \text{ m/s}^2.}$$

***Q.No.14*: Compute the acceleration due to gravity on the surface of the moon where a simple pendulum 1.5 m long makes 100 vibrations in 605 seconds.**

(***2002 Karachi Board***)

***Data*:**

Length of the pendulum L = 1.5m
Number of vibrations = 100
Time taken = 605 sec.
Time period $T = \frac{605}{100} = 6.05$ s
Acceleration due to gravity on the surface of moon $g_m$ = ?

***Solution*:**

Acceleration due to gravity on the surface of moon is given by:

$$T = 2\pi\sqrt{\frac{L}{g_m}}$$

Squaring both sides we get:

$$T^2 = 4\pi^2 \frac{L}{g_m}$$

$$(6.05)^2 = 4\pi^2 \times \frac{1.5}{g_m}$$

$$g_m = \frac{4\pi^2 \times 1.5}{(6.05)^2} = \frac{59.218}{36.6025}$$

$$\boxed{g_m = 1.618 \text{ m/s}^2}$$

***Q.No.15*: A mass at the end of a spring oscillates with a simple harmonic motion with a period of 0.40 sec. Find the acceleration when the displacement is 4.0 cm.** (***2003 Karachi Board***)

***Data*:**

Time period T = 0.40 sec.
Displacement $x$ = 4.0 cm= 0.04 m
Acceleration a =?

***Solution***:

$$T = 2\pi\sqrt{\frac{m}{K}}$$

Squaring both sides we get:

$$(0.40)^2 = 4\pi^2 \frac{m}{K}$$

$$0.16 = 4\pi^2 \frac{m}{K}$$

$$\frac{m}{K} = \frac{0.16}{4\pi^2}$$

But $F = m\,a = K\,x$

$$\frac{m}{K} = \frac{x}{a}$$

$$\therefore \quad 0.16 = 4\pi^2 \frac{x}{a}$$

$$a = \frac{4\pi^2 \times x}{0.16} = \frac{4\pi^2 \times 0.04}{0.16}$$

$$\therefore \quad \boxed{a = 9.87\ m/s^2.}$$

***Q.No.16***: **The period of vibration of a body of mass 25 gm. attached to a spring, vibrating on a smooth horizontal surface, when it is displaced 10 cm to the right of its extreme position, is 1.57 sec. and the velocity at the end of the displacement is 0.4 m/s. Determine the (a) spring constant (b)Total energy and (c) amplitude.**
(***2004 Karachi Board***)

***Data***:

| | |
|---|---|
| Mass of body | m =25gm=0.025 kg |
| Displacement | $x$ =10 cm=0.10 m |
| Time period | T=1.57 sec |
| Velocity | v = 0.4 m/s |
| Spring constant | K =? |
| Total energy | E =? |
| Amplitude | $x_o$=? |

***Solution***:

Time period of a mass attached to an elastic spring executing S.H.M is given by:

$$T = 2\pi\sqrt{\frac{m}{K}}$$ squaring both sides

$$T^2 = 4\pi^2 \frac{m}{K}$$

$$(1.57)^2 = 4\pi^2 \frac{0.025}{K}$$

$$K = \frac{4\pi^2 \times 0.025}{(1.57)^2} \qquad \boxed{K = 0.4\ N/m}$$

Spring constant is ***0.4 N/m***.

Energy of the system at any instant is partially K.E and partially P.E, hence:

Total energy E = K.E + P.E

$$E = \tfrac{1}{2} m v^2 + \tfrac{1}{2} K x^2$$

$$E = \tfrac{1}{2}\times 0.025\times(0.4)^2 + \tfrac{1}{2}\times 0.4\times(0.1)^2$$

$$E = 2\times10^{-3} + 2\times10^{-3}$$

$$\boxed{E = 4\times10^{-3}\ J.}$$

Energy of the system is ***4×10⁻³J***.

But $E = \tfrac{1}{2} k\, x_o^2$

$$\therefore \quad 4\times10^{-3} = \tfrac{1}{2}\times 0.4\, x_o^2$$

$$x_o^2 = \frac{4\times10^{-3}\times 2}{0.4} = 0.02$$

$$\therefore \quad \boxed{x_o = 0.1414\ m.}$$

Amplitude of oscillation is ***0.1414 m***

***Q.No.17***: **A body of mass 32 gm. attached to an elastic spring is performing SHM. Its velocity is 0.4 m/s when the displacement is 8 cm. towards right. If the spring constant is 0.4 N m⁻¹, Calculate: (*i*) Total energy. (*ii*) The amplitude of its motion**
(***2008,2000,1995,93 Karachi Board***)

***Data***:

| | |
|---|---|
| Mass | m = 32 gm.= 0.032 kg |
| Velocity | v = 0.4 m/s |
| Displacement | $x$ = 8 cm. = 0.08m |
| Spring constant | K = 0.4 N m⁻¹ |
| (*i*) Total energy | E = ? |
| (*ii*) The amplitude of motion | $x_o$= ? |

***Solution***:

Total energy of mass attached to an elastic spring and executing SHM is given by:

$$E = K.E + P.E = \tfrac{1}{2} mv^2 + \tfrac{1}{2} K x^2$$

$$= \tfrac{1}{2}\times 0.032\times(0.4)^2 + \tfrac{1}{2}\times 0.4\times(0.08)^2$$

$$= 2.56\times10^{-3} + 1.28\times10^{-3}$$

$$\therefore \quad \boxed{E = 3.84\times10^{-3}\ J}$$

But Total energy $E = \tfrac{1}{2} k x_o^2$

$$\therefore \quad 3.84\times10^{-3} = \tfrac{1}{2}\times 0.4\times x_o^2$$

$$x_o^2 = \frac{2\times 3.84\times10^{-3}}{0.4} = 0.0192$$

$$\therefore \quad x_o = \sqrt{0.0192} \qquad \boxed{x_o = 0.1386\ m}$$

The amplitude of vibrations is ***0.1386m.***

***Q.No.17***: **A sonometer wire of length 1m, when plucked at the center, vibrates with a frequency of 256 Hz.**

**Calculate the wavelength and the speed of the waves in the wire.**

(***2009, 2005 Karachi Board***)

***Data***:

| | |
|---|---|
| Length of the wire | L = 1m |
| Frequency | $f$ = 256 Hz. |
| Wave length of wave | $\lambda$ = ? |
| Speed of waves | v = ? |

***Solution***:

When a wire is plucked from its center it vibrates with the fundamental frequency, forming one loop. Hence wave length of the wave formed in it:

$\lambda = 2L = 2\times 1$ $\boxed{\lambda = 2\ m}$

Similarly speed of wave:

$v = f\lambda = 256 \times 2$ $\boxed{v = 512\ m/s}$

***Q.No.18***: **A string 2 m long and a mass of 0.004 kg. is stretched horizontally by passing one of its ends over a pulley and the string is attached with one kg. mass to it vertically.**

**Find the speed of the transverse wave on the string and the frequency of the fundamental and fifth harmonic at which the string vibrates.**

(***2010,02,03 Karachi Board***)

***Data***:

| | |
|---|---|
| Length of the string | L = 2 m |
| Mass of the string | m= 0.004 kg |
| Mass suspended | M = 1 kg |
| Speed of the wave | V = ? |
| Fundamental frequency | $f_1$ =? |
| Frequency of the fifth harmonic | $f_5$ = ? |

***Solution***:

Speed of the transverse wave on a string is given by:

$V = \sqrt{\dfrac{TL}{m}}$ Since T = Mg

$\therefore$ $V = \sqrt{\dfrac{MgL}{m}} = \sqrt{\dfrac{1\times 9.8\times 2}{0.004}}$

$V = \sqrt{\dfrac{19.6}{0.004}} = \sqrt{4900}$

$\boxed{V = 70\ m/s.}$

Speed of the transverse waves on the string is ***70 m/s***.

Fundamental frequency is given by:

$f_1 = \dfrac{V}{2L}$

$f_1 = \dfrac{70}{2\times 2}$ $\boxed{f_1 = 17.5\ Hertz}$

Fundamental frequency is ***17.5 Hertz***.

Frequency for n segments is given by:

$f_n = n\dfrac{V}{2L}$ or $f_5 = 5\times\dfrac{70}{2\times 2}$

$\boxed{f_5 = 87.5\ Hertz}$

(Or $f_5 = 5 f_1 = 5\times 17.5 = 87.5$ Hertz)

Frequency for 5th harmonic is ***87.5 Hertz***.

***Q.No.19***: **A simple pendulum completes 4 vibrations in 8 seconds on the surface of the earth. Find the time period on the surface of the moon where the acceleration due to gravity is one sixth of that on the earth.**(***2010,09,07,1993Karachi Board***)

***Data***:

Time taken for 4 vib. t = 8 sec.

Time period on earth $T_e = \dfrac{8}{4}$ s = 2 s.

Time period on moon $T_m$ = ?

Acceleration due to gravity on moon

$g_m = 1/6\ g_e$

***Solution***:

Time period of a pendulum on the surface of earth is given by:

$T_e = 2\pi\sqrt{\dfrac{L}{g_e}}$

OR $2 = 2\pi\sqrt{\dfrac{L}{9.8}}$

Squaring both sides we get:

$4 = 4\pi^2\dfrac{L}{9.8}$ $\therefore$ $L = \dfrac{4\times 9.8}{4\pi^2} = 0.99$ m

Time period on the surface of the moon:

$T_m = 2\pi\sqrt{\dfrac{L}{g_m}}$

$T_m = 2\pi\sqrt{\dfrac{0.99}{1/6\ g_e}}$ OR $T_m = 2\pi\sqrt{\dfrac{0.99}{1.633}}$

$T_m = 2\pi\times 0.7785$ $\boxed{T_m = 4.89\ sec}$

Time period of the given pendulum on the surface of moon is ***4.89 sec***.

**Q.No.20**: **A string 1 m long and of mass 0.004 kg. is stretched with a force.**

**Calculate the force if the speed of the wave in the string is 140 m/s.**

(***2012 Karachi Board***)

**Data**:

Length of the string $L = 1$ m.
Mass of the string $m = 0.004$ kg
Speed of the wave $V = 140$ m/s
Stretching force = tension in the string $T = ?$

**Solution**:

Speed of a wave in a string is given by:

$$V = \sqrt{\frac{T\,L}{m}}$$

$$140 = \sqrt{\frac{T \times 1}{0.004}}$$

Squaring both sides, we get:

$$(140)^2 = \frac{T}{0.004}$$

$$19600 = \frac{T}{0.004}$$

$$T = 19600 \times 0.004 \qquad \boxed{T = 78.4\ N}$$

**Q.No.21**: **A body hanging from a spring is set into motion and the period of oscillation is found to be 0.5 sec. After the body has come to rest, it is removed. How much shorter will the spring be when it comes to rest.**

(***2015, 2013, 02 Karachi Board***)

**Data**:

Time period $T = 0.5$ s.
Decrease In length $x = ?$

**Solution**:

Force constant:

$$k = \frac{F}{x} = \frac{m\,g}{x}$$

But the time period of the oscillating body attached to a spring is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Therefore, $$T = 2\pi\sqrt{\frac{m}{m\,g/x}}$$

$$T = 2\pi\sqrt{\frac{x}{g}}$$

Squaring both sides we get:

$$T^2 = 4\pi^2 \frac{x}{g}$$

$$x = \frac{T^2 g}{4\pi^2}$$

$$x = \frac{(0.5)^2 \times 9.8}{4\pi^2} \qquad \boxed{x = 0.062\ m}$$

The spring will be shorter by ***0.062 m*** after the oscillating suspended body has been removed from it.

**Q.No.22**: **A 100 cm long string vibrates in 4 loops at 50 Hz. The linear density of the string is $4\times10^{-4}$ gram./cm.**

**Calculate the tension in the string.**

(***2017 Karachi Board***)

**Data**:

Length $L = 100$ cm $= 1$ m
Frequency $f_4 = 50$ Hz.
Number of loops $= 4$
Linear density $\mu = 4\times10^{-4}$ g/cm
$= 4\times10^{-4}\times\frac{100}{1000}$ kg/m $= 4\times10^{-5}$ kg/m
Tension $T = ?$

**Solution**:

Since $f_4 = 4 f_1$

$\therefore \quad f_1 = f_4 / 4 = 50 / 4 = 12.5$ Hz.

But $\quad f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$$12.5 = \frac{1}{2\times1}\sqrt{\frac{T}{4\times10^{-5}}}$$

$$25 = \sqrt{\frac{T}{4\times10^{-5}}}$$ squaring both sides, we get

$$(25)^2 = \frac{T}{4\times10^{-5}}$$

$$T = 625 \times 4 \times10^{-5} \qquad \boxed{T = 0.025\ N}$$

***Alternate method***:

Speed v of waves is given by:

$$v = \lambda f = 0.5 \times 50 = 25 \text{ m/s}$$

But speed of stationary waves in a stretched string is given by:

$$v = \sqrt{\frac{T}{\mu}} \qquad v^2 \times \mu = T$$

$$T = (25)^2\times 4 \times10^{-5}$$

$$T = 625 \times 4 \times10^{-5} \qquad \boxed{T = 0.025\ N}$$

Tension in the string is ***0.025 N.***

***Q.No.23***: **A 15 kg. block is suspended by a spring of spring constant $5\times10^3$ N/m.**

**Calculate the frequency of vibrations of the block displaced from its equilibrium position and then released.**

(***2018 Karachi Board***)

***Data***:

m = 15 kg. k = $5\times10^3$ N/m. $f$ = ?

***Solution***:

Time period of the vibrating block:

$T = 2\pi\sqrt{\frac{m}{k}}$ $T = 2\pi\sqrt{\frac{15}{5\times10^3}}$

But $f = \frac{1}{T}$ $\therefore$ $f = \frac{1}{2\pi}\sqrt{\frac{5\times10^3}{15}}$

$\therefore$ $f = \frac{1}{2\pi}\sqrt{333.3}$ $f = \frac{18.26}{2\pi}$

$$\boxed{f = 2.91\ \text{Hz.}}$$

***Q.No.24***: **A string 2 m long and of mass 0.004 kg. is stretched horizontally by passing one end over a pulley and attaching a 1 kg mass to it.**

**Find the speed of transverse wave on the string and the frequency of the second harmonic.**

(***2019, 2016 Karachi Board***)

***Data***:

| | |
|---|---|
| Length of the string | L = 2 m |
| Mass of the string | m =0.004 kg |
| Mass suspended | M = 1 kg |
| Speed of the wave | V = ? |

Frequency of the second harmonic $f_2$=?

***Solution***:

Speed of the transverse wave on a string is given by:

$V = \sqrt{\frac{T\,L}{m}}$ Since T = Mg

$\therefore$ $V = \sqrt{\frac{M\,g\,L}{m}} = \sqrt{\frac{1\times 9.8\times2}{0.004}}$

$V = \sqrt{\frac{19.6}{0.004}} = \sqrt{4900}$ $\boxed{V = 70\ m/s.}$

Speed of the transverse waves on the string is ***70 m/s.***

Frequency for n segments is given by:

$f_n = n\ \frac{V}{2\,L}$ or $f_2 = 2\times\frac{70}{2\times2}$

$$\boxed{f_2 = 35\ \text{Hertz}}$$

Frequency for second harmonic is ***35 Hertz.***

***Q.No.25***: **A mass at the end of spring oscillates with a period of 0.4 sec. Find the acceleration when displacement is 6 cm.**

(***2019, 2003 Karachi Board***)

***Data***:

| | |
|---|---|
| Time period | T = 0.4 sec. |
| Displacement | $x$ = 6 cm = 0.06 m |
| Acceleration | a =? |

***Solution***:

$T = 2\,\pi\sqrt{\frac{m}{K}}$

Squaring both sides we get:

$(0.4)^2 = 4\,\pi^2\,\frac{m}{K}$ $0.16 = 4\,\pi^2\,\frac{m}{K}$

$\frac{m}{K} = \frac{0.16}{4\,\pi^2}$

But $F = m\,a = K\,x$

$\frac{m}{K} = \frac{x}{a}$

$\therefore$ $0.16 = 4\,\pi^2\,\frac{x}{a}$

$a = \frac{4\,\pi^2\times x}{0.16}$ or $a = \frac{4\,\pi^2\times 0.06}{0.16}$

$\therefore$ $\boxed{a = 14.80\ m/s^2.}$

***Q.No.26***: **A body of mass 5 gram oscillates as a simple pendulum. Calculate the time period of the pendulum if its length is 1.2 m.**

(***2025 Karachi Board***)

***Data***:

Mass of the body m = 5 gm.

Length of the pendulum L = 1.2 m

Time period T = ?

***Solution***:

Time period of a simple pendulum is independent of its mass, it is given by:

$T = 2\,\pi\sqrt{\frac{L}{g}} = 2\,\pi\sqrt{\frac{1.2}{9.8}}$

Squaring both sides we get:

$T^2 = 4\,\pi^2\,\frac{1.2}{9.8} = 4.834$

$T = \sqrt{4.834}$ = **2.198 sec.**

***Unit.12*****.....** ✍

# Acoustics

## Sound:

***Q.No.1*****: *What is sound?***

***Ans*****:** Sound is a ***form of energy*** which produces sense of hearing.

OR

Sound is a ***form of energy*** which produces a sensation in our auditory system.

Sound travels in the form of ***longitudinal, mechanical waves.***

***Longitudinal waves*** are those waves which **require a material medium** (solid, liquid or a gas) for its propagation.

Mechanical waves **cannot travel** through vacuum.

A mechanical wave originates in the displacement of some portion of an elastic medium, the disturbance is transmitted from one layer to the next. In this way the wave propagates through the medium. The medium itself **does not** move as a whole along with the wave.

Sound waves transfer energy from the vibrating body (source of sound) to the surface on which it falls (e.g. ear drum).

Since sound waves cannot travel through vacuum and there is vast vacuum between the earth and the sun, hence explosions taking place on the surface of the sun **cannot be heard** on the earth.

Similarly, astronauts communicate with each other on the surface of the moon through microphone and a receiver attached to their helmet. This microphone converts sound waves into electromagnetic waves (which **do not require a medium**) and the receiver converts them back into sound waves. (There is no atmosphere i.e. no medium on the moon).

***Q.No.2*****: *What is audible sound*?**

***Ans*****:** Human ear responds to sound waves of a limited frequency range.

Sound which can be heard by a normal human ear is called **audible sound**. Normal human ear can hear sound of frequency ***between 20 Hz. and 20,000 Hz*** (or ***20 kHz***.), hence sound of this frequency range is called **audible sound** and this range of frequencies is called **audible frequency range**.

***Q.No.3*****: *What are infrasonic waves*?**

***Ans*****:** Sound waves (Longitudinal waves) of frequency ***less than 20 hz.*** are called infrasonic waves.

***Q.No.4*****: *What are ultrasonic waves*?**

***Ans*****:** Longitudinal waves whose frequency is ***more than 20 kHz*** are called ultrasonic waves.

Normal human ear cannot detect infrasonic and ultrasonic waves; few animals such as whales and bats communicate with each other using ultrasonic waves.

Nowadays ultrasonic waves are widely used in medical field as a diagnostic tool (ultrasound).

# Speed of sound:

***Q.No.1: What are the factors on which does speed of sound in a medium depend?***

***Ans:*** Sound waves travel in the form of **longitudinal compressional** waves. Hence a compressible medium is needed for the propagation of sound waves. Since compressibility of a medium depends upon ***elasticity*** of the medium, therefore, speed of sound depends upon ***elastic*** as well as ***inertial properties*** of the medium.

Speed of sound or any mechanical wave in an elastic medium is basically:

$$\text{Speed of sound} = \sqrt{\frac{\text{Modulus of elasticity of the medium}}{\text{Density of the medium}}}$$

$$v = \sqrt{\frac{E}{\rho}}$$

Where 'E' is the ***modulus of elasticity*** of the medium (an elastic property) and '$\rho$' is the ***density*** (an inertial property) of the medium.

# Newton's formula for speed of sound:

***Q.No.2: Give Newton's formula for speed of sound.***
***On what basic assumption does Newton's formula for speed of sound depend?***

***Ans:*** Newton assumed that when sound waves travel through a medium the temperature of the surrounding medium ***does not change***. A process in which temperature remains constant is called ***isothermal process***.

***In other words***, according to Newton sound waves travel from one point to another under ***isothermal conditions*** i.e. during the rapid production of compressions and rarefactions temperature of the surrounding air ***does not change***.

Speed of sound in a gas depends upon its bulk modulus "B", and under isothermal conditions the bulk modulus "B" of a gaseous medium such as air, is equal to its pressure "P". Therefore, the speed of sound in air is given by:

$$v = \sqrt{\frac{P}{\rho}}$$

This formula for speed of sound is known as ***Newton's formula***.

But for mercury barometer $P = \rho g h$ (note that $\rho$ is the density of mercury)

**Speed of sound in air as calculated by Newton's formula:**

Speed of sound in air is given by:

$$v = \sqrt{\frac{\rho_{mercury}\, g\, h}{\rho_{medium\ (air)}}}$$

**To calculate** speed of sound by Newton's formula at standard temperature and pressure (STP). We know that normal atmospheric pressure is enough to support 76 cm. column of mercury in a barometer. The corresponding atmospheric pressure will be:

$$P = \rho_{mercury}\, g\, h$$

Where $\rho_{mercury}$ = 13.6 gm/cm$^3$, g = 980 cm./s$^2$ and h = 76 cm. (in CGS units)

$\therefore$ P = 13.6 × 980 × 76 = **1,012,928 dyne/cm²**.

Density of air at STP is $\rho_{air} = 1.293\times10^{-3}$ gm./m$^3$.

Substituting these values in Newton's formula we get:

$$\upsilon = \sqrt{\frac{1{,}012{,}928}{1.293\times10^{-3}}} = \mathbf{27989.17\ cm./s = 279.8917\ m/s.}$$

Speed of sound in air by Newton's formula comes out to be **279.8917 m/s.** Speed of sound at STP as measured accurately by different experiments is about **320 m/s.**

Speed of sound as calculated by Newton's formula, even with highest degree of accuracy, gave a lesser value as compared to experimentally determined value.

# ***Laplace's correction:***

***Q.No.3: How did Laplace correct Newton's formula for speed of sound? What is the basic assumption on which does the Laplace's correction to Newton's formula for speed of sound depend?***

***Ans:*** The basic assumption on which Laplace's correction to Newton's formula for speed of sound is based is that the propagation of sound takes place under ***adiabatic conditions***, i.e. during the rapid production of compressions and rarefactions, temperature of the surrounding air ***does not remain constant***.

According to Laplace, compressions and rarefactions are produced so **rapidly** and that the **thermal conductivity** of the air is so **poor** (Air is a bad conductor of heat) that the temperature of the surrounding air **does not remain constant**. Due to which propagation of sound through air takes place under ***adiabatic conditions***. Under adiabatic conditions, Bulk modulus "B" of a gaseous medium such as (air) is equal to "γ P". Therefore,

$$\boxed{\upsilon = \sqrt{\frac{\gamma P}{\rho}}}$$

Where $\gamma = \dfrac{\text{molar specific heat at constant pressure}}{\text{molar specific heat at constant volume}} = \dfrac{C_p}{C_V}$, for air $\gamma = 1.41$

***Q.No.4: Derive Laplace's formula for speed of sound in air?***

***Ans:*** According to Laplace, propagation of sound through air takes place under ***adiabatic conditions***.

Under adiabatic conditions, Bulk modulus 'B' of a gaseous medium such as (air) is equal to γ P. Where 'γ' is the ratio of molar specific heats of a gas at constant pressure '$C_p$' and at constant volume '$C_V$'.

Therefore, the speed of sound in a gaseous medium is given by:

$$\boxed{\upsilon = \sqrt{\frac{\gamma P}{\rho}}}$$

This formula gives us the speed of sound in a gaseous medium such as air; it is known as ***Laplace's formula for speed of sound***.

Substituting the values of $\rho_{mercury}$ = 13.6 gm/cm$^3$, g = 980 cm./s$^2$, h = 76 cm. and $\gamma$ = 1.41 for air, Laplace's formula for speed of sound in air at STP will give us:

$$\upsilon = \sqrt{\frac{1.41\times13.6\times980\times76}{1.293\times10^{-3}}} = \mathbf{33,235.2\ cm./s = 332.352\ m/s.}$$

Speed of sound in air at STP as calculated by Laplace's formula gives us accurate value which is in agreement with experimentally determined value.

## ***Factors affecting speed of sound*:**

1. Speed of sound is affected by **density** of the surrounding air, It is inversely proportional to square root of density of the medium (air).
2. Speed of sound is affected by the presence of **moisture** in the surrounding air. Hence speed of sound in damp air (having more moisture) is greater than in dry air (having less moisture).
3. Speed of sound in air is ***not affected*** by ***pitch*** and ***loudness*** of sound.
4. It is ***not affected*** by changes in ***pressure,*** because, according to Laplace formula, speed of sound in air depends upon $\sqrt{\gamma P/\rho}$. According to Boyle's law, pressure "P" of a given mass of a gas at constant temperature is inversely proportional to its volume "V", hence if pressure of a gas is doubled its volume will reduce to half but under this condition the density "$\rho$" (mass per unit volume) of the gas will become double its initial value. ***In other words,*** if "P" is doubled "$\rho$" will also be doubled so that the ratio $\sqrt{P/\rho}$ will remain unchanged. Hence changing pressure "P" of the gas will also change its density "$\rho$" such that the speed of sound remains the ***same***.
5. Speed of sound in air ***will change*** if "$\rho$" is changed without changing pressure "P" of the gas. This happens when **temperature "T"** of the gas is changed at constant pressure. According to Charles's law; $V \propto T$ (at constant P). When temperature "T" of air increases its ***volume also increases*** (thermal expansion of gases) due to which its ***density "$\rho$" decreases***. As a result of which its ***speed will increase***. ***In other words,*** speed of sound in air ***increases*** with a ***rise in temperature***. It has been found that speed of sound in air increases by 0.61 m/s per degree rise in temperature. Hence if '$V_o$' is the speed of sound in air at 0°C then its speed '$V_T$' at 'T °C' is given by:

$$V_T = V_o + 0.61\ T$$

7. Speed of sound also changes with wind speed "$V_W$". Speed of sound as received by an observer is $(V + V_W)$ in the direction of wind and is $(V - V_W)$ against the wind.

# ***Principle of superposition of waves*:**

***Q.No.1*: *State and explain the principle of superposition of waves?***

***Ans*:** According to the principle of superposition of waves:

***"When two or more waves superpose (overlap) each other then the net displacement at any point will be equal to the algebraic sum of displacements due to individual wave at that point".***

Let $\pm y_1, \pm y_2, \pm y_3$ …….. etc. be the displacements due to first second, third, ……… etc. waves then the net displacement 'Y' at a particular point will be:

$$Y = \pm y_1 \pm y_2 \pm y_3 \pm \ldots\ldots\ldots\ldots\ldots\ldots$$

Displacement is taken as $+y$ for a crest, whereas, it is taken as $-y$ for a trough.

***In other words***, if a medium is disturbed simultaneously by two crests then the net displacement will be equal to sum of individual displacements, hence a bigger crest will be formed. Similarly, two troughs will form a deeper trough. But a crest and a trough tend to cancel each other's effect.

If amplitude of the crest is more than the amplitude of the trough then a small crest will remain during the overlapping of a crest and a trough, and vice versa.

After the overlapping, the overlapping waves continue their motion with their original amplitude and shape.

## ***Interference of sound waves***:

Two waves are said to be coherent if they are of same frequency.

Two **coherent** sound waves having the **same phase** are said to **interfere constructively** if crest of one wave overlaps the crest of other, and trough of one wave overlaps the trough of the other. Under this condition points at which the two crests overlap a **bigger crest** is formed and the points at which troughs of waves overlap a **deeper trough** is formed. This happens according to the principle of superposition of waves.

Similarly, two **coherent** sound waves having the **same phase** are said to **interfere destructively** if crest of one wave overlaps trough of the other. Under this condition, at these points **net displacement** will be equal to the difference of amplitude of waves. Under this condition displacement due to the crest of one wave tend to cancel the displacement due to trough of the other wave. This happens according to the principle of superposition of waves.

## ***Beats***:

***Q.No.1: What are beats?***

***Ans***: "The periodic variation in the loudness of sound, produced by two different sources of sound, having **slightly different frequency**, when both are tuned at the same time, is known as **beats**".

Beats are heard because of superposition of sound waves of slightly different frequencies produced by two different sources.

Beats are produced because of slow change in **phase difference** between the two waves.

To understand phenomenon of beats, let us imagine two waves at an instant when waves are in phase, at this instant, waves will interfere constructively and the net amplitude will be maximum, producing loud sound. Since frequency of one of the waves is slightly more than that of the other wave, therefore, this higher frequency wave gets ahead of the other wave due to which phase difference between the waves **increases**. Ultimately the phase difference becomes $180^{o}$ at some instant, now crest of one wave overlaps trough of the other. Now waves will interfere destructively as a result of which resultant amplitude is **minimum**. As time goes on phase difference between the two waves increases and the net amplitude keeps on increasing so that again the two waves interfere constructively. This process goes on regularly producing **changing amplitude**, due to which **beats** are heard.

***Q.No.2: What is beat frequency?***

***Ans***: Beat frequency is the number of beats heard per second.

Beat frequency is equal to the difference in frequency of the two sounds.

$$\text{Beat frequency (No. of beats heard per sec.)} = f_1 - f_2$$

Beats can be heard if the difference in frequency (or beat frequency) is ***not too large***.

The maximum number of beats per second that a normal human ear can detect is 7 beats per second. (In other words, for hearing beats the difference in frequency must <u>not be more than 7 Hertz</u>)

***Q.No.3*: What is the main cause of production of beats?**

***Ans*:** Beats are caused because of interference of sound waves of slightly different frequencies. When compressions (Or rarefactions) of the two sounds superpose each other a <u>loud sound</u> is heard but if a compression of one sound overlaps a rarefaction of the other then a <u>low sound</u> is heard.

# Standing or stationary waves:

***Q.No.1*: What are standing or stationary waves? How are they formed?**

***Ans*:** Waves which do not appear to move or appear to be stationary are called ***standing*** or ***stationary waves***.

Points at which a string vibrates with **maximum amplitude** are called ***antinodes***, whereas, the points of **zero amplitude** are called ***nodes***.

- Length of a loop is equal to length of string between two adjacent nodes and is $\lambda/2$ .

***Stationary waves*** are produced when two exactly similar waves travel in opposite direction along the same string and the conditions for superposition of waves are satisfied.

***Q.No.2*: How is one loop formed on a string under tension?**

***Ans*:** When a string under tension is plucked from its midpoint, as soon as the string is released two ***half crests*** travel along the string in opposite directions. One towards **right** and the other towards **left**. These two half crests are reflected from their respective ends as two ***half troughs***. These two half troughs meet in the middle forming a full trough.

***In other words***, after reflection ***a crest becomes a trough*** and vice versa.

The string, therefore, vibrates such that at one instant there is a crest and at the next instant there is a trough on it.

Crest and trough vibrate so rapidly that one loop seems to remain stationary.

Ends of the loop, where displacement of the string is minimum (= 0) are called ***nodes*** and the midpoint of the loop, where displacement of the string is maximum, is called ***antinode***.

Frequency with which the string vibrates so that a loop is formed on it is called ***fundamental frequency*** or ***first harmonic***.

***Q.No.3*: What is fundamental frequency or first harmonic?**

***Ans*: *Fundamental frequency*** or ***first harmonic*** is the ***minimum frequency*** required to produce a stationary wave. If frequency of waves is less than the fundamental frequency no stationary waves are produced.

# Derivation:

***Q.No.4*: Derive a formula for fundamental frequency (or frequency required to produce one loop on the string)?**

***Ans*:** Consider a string of length 'L' and of mass 'm' stretched and fixed from both ends. Let 'T' be the tension in the string. If such a string is plucked from its center a stationary loop is formed and the string is said to vibrate in **<u>one loop</u>**. Wave length of the waves when the string vibrates in one loop is given by:

$$\lambda_1 = 2\,L$$

(Length of one loop is half the wavelength $\lambda_1/2 = L$, two loops make one wave)

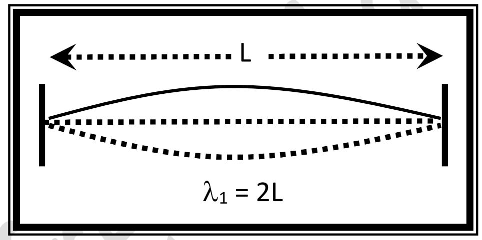
But the frequency with which a string vibrates is given by:

$$f_1 = \frac{v}{\lambda_1} \qquad \text{(For any type of waves } f\lambda = v\text{)}$$

Frequency '$f_1$' with which a string must vibrate so that one loop is formed on it, is called ***fundamental frequency*** or ***first harmonic***. It is the minimum frequency required to produce a stationary wave.

Velocity 'v' with which waves travel along a string, whatever their frequency and wave length is, is given by:

$$v = \sqrt{\frac{T\,L}{m}}$$




Therefore,

$$f_1 = \frac{1}{2\,L}\sqrt{\frac{T\,L}{m}}$$

This formula gives us the frequency required to produce one loop on the string of length 'L' and mass 'm' when the tension in it is 'T'.

***Note that*** if the string vibrates with a frequency other than as calculated by the above formula (slightly higher or lesser) then no stationary wave will be produced and therefore, no well-defined loop will be formed.

Mass per unit length $m/L = \mu$ of a string is called its ***linear density***.

If '$\mu$' is the linear density of the given string then the fundamental frequency is given by:

$$f_1 = \frac{1}{2\,L}\sqrt{\frac{T}{\mu}}$$

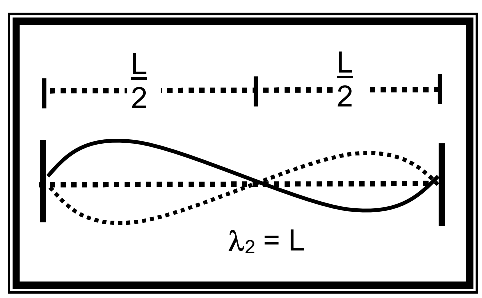
***Q.No.5: Derive a formula for frequency required to form two loops on the string (or second harmonic).***

***Ans:*** If a string is properly tuned such that two loops are formed on it then the frequency '$f_2$' with which it now vibrates is called ***second harmonic*** or ***first overtone***. It is given by:

$$f_2 = \frac{v}{\lambda_2}$$

But in this case wave length of stationary waves is $\lambda_2 = L$

(Length of each loop L/2 Therefore, $\lambda_2 = L/2 + L/2$).,




$$f_2 = \frac{1}{L}\sqrt{\frac{T\,L}{m}}$$

This equation can also be written as:

$$f_2 = \frac{2}{2\,L}\sqrt{\frac{T\,L}{m}}$$

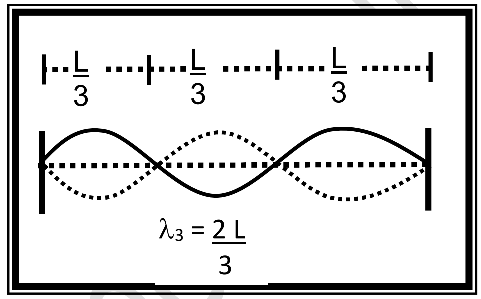
But $\quad f_1 = \dfrac{1}{2\,L}\sqrt{\dfrac{T\,L}{m}}$

Therefore, $f_2 = 2 f_1$

Hence the frequency required to produce **two loops** (or second harmonic or first overtone) is **twice** the **fundamental frequency**.

***Q.No.6*: *Derive a formula for frequency required to produce three loops on the string (or third harmonic).***

***Ans*:** If the string is properly tuned such that three loops are formed on it then the frequency '$f_3$' with which it now vibrates is called ***third harmonic*** or ***second overtone***. It is given by:




$$f_3 = \frac{v}{\lambda_3}$$

But in this case wave length of stationary waves is $\lambda_3 = \frac{2L}{3}$.(Length of each loop L/3, λ= Length of two loops)

Therefore,

$$f_3 = \frac{3}{2L}\sqrt{\frac{TL}{m}}$$

But $f_1 = \frac{1}{2L}\sqrt{\frac{TL}{m}}$

Therefore, $f_3 = 3 f_1$

Hence the frequency required to produce three loops (or third harmonic or second overtone) is three times the fundamental frequency.

***Q.No.7*: *Derive a formula for frequency required to produce any number of loops on a string.***

***Ans*:** Frequency with which a string vibrates to produce any number of loops is an integral multiple of ***fundamental frequency***. Hence frequency required to produce 'n' loops will be:

$$f_n = n f_1$$

Where $f_1 = \frac{1}{2L}\sqrt{\frac{TL}{m}}$ OR $f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

Similarly, if 'L' is length of a string then the wavelength of waves when any number of loops 'n' are formed on it is given by:

$$\lambda_n = \frac{2L}{n}$$

***Q.No.8*: *What are harmonics?***

***Ans*: *Harmonics*** are the series of frequencies required to produce stationary waves. Minimum frequency required for producing a stationary wave is called ***fundamental frequency***.

Similarly, frequencies required to produce two, three, four, ……..etc. loops are called second, third, fourth, …….. etc. harmonics.

Harmonics are integral multiples of fundamental frequency. If frequency is between two harmonics no stationary waves (i.e. no loops) are formed.

***Q.No.9*: If tension in a string is doubled what will be the effect on the speed of standing waves in a string?** ***(2010 Karachi Board)***

***Ans*:** Speed 'v' of a standing or stationary wave in a stretched string is given by:

$$v = \sqrt{\frac{TL}{m}}$$

This formula shows that speed of waves produced on a stretched string is directly proportional to $\sqrt{T}$. Hence if tension in a string is doubled then speed of stationary waves produced on it increases by $\sqrt{2}$ times its initial value.

## ***Quantization of frequencies*:**

Frequencies required to produce stationary waves on a string are always integral multiples of fundamental frequency. This fact is known as **quantization of frequencies** for stationary waves on a string.

# ***Standing waves in open and closed pipes*:**

- **Just to remined you** that standing or stationary waves are formed only due to the interference of two waves of **equal frequency** traveling in **opposite direction**. Although the interfering waves **travel / propagate** in opposite directions but the standing wave **does not travel / propagate** and it seems to be **stationary**, that is why it is called **standing** or **stationary wave**.
- Stationary waves can be produced in a string under tension, open and closed pipes etc.
- Minimum frequency required to produce a stationary wave is called **first harmonic** or **fundamental frequency.** If frequency of interfering waves is less than the fundamental frequency **no** stationary waves are produced.

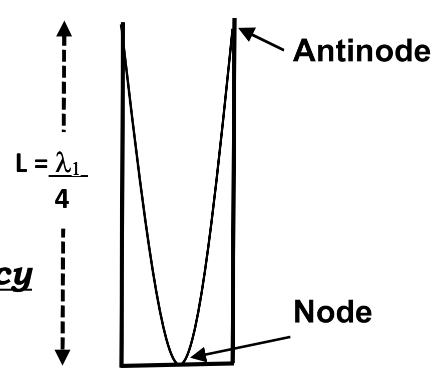
## ***Standing waves in closed pipe*:**

### ***Idea*:**

When sound waves of a particular frequency are produced in a pipe closed at one end and open at the Other, air column in the pipe starts vibrating in such a manner that stationary waves are produced, at that instant loud sound is heard. This happens only when at the open mouth an **antinode** and at the closed end a **node** is formed.

- **Node** is that point at which particles of air do not vibrate at all they have **zero amplitude** (due to **destructive interference**), whereas **antinode** is that point at which particles of air vibrate with **maximum amplitude** (due to **constructive interference**).
- When a sound wave traveling in a pipe reaches closed end of the pipe, it is reflected such that if **crest** strikes the closed end then it will be reflected as a **trough** and **trough** is reflected as a **crest**.
- A stationary wave in a pipe closed at one end **always has a node** at the **closed end** and **an antinode** at the **open end.**

***Fundamental frequency***

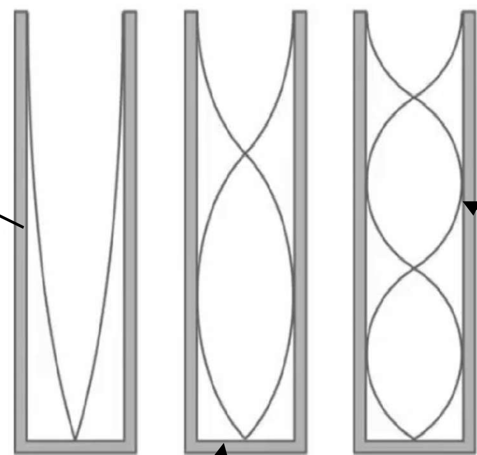


- When only **one node** and **an antinode** are formed in a closed pipe, half crest or half trough is formed or **in other words** only a quarter wave is formed (as in this case). (**Note that** a wave has a complete crest and a complete trough),

If L is length of the pipe then in this case, wave length "$\lambda_1$" of the wave will be equal to "4L". At this instant a loud sound is heard. Under this condition air inside the pipe is said to vibrate with **fundamental frequency** '$f_1$'. Hence $L = \lambda_1 /4$ or $\lambda_1 = 4L$

If 'V' is the speed of sound in air then $V = \lambda_1 f_1$

Hence, $f_1 = \frac{V}{\lambda_1}$ OR $\boxed{f_1 = \frac{V}{4L}}$




- When **two nodes** and **two antinodes** are formed in a closed pipe, such that the **first node** is at the closed end and the **second antinode** is at the open end, In other words, three quarter of a wave is formed. Length of the pipe will be equal to "$3\lambda_2/4$" where "$\lambda_2$" is the wavelength of stationary wave produced. At this instant a loud sound is heard. Under this condition air is said to vibrate in **second harmonic**. Hence $L = 3\lambda_2/4$ OR $\lambda_2 = 4L/3$

Since $V = \lambda_2 f_2$ $\therefore$ $f_2 = \frac{V}{\lambda_2}$ OR $\boxed{f_2 = \frac{3V}{4L}}$ OR $\boxed{f_2 = 3f_1}$

- Similarly, when **three nodes** and **three antinodes** are formed, length of the pipe will be equal to "$5\lambda_3/4$" where "$\lambda_3$" is the wavelength of the sound waves produced. At this instant again a loud sound is heard. Under this condition air is said to vibrate in **third harmonic**, and so on. Hence in this case $L = 5\lambda_3/4$ OR $\lambda_3 = 4L/5$

Since $V = \lambda_3 f_3$ $\therefore$ $f_3 = \frac{V}{\lambda_3}$ OR $\boxed{f_3 = \frac{5V}{4L}}$ OR $\boxed{f_3 = 5f_1}$

❖ **We notice that** stationary waves can be produced in a closed pipe when frequency of interfering waves is an odd multiple of ***fundamental frequency*** '$f_1$'.

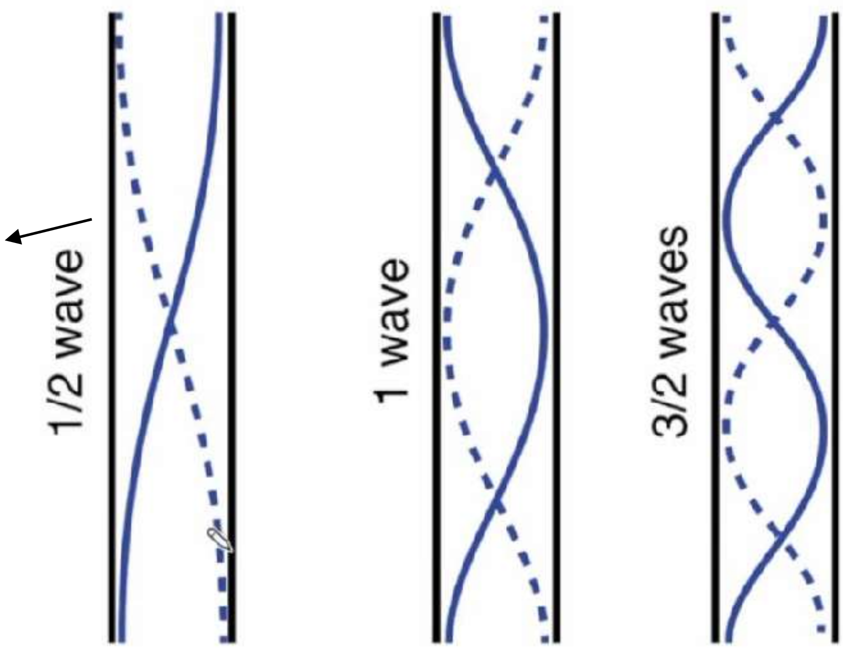
Hence $f_2 = 3f_1$, $f_3 = 5f_1$, $f_4 = 7f_1$, ……

***OR*** $f_n = n\, f_1$ Where n = 1, 3, 5, 7, …..

**Also note that** the fundamental wave is of **longest wave length** and of **smallest frequency**.

## ***Standing waves in open pipe*:**

***Idea*:** In a pipe open at both ends, stationary waves (Loud sound) can be produced only when **antinodes** are formed at both ends.




***Fundamental mode or first harmonic***

$\lambda_1 = 2\,L$

$f_1 = \dfrac{V}{2\,L}$

***Second harmonic***

$\lambda_2 = L$

$f_2 = \dfrac{V}{L}$

***Third harmonic***

$\lambda_3 = \dfrac{2\,L}{3}$

$f_3 = \dfrac{3\,V}{2\,L}$

- When we send sound waves in a pipe open at both ends, starting from a certain low frequency, then a stage will be reached when we hear a loud sound. At this instant a **node** is formed exactly in the middle and **antinodes** are formed at each open end. Stationary wave formed at this instant is said to have ***fundamental frequency***. (**minimum frequency** required to produce a stationary wave). We see that only half wave is formed. ***It means that*** the wave length "$\lambda_1$" of the wave formed will be equal to twice the length "2L" of the open pipe ($\lambda_1$= 2L). Since frequency of a wave is given by:

$$f = \frac{V}{\lambda} \qquad \text{(V is velocity of sound = constant)}$$

Therefore, ***fundamental frequency*** "$f_1$" is given by:

$$f_1 = \frac{V}{\lambda_1} \qquad \text{OR} \qquad \boxed{f_1 = \frac{V}{2\,L}}$$

- If the frequency of sound waves is gradually increased, then after reaching a certain particular frequency a loud sound is heard again. This is the ***second harmonic***, with two **nodes** inside and two **antinodes** at the open ends of the pipe. This time a full stationary wave is formed. **In other words**, wave length "$\lambda_2$" of the stationary wave formed will equal to "L",($\lambda_2$ = L).

Therefore, ***frequency for second harmonic*** "$f_2$" is given by:

$$f_2 = \frac{V}{\lambda_2} \quad \text{OR} \quad \boxed{f_2 = \frac{V}{L}} \quad \text{OR} \quad \boxed{f_2 = 2f_1}$$

- Similarly for ***third harmonic*** standing wave has one and half wave, for which wave length will be $\lambda_3 = 2L/3$ and frequency required will be :

$$f_3 = \frac{3V}{2L} \quad \text{OR} \quad \boxed{f_3 = 3f_1}$$

And so on. Hence we conclude that:

$$\boxed{f_n = n f_1} \quad \text{and} \quad \boxed{\lambda_n = \frac{2L}{n}}$$

Where n = 1,2,3,4……

# Doppler's effect:

***Q.No.1*: What is Doppler's effect?**

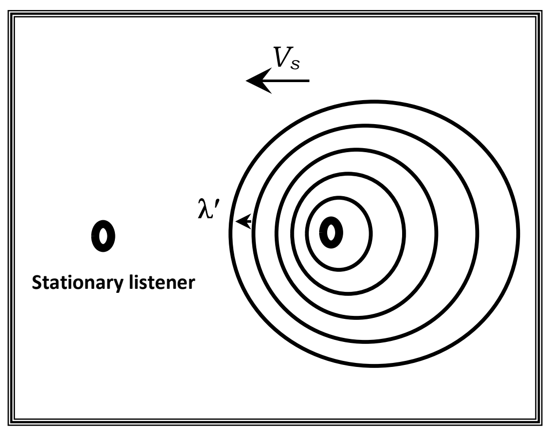
***Ans*:** Doppler's effect is:

***"The apparent change in pitch of sound due to the relative motion between a source of sound and a listener".***

Pitch of sound depends upon its **frequency**, hence due to the relative motion between its source and the listener, frequency of sound heard by the listener appears to have changed, as a result of which pitch of sound heard by the listener changes.

## Case.1:

***Q.No.2*: Derive a formula for frequency of sound heard when the source of sound moves towards a stationary listener.**




***Ans*:** When a source of sound emitting sound waves of frequency "$f_s$" moves towards a stationary listener with speed "$V_s$" then the sound waves present between the listener and the source are compressed, hence, their wave length decreases.

Wave length "$\lambda'$" of sound heard by the listener is less than the wave length of sound emitted by the source and is given by:

$$\lambda' = \frac{V - V_s}{f_s}$$

The frequency "$f_L$" of sound heard by the listener is given by:

$$f_L = \frac{V}{\lambda'}$$

$$f_L = \frac{V}{\frac{V - V_s}{f_s}}$$

$$f_L = \frac{V}{V - V_S} \times f_S$$

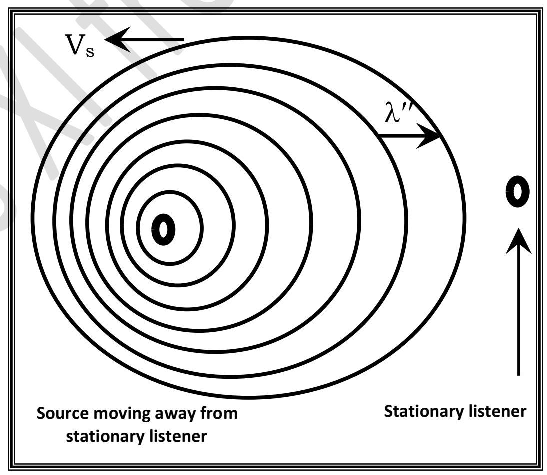
It shows that the frequency of sound "$f_L$" heard by the stationary listener will be ***greater*** than the frequency of sound "$f_S$" emitted by the moving source. The increase in frequency heard depends upon the speed "$V_S$" with which source approaches the listener. As a result of this increase in frequency, the pitch of sound heard by the listener appears to have ***increased*** (Sound heard becomes more and more shrill as the difference in frequency increases).

## *Case.2*:

***Q.No.3*: Derive a formula for frequency of sound heard when the source of sound moves away from a stationary listener.**

***Ans*:** When a source of sound emitting sound waves of frequency "$f_s$" moves away from a stationary listener with a velocity "$V_s$", then the wave length of sound heard by the listener **increases** (as the sound waves present between the source and the listener are elongated),and is given by:

$$\lambda'' = \frac{V + V_s}{f_s}$$




The frequency "$f_L$" of sound heard by the listener is given by:

$$f_L = \frac{V}{\lambda''}$$

$$f_L = \frac{V}{\frac{V + V_S}{f_s}}$$

$$f_L = \frac{V}{V + V_s} \times f_s$$

Frequency of sound heard by the listener is ***less*** than the frequency produced by the source, hence pitch of sound as heard by the stationary listener also ***decreases*** as a result sound becomes ***grave*** or ***flat***.

***Note that*** due to the motion of the source (above two cases), **wave length** (and not the relative velocity) with which sound waves reach the listener changes, as a result of which frequency and therefore, the pitch of sound heard by the listener will change.

In these two cases, the main cause of change in frequency heard by the listener is the change in **wavelength** of sound reaching the listener.

## *Case 3*:

***Q.No.4*: Derive a formula for frequency of sound heard when the listener approaches a stationary source of sound.**

***Ans*:** Let a source of sound emit sound waves of frequency "$f_s$" and wave length "λ", given by

$$\lambda = \frac{V}{f_s}$$

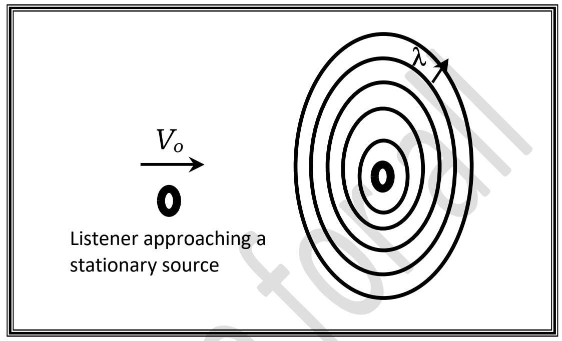
Where "V" is the speed of sound waves.

Let a listener approach the stationary source with a speed "$V_L$" then the relative speed with which sound waves reach him will be "$V + V_L$". Under this condition the frequency of sound "$f_L$" heard by the moving listener will be: (Frequency of waves = speed / wavelength)

$$\therefore \quad f_L = \frac{V + V_L}{\lambda} \qquad \text{But } \lambda = \frac{V}{f}$$

Therefore,
$$f_L = \frac{V + V_L}{\frac{V}{f}}$$

$$\boxed{f_L = \frac{V + V_L}{V} \times f_S}$$




$f$ is the frequency of sound emitted by the source and $f_L$ is the frequency of sound heard by a listener moving with speed $V_L$ towards a stationary source.

It shows that the frequency of sound "$f_L$" heard by the approaching listener will be ***greater*** than the frequency of sound "$f_S$" emitted by the stationary source. The increase in frequency heard depends upon the speed "$V_L$" with which listener approaches the source. As a result of this increase in frequency the pitch of sound heard by the listener appears to have ***increased*** (Sound heard becomes more and more shrill as the difference in frequency increases).

## ***Case 4*:**

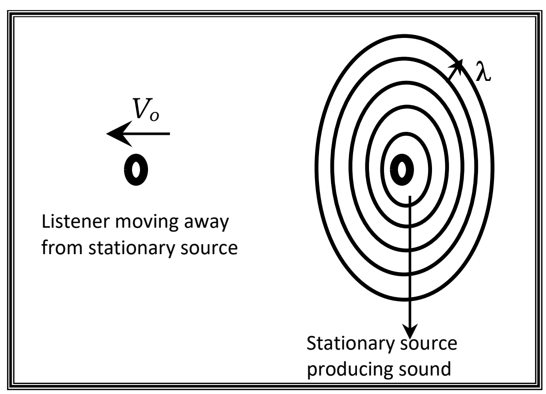
***Q.No.5*: Derive a formula for frequency of sound heard when the listener moves away from a stationary source of sound.**

***Ans*:** When a listener moves away from a stationary source with a speed "$V_L$" then the relative velocity with which sound waves reach him will be "$V - V_L$".
Hence the sound heard by the listener will be of frequency "$f_L$", given by:

$$\therefore \quad f_L = \frac{V - V_L}{\lambda} \qquad \text{But } \lambda = \frac{V}{f_S}$$

Therefore,
$$f_L = \frac{V - V_L}{\frac{V}{f_S}}$$

$$\boxed{f_L = \frac{V - V_L}{V} \times f_S}$$




It shows that the frequency of sound "$f_L$" heard by the moving listener will be ***less*** than the frequency of sound "$f_S$" emitted by the stationary source. The decrease in frequency heard depends upon the speed "$V_L$" with which listener moves away from the source. As a result of this decrease in frequency the pitch of sound heard by the listener appears to have ***decreased*** (Sound heard becomes grave or less and less shrill as the frequency of sound heard decreases as compared with the frequency emitted).

***Note that*** due to the motion of the listener (above two cases), **relative velocity** (wave length is same) with which sound waves reach the listener changes, as a result of which frequency and therefore, the pitch of sound heard by the listener will change.

The **change in relative velocity** of sound reaching the listener due to the relative motion of the listener is the main cause of change in pitch.

## Case. 5:

***Q.No.6: Derive a formula for frequency of sound heard when the source of sound and the listener both approach each other while moving along the same line.***

***Ans***: If a source and a listener both are moving towards each other with velocities "$V_S$" and "$V_L$" respectively then the frequency "$f_L$" heard by the listener is given by:

$$f_L = \frac{V + V_L}{V - V_S} \times f_S$$

In this case $f_L > f_S$, therefore pitch of sound heard by the listener **increase**s due to which sound heard becomes shrill.

## Case 6:

***Q.No.7: Derive a formula for frequency of sound heard when the source of sound and the listener both move away from each other while moving along the same line.***

***Ans***: If a source and a listener both are moving away from each other with velocities "$V_S$" and "$V_L$" respectively then the frequency "$f_L$" heard by the listener is given by:

$$f_L = \frac{V - V_L}{V + V_s} \times f_S$$

In this case frequency of sound $f_L$ heard by the listener will be less than the frequency '$f_S$' of sound emitted by the source, therefore pitch of sound heard by the listener **decreases** due to which sound heard becomes grave or less shrill.



- If a source of sound moves when a listener is at rest, **frequency** of sound heard changes due to a change in its **wavelength**.
- If a listener moves when the source of sound is at rest, ***frequency*** of sound heard changes due to a change in its **velocity**.

***General formula for frequency of sound heard when the***
***source of sound is in motion***
***and the listener is at rest is:***

$$f_L = f_S \left\{ \frac{V}{V \pm V_S} \right\}$$

− *when source of sound approaches*
+ *when source of sound moves away*

***General formula for frequency of sound heard when the***
***source of sound is at rest***
***and the listener is in motion is:***

$$f_L = f_S \left\{ \frac{V \pm V_L}{V} \right\}$$

+ ***when listener approaches***
− ***when listener moves away***

***Note that:*** Frequency of a wave depends upon its **wave length** and its **velocity**.
Frequency of a wave changes if either its **wave length** is changed at **constant velocity,** its **velocity** is changed keeping its **wavelength** constant or both its **wavelength** as well as **velocity** are changed.

***Doppler effect*** takes place in case of light waves also.
By analyzing wave length of light coming from distant stars and galaxies, scientists observed Doppler effect in light waves. They found that wavelength of light coming from distant stars and galaxies **increases** over a long period of time. Wave length of light is found to shift towards longer wavelength, they call it ***red shift***. Observing red shift, they concluded that our universe is ***expanding***.

***Q.No.8: What happens to the pitch of sound of siren of an ambulance which approaches a person standing on the road side?***

***Ans:*** As an ambulance sounding its siren approaches a person standing on the road side, the frequency of sound heard by the person ***increases***, as a result of which pitch of sound also increases. Due to increase in pitch of sound, sound of siren heard becomes more and more shrill as the ambulance approaches the person.

***Reason:*** ***wavelength*** of sound waves presents between the ambulance and the person ***decreases*** as the ambulance approaches the person, due to which ***frequency*** of sound of siren heard by the person standing by the road side appears to ***increase*** and the sound heard becomes ***more and more shrill***.

***<u>Q.No.9</u>: What happens to the pitch of sound of siren of an ambulance heard by a person traveling in a car such that the car and the ambulance both move <u>towards each other</u>?***

***<u>Ans</u>*:** If an ambulance sounding its siren approaches a person travelling in a car moving towards the ambulance, the frequency of sound of siren heard by the person ***increases***, as a result of which pitch of sound heard will also ***increase***. Sound heard will become ***more and more shrill*** as the ambulance and the person both approach each other.

***<u>Q.No.10</u>: What happens to the pitch of sound of siren of an ambulance heard by a person traveling in a car such that the car and the ambulance move <u>away from each other</u>?***

***<u>Ans</u>*:** If an ambulance sounding its siren and a person travelling in a car move away from each other, the frequency of sound of siren heard by the person ***decreases***, as a result of which pitch of sound heard will also ***decrease***. Sound heard will become ***less and less shrill*** as the ambulance and the person both move away from each other.

# ***<u>Applications of Doppler's effect</u>:***

***<u>Q.No.11</u>: Give some applications of Doppler's effect.***

***<u>Ans</u>*:**

- With the help of Doppler's effect speed of approaching or receding vehicles can be found. For this purpose, traffic police use a device based on this idea.
- Radar waves after reflection from a plane etc. on reaching its antenna show a change in frequency. This change in frequency depends upon the speed with which a plane approaches or moves away from the radar station. Hence a radar operator can easily determine the movement and intention of the plane.
- Ships are equipped with a device called "SONAR". It works on the principle of Doppler's effect. It detects the presence of underground rocks and submarines etc.
- Light also show Doppler effect provided speed of source of light or that of the observer is very high.
- Currently ultrasound radiations have been used for the diagnosis of various medical problems. It is based on Doppler's effect.

# ***<u>Sonic boom</u>:***

***<u>Q.No.1</u>: What is sonic boom? How is sonic boom produced?***

***<u>Ans</u>*:** When speed of a source of sound '$V_S$' is comparable to the speed 'V' of sound then the source of sound and the waves of sound move together in the same direction. As a result of which the number of waves produced in front of the source keeps on ***increasing***. Due to which a plane wave front of highly compressed air is formed which moves in front of the source of sound.

If the source of sound moves faster than sound i.e. $V_S > V$, then the plane wave front takes the shape of a cone with the fast-moving source at its apex. Air in the cone shaped wave front is highly compressed. This cone shaped wave front of highly compressed air is known as ***shock wave.***

Hence supersonic air crafts, such as F-16 or Mirage fighter planes, produce shock waves of such a strength that a loud explosion is heard. This loud explosion is called '***Sonic boom***'. Supper sonic planes produce double sonic booms, one from the shock waves produced in the front and the other to the rear of the plane.

## ***Section–A Multiple-Choice Questions MCQs*:**

***Q.No.1*:** The speed v of a wave represented by $y = A \sin (k x - \omega t)$ is:

(a) k/ω (b) ω /k (c) ω k (d) 1/ ω k

***Q.No.2*:** Two sound waves are

$y = \sin (k x - \omega t)$ and $y = \cos (k x - \omega t)$.

Phase difference between the two waves is:

(a) π/2 (b) π /4 (c) π (d) 0

***Q.No.3*:** If $v_a$, $v_b$ and $v_m$ are speed of sound in air, in hydrogen gas and in a metal at the same temperature, then:

(a) $v_a > v_h > v_m$ (b) $v_m > v_h > v_a$

(c) $v_h > v_m > v_a$ (d) $v_h > v_a > v_m$

***Q.No.4*:** Speed of sound in air at STP is 332 m/s. If the pressure becomes double at the same temperature, the speed of sound becomes:

(a) 1382 m/s. (b) 664 m/s.

(c) 332 m/s. (d) 166 m/s.`

***Q.No.5*:** How does the speed of sound "v" in air depend on the atmospheric pressure "P":

(a) $v \propto P^0$ (b) $v \propto P^{-1}$ (c) $v \propto P^2$ (d) $v \propto P^1$

***Q.No.6*:** The speed of sound in a gas is proportional to:

(a) square root of isotheral elasticity.

(b) square root of adiabatic elasticity.

(c) isotheral elasticity.

(d) adiabatic elasticity.

***Q.No.7*:** Length of a pipe closed at one end is "L". In the standing waves whose frequency is 7 time the fundamental frequency, what is the closest distance between nodes?

(a) $\frac{1}{14}$ L (b) $\frac{1}{7}$ L (c) $\frac{2}{7}$ L (d) $\frac{4}{7}$ L

***Q.No.8*:** A 620 Hz frequency song of an ice cream trolley approaches with speed "v" to a boy standing at the door of his house is heard with frequency "$f_1$". If the trolley is stopped and the boy approaches the ice cream trolley with the same speed "v" the boy now hears sound of frequency "$f_2$", choose the correct statement

(a) $f_1 = f_2$ both are greater than 620 Hz.

(b) $f_2 > f_1 > 620$ Hz.

(c) $f_1 = f_2$ both are lesser than 620 Hz.

(d) $f_1 > f_2 > 620$ Hz.

***Q.No.9*:** The speed of sound in a gas in which two waves of wavelength 50 cm and 50.4 cm produce 6 beats per second is:

(a) 338m/s (b) 350m/s (c) 378 m/s (d) 400 m/s

***Q.No.10*:** The speed of a wave in a medium is 760 m/s. If 3600 waves are passing through a point in the medium in 2 minutes, then its wave length is:

(a) 13.85 m. (b)25.3 m. (c)41.5 m. (d)57.2 m.

***Q.No.11*:** Human beings can hear sound of frequency: (***2005 Karachi Board***)

• 5 Hz. • 5000Hz. • 25000 Hz • 50000 Hz.

***Q.No.12*:** The velocity of sound in a gas increases with: (***2005 Karachi Board***)

• Temperature • Pressure

• Loudness • frequency.

***Q.No.13*:** Frequencies which are multiples of fundamental frequency are called: (***2005 Karachi Board***)

• Beat frequency • Nodal frequency.

• Harmonics • Doppler effect

***Q.No.14*:** Which of the following is not the property of sound waves? (***2005 Karachi Board***)

• Interference • Diffraction

• Polarization • Refraction.

***Q.No.15*:** Beats are produced due to: (***2006 Karachi Board***)

• Diffraction • Interference

• Polarization • Refraction

***Q.No.16*:** Which of the following represents longitudinal waves?:(***2006 Karachi Board***)

• light waves • sound waves

• Radio waves • X-rays

***Q.No.17*:** The speed of sound is highest in: (***2007 Karachi Board***)

• solid • liquid • gas • vacuum

***Q.No.18*:**The voice of two persons having the same pitch and loudness can be recognized due to the characteristic of sound known as: (***2007 Karachi Board***)

• Beats • Quality • Frequency • Intensity

**_Q.No.19_**:The frequency of a simple pendulum is given by: (***2008 Karachi Board***)

- $\nu = 2\pi\sqrt{g/L}$ • $\nu = 2\pi\sqrt{L/g}$
- $\nu = \frac{1}{2\pi}\sqrt{L/g}$ • $\nu = \frac{1}{2\pi}\sqrt{g/L}$

**_Q.No.20_**: If mass of a body suspended from a spring is increased to 4 times, the period of vibration of the body will be:

(***2008, 2007 Karachi Board***)

- 4 times. • 2 times
- $\sqrt{2}$ times • same as before.

**_Q.No.21_**: If the fundamental frequency of vibration of a string fixed at both ends is 50Hz. The fourth harmonic will be:

(***2008 Karachi Board***)

• 100 Hz • 150 Hz • 200 Hz • 250 Hz

**_Q.No.22_**: If the tension of a stretched string is increased 4 times, the speed of the transverse wave in it will increase:

(***2009 Karachi Board***)

• 4 times • 8 times • 2 times •16 times

**_Q.No.23_**:The velocity of sound has maximum value in: (***2009 Karachi Board***)

• Solids • liquids • gasses • free space

**_Q.No.24_**: This is compressional wave

(***2011 Karachi Board***)

- Light waves. • Sound waves.
- Radio waves. • X-rays.

**_Q.No.25_**: If two tuning forks of frequencies 256 Hz. and 260 Hz. are sounded together, the number of beats per second will be:

(***2011, 2009, 04 Karachi Board***)

• 3. • 4. • 5. • 6.

**_Q.No.26_**: The frequency of wave produced in a stretched string depends upon:

(***2012 Karachi Board***)

- Length. • Tension.
- Linear density. • All of these.

**_Q.No.27_**: The maximum number of beats that can be detected by the human ear is:

(***2013, 06,07 Karachi Board***)

• 2 • 3 • 5 • 7

**_Q.No.28_**: Two vibrating bodies having slightly different frequencies produce:

(***2016 Karachi Board***)

• Echo • Beats • Resonance • Polarization

**_Q.No.29_**: The velocity of a wave of wave length 'λ' and frequency 'ν' is given by:

(***2015 Karachi Board***)

• $\nu/\lambda$ • $\lambda/\nu$ • $\nu\lambda$ • $1/\nu\lambda$

**_Q.No.30_**: If the frequency of fifth harmonic of a vibrating string is 200 Hz. its fundamental frequency is: (***2017 Karachi Board***)

• 5 Hz. • 25 Hz. • 40 Hz. • 100 Hz.

**_Q.No.31_**: The speed of sound in vacuum is:

(***2017, 2010 Karachi Board***)

• zero • 332 $ms^{-1}$ • 33200cm $s^{-1}$ • $3\times10^8 ms^{-1}$

**_Q.No.32_**: Intensity of sound is measured in:

(***2018 Karachi Board***)

• watt/$m^2$ • joule/m • watt/s • watt/m

**_Q.No.33_**: When temperature of air rises the speed of sound waves increases, because:

(***2018, 08 Karachi Board***)

- The frequency of waves increases.
- The wavelength of waves increases
- Both the frequency and wavelength increases
- Neither frequency nor wavelength increases

**_Q.No.34_**: Beats are produced due to:

(***2019, 08 Karachi Board***)

- diffraction of waves in time.
- reflection of waves in time.
- Polarization of waves in time.
- interference of waves in time.

**_Q.No.35_**: The product of frequency and time period is: (***2019 Karachi Board***)

• 1 • 2 • 3 • 4

**_Q.No.36_**: The range of audible sound is:

(***2022, 2016 Karachi Board***)

- 1 Hz. to 10 Hz. • 20 Hz. to 20,000 Hz.
- 21,000 Hz. to 24,000 Hz.
- 25,000 Hz. and onwards.

**_Q.No.37_**: Pitch of sound depends upon:

(***2022 Karachi Board***)

- Amplitude • Intensity
- Frequency • Loudness

**_Q.No.38_**: Distance between two consecutive nodes in a standing wave is:

(***2022, 2017,14, 06 Karachi Board***)

• $\lambda/4$ • $\lambda$ • $\lambda/2$ • $\lambda/3$

***Q.No.39*:** Speed of sound in air at STP is 332 m/s. If the air pressure becomes double at the same temperature, the speed of sound will become: (***2025 Karachi Board***)

- 1382 m/s
- 664 m/s
- 332 m/s
- 166 m/s

## ***Answers:***

(1) $\omega /k$
(2) $\pi/2$
(3) $v_m > v_h > v_a$
(4) 332 m/s.
(5) $v \propto P^0$
(6) Square root of adiabatic elasticity.
(7) $\frac{1}{7}$ L
(8) $f_1 = f_2$ both are greater than 620 Hz.
(9) 378 m/s.
(10) 25.3 m.
(11) 5000 Hz.
(12) Temperature.
(13) Harmonics.
(14) Polarization
(15) Interference
(16) sound waves.
(17) Solid.
(18) Quality.
(19) $\nu = \frac{1}{2\pi}\sqrt{g/L}$
(20) $\sqrt{2}$ times.
(21) 200 Hz.
(22) 2 times.
(23) Solids.
(24) Sound waves.
(25) 4.
(26) All of these.
(27) 7.
(28) Beats.
(29) $\nu \lambda$
(30) 40 Hz.
(31) zero
(32) watt/m$^2$
(33)The wavelength of waves increases
(34) interference of waves in time.
(35) 1
(36) 20 Hz. to 20,000 Hz.
(37) Frequency
(38) $\lambda /2$
(39) 332 m/s.

# Numericals:

***Q.No.4*: An increase in pressure of 100 kPa causes a certain volume of water to decrease by $5\times10^{-3}$ percent of its original volume.**

**Find: (a) Bulk modulus of water.**

**(b) What is the speed of sound in water?**

***Data*:**

Increase in pressure $\Delta P = 100$ kPa

$= 100\times1000$ Pa $= 10^5$ Pa

Decrease in volume $\Delta V = 5\times10^{-3}$ % of V.

(V is the original volume)

$\therefore \quad \Delta V/V = 5\times10^{-3}$ %

$\Delta V/V = \dfrac{5\times10^{-3}}{100} = 5\times10^{-5}$

(a) Bulk modulus of water =?

(b)Speed of sound in water v =?

***Solution*:**

(a) Bulk modulus is given by:

Bulk modulus $B = \dfrac{\Delta P}{\Delta V/V} = \dfrac{10^5}{5\times10^{-5}} = 0.2\times10^5\times10^5$

Bulk modulus $B = 0.2\times10^{10}$ Pa.

$= 2000\times10^6$ Pa $=$ **2000 MPa**

(b) Speed of sound is given by:

$V = \sqrt{\dfrac{B}{\rho}} = \sqrt{\dfrac{2000\times10^6}{10^3}} = \sqrt{2000\times10^3}$

($\rho$ is the density of water)

V = **1414 m/s.**

🗣 Bulk modulus is **<u>2000 MPa</u>** and the speed of sound in water is **<u>1414 m/s</u>.**

***Q.No.5*: A uniform string of length 10.0 m and weighing 0.25 N is attached to the ceiling. A weight of 1.00 kN hangs from its lower end. The lower end of the string is suddenly displaced horizontally. How long does it take the resulting wave pulse to travel to upper end.**

**(Neglect the weight of string in comparison to hanging mass).**

***Data*:**

Length of string $L = 10.0$ m.

Weight of string $W_1 = 0.25$ N.

Mass of string $m = \dfrac{W}{g} = \dfrac{0.25}{9.8} = 0.0255$ kg.

Weight suspended $W_2 = T = 1.0$ kN $= 10^3$ N

(T is tension in the string = weight suspended)

Time to travel to upper end $t$ =?

***Solution*:**

Distance travelled is given by:

$S = Vt$ OR $t = \dfrac{S}{V}$

But speed V of a wave in string is given by:

$V = \sqrt{\dfrac{TL}{m}} = \sqrt{\dfrac{10^3\times10}{0.0255}} = 626.22$ m/s.

Hence: $t = \dfrac{S\text{ (or L)}}{V} = \dfrac{10}{626.22} = 0.015969$s

**= 15. 969 ms = 16 ms.** (ms = millisecond)

🗣 Wave pulse will take about **<u>16 ms.</u>** to travel to the upper end.

***Q.No.6*: A travelling sin wave is the result of the superposition of two other sin waves with equal amplitude, wavelength and frequencies. The two component waves each have amplitude 5.00 cm. If the resultant wave has amplitude 6.69 cm.**

**What is the phase difference "$\varphi$" between the component waves?**

***Data*:**

Amplitude of each wave $A_o = 5$ cm.

Amplitude of resultant wave $A = 6.69$ cm

Phase difference $\varphi$ = ?

***Solution*:**

Amplitude of resultant wave is given by:

$A = 2A_o \cos(\varphi/2)$

$6.69 = 2\times5\cos(\varphi/2)$

$\dfrac{6.69}{10} = \cos(\varphi/2) \qquad \cos(\varphi/2) = 0.669$

$(\varphi/2) = \cos^{-1} 0.669 \qquad (\varphi/2) = 48°$

$\therefore \quad \varphi = 48\times2 = 96°$

🗣 Phase difference between the waves is **<u>96°</u>**

***Q.No.7*: In order to decrease the fundamental frequency of a guitar string by 4.0 %, by what percentage should you reduce the tension?**

***Data*:**

Decrease in frequency = 4% of '$f$'

(where '$f$' is the fundamental frequency)

% decrease in tension =?

***Solution*:**

Let '$f$' be the fundamental frequency, the new frequency is given by:

$f' = (100 - 4)\%$ of $f = 96\%$ of $f = 0.96 f$

Fundamental frequency of a wave in string is given by: (M is mass per unit length)

$$f = \frac{1}{2L}\sqrt{\frac{T}{M}} \quad \ldots\ldots\ldots\ldots (i)$$

New frequency in the string is given by:

$$f' = \frac{1}{2L}\sqrt{\frac{T'}{M}} \quad \text{(M is mass per unit length)}$$

$$0.96 f = \frac{1}{2L}\sqrt{\frac{T'}{M}} \quad \ldots\ldots\ldots\ldots (ii)$$

Dividing $(ii)$ by $(i)$ we get:

$$\frac{0.96 f}{f} = \frac{1}{2L}\sqrt{\frac{T'}{M}} \Big/ \frac{1}{2L}\sqrt{\frac{T}{M}}$$

$$0.96 = \sqrt{\frac{T'}{T}}$$

Squaring both sides and re-arranging:

$0.9216\ T = T'$

But Percentage decrease in T= $\frac{\text{Decrease in T}}{T} \times 100$

Percentage decrease in T = $\frac{T - T'}{T} \times 100\%$

Percentage decrease in T = $\frac{T - 0.9216\ T}{T} \times 100\%$

Percentage decrease in T= $\frac{\cancel{T}(1 - 0.9216)}{\cancel{T}} \times 100\%$

Percentage decrease in T=$0.0784 \times 100\%$=**78.4%**

🗣Percentage decrease in tension will be **78.4%**.

***Q.No.8*: A string 2.0 m long is fixed at both ends. If a sharp blow is applied to the string at its center, it takes 0.050 s for the pulse to travel to both ends of the string and return to its middle.**

**What is the fundamental frequency of oscillation for this string?**

***Data*:**

Length of the string $L = 2$ m.
Time taken $t = 0.05$ s.
Fundamental frequency $f = ?$

***Solution*:**

When a string fixed at both ends is disturbed from its center, one loop is formed, it is said to vibrate with fundamental frequency, which is given by:

$$f = \frac{v}{2L} \qquad \text{But } v = \frac{L}{t}$$

$$f = \frac{L/t}{2L} = \frac{\cancel{L}}{2\cancel{L}t} = \frac{1}{2t} = \frac{1}{2 \times 0.05} = \mathbf{10 Hz.}$$

🗣 Fundamental frequency is **10 Hz.**

***Q.No.9*: A sound source of frequency "$f_o$" and an observer are located at a fixed distance apart. Both the source and observer are at rest. However, the propagation medium (through which the sound waves travel at speed v) is moving at a uniform velocity "$v_m$" in an arbitrary direction. Find the frequency detected by the observer giving physical explanation.**

***Data*:**

Speed of sound = v
Speed of the medium = $v_m$.

Frequency detected when medium flows in the direction of sound waves $f_1$ =?

Frequency detected when medium flows against the direction of sound waves $f_2$ =?

***Solution*:**

Speed with which sound waves reach the observer depends not only upon its own speed "v" but also upon the magnitude and direction of speed "$v_m$" of the medium. Hence:

- ❖ When the source of sound and an observer are **at rest relative to each other**, frequency of sound emitted will be equal to the frequency of sound received.
- ❖ When medium flows **in the direction of sound waves** Speed with which sound waves are received by the observer will be "$v + v_m$", now the frequency of sound received is given by:

$$\boxed{f_1 = \frac{v + v_m}{\lambda}} \qquad \text{(since } f\lambda = v)$$

- ❖ When medium flows **against the direction of sound waves,** Speed with which sound waves are received by the observer will be

"$v - v_m$", now the frequency of sound received is given by:

$$f_2 = \frac{v - v_m}{\lambda} \quad \text{(since } f\lambda = v\text{)}$$

***Note that*:** frequency of sound heard by a listener changes (*i*) due to a **change in its wave length** when the source of sound moves.
OR (*ii*) due to a **change in its speed** with which sound waves reach the listener when either the listener or the medium or both move

***Q.No.10*: A train sounds its whistle while passing by a railroad crossing. An observer at the crossing measures a frequency of 219 Hz. as the train approaches the crossing and a frequency of 184 Hz as the train leaves. The speed of sound is 340 m/s. Find the speed of the train and frequency of its whistle.**

***Data*:**

Frequency of sound heard when source **approaches** the listener $f' = 219$ Hz.
Frequency of sound heard when source **moves away** from the listener $f'' = 184$ Hz.
Speed of sound $V = 340$ m/s.
Speed of the train $V_s = ?$
Frequency of the whistle $f = ?$

***Solution*:**

Frequency of sound heard by a stationary listener when the train **approaches** him is given by:

$$f' = \frac{V}{V - V_s} \times f$$

$$219 = \frac{340}{340 - V_s} \times f$$

$$219\,(340 - V_s) = 340\,f$$

$$219 \times 340 - 219\,V_s = 340\,f \,\ldots\ldots\ldots(i)$$

Frequency of sound heard by a stationary listener when the train **moves away** from him is given by:

$$f'' = \frac{V}{V + V_s} \times f$$

$$184 = \frac{340}{340 + V_s} \times f$$

$$184\,(340 + V_s) = 340\,f$$

$$184 \times 340 + 184\,V_s = 340\,f \,\ldots\ldots\ldots(ii)$$

LHS of (*i*) and (*ii*) are equal, therefore:

$$184 \times 340 + 184\,V_s = 219 \times 340 - 219\,V_s$$

$$184\,V_s + 219\,V_s = 219 \times 340 - 184 \times 340$$

$$403\,V_s = 11900$$

$$V_s = \frac{11900}{403} = \mathbf{29.53\ m/s.}$$

Put the value of $V_s$ in equation (*ii*) we get:

$$184 \times 340 + 184 \times 29.53 = 340\,f$$

$$f = \frac{184 \times 340 + 184 \times 29.53}{340} = \mathbf{200\ Hz.}$$

🗣 Speed of the train is **29.53 m/s** and frequency of its whistle is **200 Hz**.

***Q.No.11*: A guitar string has a linear density of 7.16 g/m and under tension of 152 N. The fixed supports of the string are 89.4 cm apart. If it vibrates in three segments, calculate the speed, the wave length and the frequency of the standing wave.** (***1996 Karachi Board***)

***Data*:**

Linear density $\mu = 7.16$ g/m $= 7.16 \times 10^{-3}$ kg./m
Tension in the string T = 152 N
Length of three segments
$l = 89.4$ cm $= 0.894$ m
Speed of the wave $V = ?$
Frequency $f = ?$
Wave length $\lambda = ?$

***Solution*:**

Speed of waves in a string under tension is given by:

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{152}{7.16 \times 10^{-3}}}$$

$$V = \sqrt{21229.05} \qquad \boxed{V = 145.7\ m/s}$$

Wave length of the wave when the string vibrates in three segments is given by:

$$\lambda = \frac{2\,l}{3} = \frac{2 \times 0.894}{3} = \frac{1.788}{3}$$

$$\boxed{\lambda = 0.596\ m.}$$

Frequency with which the string vibrates is given by:

$$f = \frac{V}{\lambda} = \frac{145.7}{0.596} \qquad \boxed{f = 244.5\ Hz.}$$

***Q.No.12***: **The siren of an ambulance at rest is producing a frequency of 2000 hertz.**

**What frequency will be heard by a listener moving towards the ambulance with a velocity of 100 km/hour? Velocity of sound in air=1200 km/hour.**

(***2001 Karachi Board***)

***Data***:

Frequency of sound produced $f$ =2000 Hz.
Frequency heard $f' = ?$
Velocity of listener $V_o$ = 100 km/h
Velocity of sound V = 1200 km/h

***Solution***:

Frequency of sound heard by a listener approaching a stationary source is given by:

$$f' = \frac{V + V_o}{V} \times f = \frac{1200 + 100}{1200} \times 2000$$

$$f' = \frac{1300}{1200} \times 2000 \qquad \boxed{f' = 2166.7\ Hz.}$$

***Q.No.13***: **Calculate the speed of sound in air at S.T.P What is the speed of sound at 37°C?**

**(Given: density of air = 1.29 Kg/m$^2$ $\gamma$ for air = 1.42, 1atm. = 1.01×10$^5$ N/m$^2$)**

(***2002 Karachi Board***)

***Data***:

Speed of sound at S.T.P V= ?
Speed of sound at 37° $V_{37}$ =?
Temperature t = 37°
Density of air $\rho$ = 1.29 Kg/m$^2$
For air $\gamma$ = 1.42
1atm. = 1.01×10$^5$ N/m$^2$

***Solution***:

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

$$V = \sqrt{\frac{1.42 \times 1.01 \times 10^5}{1.29}} = \sqrt{111178.29}$$

$$\boxed{V = 333.43\ m/s}$$

Speed of sound at any temperature t is given by

$$V_t = V\sqrt{\frac{t + 273}{273}}$$

$$V_{37} = 333.43\sqrt{\frac{37 + 273}{273}} \qquad \text{(at 37°C)}$$

$$V_{37} = 333.43 \times \sqrt{1.1355}$$

$$V_{37} = 333.43 \times 1.066$$

$$\boxed{V_{37} = 355.31\ m/s}$$

Speed of sound at STP is ***333.43 m/s*** and at 37°C is ***355.31 m/s***.

***Q.No.14***: **Two cars are moving straight to each other from opposite directions with the same speed. The horn of one is blowing with the frequency of 3000 Hz. and is heard by people in the other car with the frequency of 3400 Hz.**

**Find the speed of the car if speed of sound in air is 340 m/s.**

(***2003 Karachi Board***)

***Data***:

Frequency produced $f$ = 3000 Hz
Frequency heard $f'$=3400 Hz.
Speed of sound V = 340 m/s
Speed of each car $V_o = V_S$ =?

***Solution***:

Since both the cars, one producing sound (source) and in the other there is a listener, are moving opposite to each other. Hence frequency of sound heard is given by:

$$f' = \frac{V + V_o}{V - V_S} \times f$$

Both cars are approaching each other with same speed, therefore, $V_s = V_o$

$$3400 = \frac{340 + V_o}{340 - V_o} \times 3000$$

$$3400\,(340 - V_o) = (340 + V_o)\,3000$$

$$3400 \times 340 - 3400\,V_o = 3000 \times 340 + 3000\,V_o$$

$$3400 \times 340 - 3000 \times 340 = 3400 \times V_o + 3000\,V_o$$

$$1156000 - 1020000 = 6400\,V_o$$

$$136000 = 6400\,V_o$$

$$V_o = \frac{136000}{6400}$$

$$\boxed{V_o = 21.25\ m/s}$$

Speed of each car is ***21.25 m/s***.

***Q.No.15***: **A standing wave is established in a 2.4 m long string fixed at both ends. The string vibrates in four segments (loops) when driven at 200 Hz Determine:**

**(1) The wave length.**

**(2) The fundamental frequency.**

(***2018, 2004, 1994 Karachi Board***)

***Data***:

Length of the string $L = 2.4$ m
Number of segments $n = 4$
Frequency $f_4 = 200$ Hz.
Wave length $\lambda_4 = ?$
Fundamental frequency $f_1 = ?$

***Solution***:

Wave length of the wave is given by:

$$\lambda_n = \frac{2L}{n}$$

$$\lambda_4 = \frac{2 \times 2.4}{4} \qquad \lambda_4 = \mathbf{1.2\ m}$$

wave length of standing waves when the string vibrates in four segments is ***1.2 m.***

Similarly: $$f_1 = \frac{f_4}{4} \qquad \left\{ f_1 = \frac{f_n}{n} \right\}$$

$$f_1 = \frac{200}{4} \qquad \boxed{f_1 = 50\text{Hz.}}$$

Fundamental frequency is ***50Hz.***

***Q.No.16***: **A note of frequency 650 Hz. Is emitted from an ambulance. What frequency will be detected by a listener if the ambulance moves**
**(*i*) at speed of 18 m/s. towards the listener.**
**(*ii*) at speed of 15 m/s away from the listener**
**(Speed of sound = 340 m/s).**
(***2008 Karachi Board***)

***Data***:

Frequency of sound $f = 650$ Hz.
Frequency of sound detected $f' = ?$
(*i*) Speed of ambulance
$V_s = 18$ m/s (towards)
(*ii*) Speed of ambulance
$V_s = 15$ m/s (away)
Speed of sound $V = 340$ m/s.

***Solution***:

Frequency of sound detected by a stationary listener when a source of sound moves ***towards him*** is given by:

$$f' = \frac{V}{V - V_s} \times f$$

$$\therefore \quad f' = \frac{340}{340 - 18} \times 650$$

$$\therefore \quad f' = \frac{340}{322} \times 650 \qquad \boxed{f' = 686.3\text{ Hz.}}$$

Frequency of sound heard by the listener when the ambulance ***moves towards him*** is ***686.3 Hz.***

Frequency of sound detected by a stationary listener when ambulance moves ***away from him*** is given by:

$$f' = \frac{V}{V + V_s} \times f$$

$$\therefore \quad f' = \frac{340}{340 + 15} \times 650$$

$$\therefore \quad f' = \frac{340}{355} \times 650 \qquad \boxed{f' = 622.54\text{ Hz.}}$$

Frequency of sound heard by the listener when the ambulance ***moves away from him*** is ***622.54 Hz.***

***Q.No.17***: **A note of frequency of 500 Hz. Is being emitted by an ambulance moving towards a listener at rest. If the listener detects a frequency of 526 Hz.**
**Calculate the speed of ambulance.**
**(Speed of sound is 340 m/s at that moment)**
(***2011, 2009, 05, 07 Karachi Board***)

***Data***:

Frequency emitted $f = 500$ Hz.
Frequency detected $f' = 526$ Hz.
Speed of sound $V = 340$ m/s.
Speed of ambulance $V_s = ?$

***Solution***:

When the ambulance approaches a stationary listener with speed $V_s$, frequency of sound heard by the listener is given by:

$$f' = \frac{V}{V - V_s} f$$

$$\therefore \quad 526 = \frac{340}{340 - V_s} \times 500$$

$$\therefore \quad 340 - V_s = \frac{340 \times 500}{526}$$

$$\therefore \quad 340 - V_s = 323.194$$

$$\therefore \quad V_s = 340 - 323.194 \qquad \boxed{V_s = 16.8\text{ m/s}}$$

Ambulance is approaching the listener with a speed of ***16.8 m/s***.

***Q.No.18***: **Find the velocity of sound in a gas when two waves of wave lengths 0.80 m and 0.81 m respectively, produce four beats per sec.** (***2012, 1992 Karachi Board***)

***Data***:

Wave length of first wave $\lambda_1 = 0.80$ m
Wave length of second wave $\lambda_2 = 0.81$ m
Number of beats per second n = 4
Velocity of sound V =?

***Solution***:

If "$f_1$" and "$f_2$" are the frequencies of the two waves, then:

$$f_1 = \frac{V}{\lambda_1} \quad \& \quad f_2 = \frac{V}{\lambda_2}$$

But the number of beats heard per second is equal to the difference of frequencies of the two waves:

∴ Number beats per second = $f_1 - f_2$

∴ Number beats per second= $\frac{V}{\lambda_1} - \frac{V}{\lambda_2}$

∴ Number beats per second = $V\left\{\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right\}$

∴ $4 = V\left\{\frac{1}{0.80} - \frac{1}{0.81}\right\}$

$4 = V(1.25 - 1.23457)$

$4 = 0.01543\ V$

$V = \frac{4}{0.01543}$ **$V=259.24$ m/s.**

Velocity of sound in the gas is ***259.24m/s.***

***Q.No.19***: **A sound wave of frequency 500 Hz. in air enters a region of temperature 25°C to a region of temperature 5°C. Calculate the percentage fractional change in wave length.** (***2013 Karachi Board***)

***Data***:

Frequency of sound $f = 500$ Hz.
Initial temperature $T_1 = 25°C$
$= 25 + 273 = 298$ k
Final temperature $T_2 = 5°C$
$= 5 + 273 = 278$ k
Percentage fractional change in wave length $\frac{\Delta\lambda}{\lambda_1}\%$= ?

***Solution***:

Let velocity of sound in air at $T_0 = 0°C$ or 273 k be $V_0 = 332$ m/s, therefore, velocity of sound at $T_1$ can be calculated by:

$$\frac{V_1}{V_0} = \sqrt{\frac{T_1}{T_0}} \quad \text{OR} \quad \frac{V_1}{332} = \sqrt{\frac{298}{273}}$$

$$\frac{V_1}{332} = \sqrt{1.0916} = 1.045$$

Therefore: $V_1 = 1.045 \times 332$

***$V_1$ = 346.87 m/s***

Hence wavelength '$\lambda_1$' of waves at temperature '$T_1$' will be:

$$\lambda_1 = \frac{V_1}{f} = \frac{346.87}{500} = \mathbf{0.694\ m}$$

Similarly, velocity at $T_2$ is given by:

$$\frac{V_2}{V_0} = \sqrt{\frac{T_2}{T_0}}$$

$$\frac{V_2}{332} = \sqrt{\frac{278}{273}}$$

$$\frac{V_2}{332} = \sqrt{1.018} = 1.0091$$

∴ $V_2 = 1.0091 \times 332$ or ***$V_2$=335.027m/s***

Hence wavelength '$\lambda_2$' of waves at temperature '$T_2$' will be:

$$\lambda_2 = \frac{V_2}{f} = \frac{335.027}{500} = \mathbf{0.670\ m}$$

Fractional change in wave length (or change in wavelength per unit wave length) is given by $\frac{\Delta\lambda}{\lambda_1} = \frac{\lambda_1 - \lambda_2}{\lambda_1}$

$$\frac{\Delta\lambda}{\lambda_1} = \frac{0.694 - 0.670}{0.694} = 0.03458$$

∴ Percentage fractional change in wave length = fractional change × 100%

Percentage fractional change in wave length = 0.03458×100%=***3.458 %.***

***Q.No.20***: **A car has a siren sounding a 2 kHz. tone . What frequency will be detected by a stationary observer as the car approaches him at 80 km./h?**

**Speed of sound = 1200 km./h**

(***2015, 2015, 1998 Karachi Board***)

***Data***:

Frequency of sound produced by the siren $f$=2 kHz.= 2000 Hz.
Speed of the car (source) $V_S = 80$ km./h
Speed of sound V = 1200 km./h
Frequency of sound heard by the listener $f' =$?

***Solution*:**

In this case a source of sound (car) approaches a stationary listener (observer), hence the frequency of sound detected by the observer will be:

$$f' = \frac{V}{V - V_S} \times f$$

$$f' = \frac{1200 \times 2000}{1120} = \frac{2400000}{112}$$

$$\boxed{f' = 2142.9\ \text{Hz.}} \quad \text{OR} \quad \boxed{f' = 2.143\ \text{kHz.}}$$

🗣Frequency of sound heard by the listener is ***2.143 kHz.***

***Q.No.21*: A source of sound and a listener are moving towards each other with velocities which are 0.5 time and 0.2 time the speed of sound respectively. If the source of sound is emitting 2 kilo-Hertz tone, calculate the frequency heard by the listener.** (***2022, 1995,93 Karachi Board***)

***Data*:**

Speed of source of sound $V_S = 0.5\ V$

Speed of listener $V_O = 0.2\ V$

Frequency of sound emitted $f = 2$ kHz.= 2000Hz

Frequency of sound heard by listener $f' = ?$

***Solution*:**

In this case source of sound and the listener both approach each other, therefore, frequency of sound heard by the listener is given by:

$$f' = \frac{V + V_O}{V - V_S} \times f$$

$$f' = \frac{V + 0.2\ V}{V - 0.5\ V} \times f$$

$$f' = \frac{1.2\ V}{0.5\ V} \times f$$

$$f' = \frac{1.2}{0.5} \times f = 2.4 \times f = 2.4 \times 2000$$

$$\boxed{f' = 4800\ \text{Hz.}} \quad \boxed{f' = 4.8\ \text{kHz.}}$$

🗣Frequency of sound heard when source of sound and the listener move towards each other is ***4800 Hz.*** OR ***4.8 kHz.*** It is more than the sound emitted, therefore, sound heard is more shrill.

***Q.No.22*: A car has been sounding 4 kHz tone. What frequency will be detected by a stationary listener as the car approaches him at 50 km./h.**

**(Speed of sound in air = 1200 km./h).**

(***2025 Karachi Board***)

***Data*:**

Speed of the car $V_S = 50$ km./h.

Speed of sound in air $V = 1200$ km./h

Frequency of sound emitted $f = 4$ kHz.
= 4000Hz

Frequency of sound heard by listener $f' = ?$

***Solution*:**

In this case a source of sound (car) approaches a stationary listener (observer), hence the frequency of sound detected by the observer will be:

$$f' = \frac{V}{V - V_S} \times f$$

$$f' = \frac{1200 \times 4000}{1200 - 50}$$

$$f' = \frac{4800000}{1150}$$

$$\boxed{f' = 4174\ \text{Hz.}} \quad \text{OR} \quad \boxed{f' = 4.174\ \text{kHz.}}$$

🗣Frequency of sound heard by the listener is ***4.174 kHz.***

***Unit.13***..... ✍

# Physical optics

## ***Nature of light***:

***Q.No.1**: What is the nature of light*?

***Ans***: Light is a form of energy, **it makes things visible**, hence when a bulb is switched off in a room there will be darkness. If the room is such that no scattered light enters into it from any side, there will be perfect darkness, nothing will be visible. We will be able to see things if the bulb is again switched on. Actually, after **irregular reflection** from objects lying in the room light enters our eye making the objects visible. We will be able to see the bulb **(source of light)** and things lying in the room. But how does light travel from the bulb to the objects and then to our eye? We don't see light itself, although it has made things visible.

Hence: **"What is the nature of light?"**

**"How does it travel from one place to another?"**

**Answer** to these and many other questions were found experimentally over last several centuries. Now we have answers to these questions.

**The answer are …..**

Light has ***dual nature*** (in other words, light has double nature).

- In experiments such as ***interference, diffraction, polarization*** etc. light behaves as **waves, In other words** from one point to another light travels as waves. These phenomena can be explained only on the basis of ***wave theory of light***.
- In experiments such as ***photoelectric effect, Compton effect*** and ***pair production*** light behaves as **particles**. These experimental observations can only be explained if light is assumed to be traveling in the form of particles. These particles are small bundles of energy called ***photons*** (sometimes a photon is called a ***quantum*** of energy)
- Hence sometimes light beaves as ***waves*** and sometimes as ***particles***. We say light has ***dual nature***.
- There is **no single experiment** in which light behaves simultaneously both as ***waves*** as well as ***particles*** (photons).

## ***Electromagnetic waves***:

***Q.No.2**: How are electromagnetic waves produced*?

***Ans***: Electromagnetic waves are produced by ***accelerated charged particles***.

When **charged particles** are made to oscillate rapidly through an antenna, varying **electric** and **magnetic** fields are produced. These electric and magnetic fields vibrate

**perpendicular** to each other, the frequency with which they vibrate is same as the frequency of alternating current, used to accelerate the charged particles. These vibrating fields produced are radiated, such that the radiated vibrating electric field induces further vibrating magnetic field and so on, in this manner varying electric and magnetic fields, vibrating perpendicular to each other are radiated as ***electromagnetic waves***. **Note that** once the electromagnetic waves are produced accelerating charged particles are no more needed as these waves travel away changing electric field induces further changing magnetic field, this process goes on and the electromagnetic waves are continuously produced. Electromagnetic waves travel **perpendicular** to the direction of vibrations of both the fields.

> ***Do you know that ……***
> Visible light ***cannot*** pass through our body that is why shadows are formed, X-rays ***can*** pass through flesh of our body but not through our bones, hence in a way shadow of our bones is formed on X-ray film. Gamma ($\gamma$) rays ***can*** pass through even our bones. Visible light, X-rays and $\gamma$-rays are All electromagnetic waves, difference between them is in their frequency. Since energy of a wave depends upon its frequency therefore, $\gamma$-rays are highly energetic because they have highest frequency and therefore, they have highest penetration power. They can penetrate even through our bones !!!!

- Light waves are ***electromagnetic***, it means that light waves have electric and magnetic fields both ***vibrating perpendicular to each other*** whereas propagation of light waves takes place perpendicular to both the fields (**Example**: For a particular light wave if electric field is vibrating along x-axis, magnetic field is vibrating along y-axis then this particular light wave will travel along z-axis).
- Electromagnetic waves **can** travel through vacuum; they **do not** need any material medium for their propagation. That is why light comes from the sun (and moon and other stars in the night) to our beautiful little earth, although there is a vast free space (vacuum) between earth and the sun (and moon and other stars).
- Electromagnetic waves are ***transverse waves***. Light waves travel from its source in **all directions**.
- Speed of all electromagnetic waves including light waves is very high, speed with which light waves travel through free space is about $3\times10^8$ m/s or about three lac kilometer per second (300,000 km/s) (actually it is $2.98\times10^8$ m/s).
- Maxwell showed that speed of light in vacuum is given by:

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}} = 3\times10^8 \text{ m/s.}$$

Where "$\varepsilon_o$" is ***permittivity*** and "$\mu_o$" is ***permeability*** of free space, they are universal constants. S.I value of "$\varepsilon_o$" is $\mathbf{8.85\times10^{-12}\ C^2/N\text{-}m^2}$. and $\mu_o$ is $\mathbf{4\pi \times10^{-7}}$ **web./amp.m**.

- Speed of light represented by "c" is a **universal constant**.

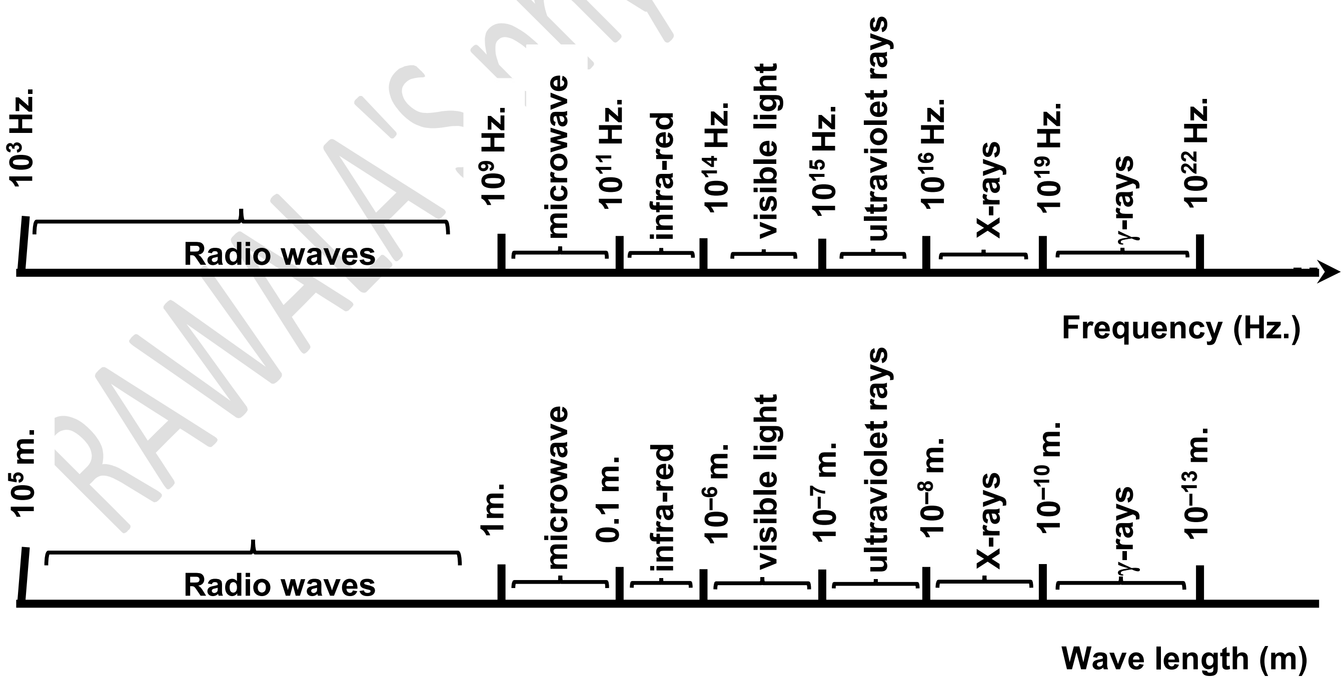
# ***Electromagnetic spectrum:***

Radio waves, Infrared rays, Visible light waves, Ultrviolet rays, X-rays and gamma rays are all electromagnetic waves. The difference between them is in their f**requency "*f*".** Since frequency "*f*" of a wave is **inversely proportional** to its wave length "$\lambda$", which means that **higher the frequency shorter is the wave length and vice versa**. But **energy** of a wave depends upon its **frequency**, higher the frequency higher is its energy. Therefore , electromagnetic waves of **high frequency** are of **high energy** but of **shorter wavelength**. In other words, **long wave length** electromagnetic waves are of **low energy**.

If we arrange all electromagnetic waves according to their frequency range (or wave length range) we will get an ***electromagnetic spectrum***.

Longest wavelength of electromagnetic waves is about $10^5$ m and shortest wavelength is about $10^{-15}$ m, corresponding to these wavelengths smallest frequency of electromagnetic waves is about $10^3$ Hz. and highest frequency is about $10^{22}$ Hz. (***In other words,*** wavelengths of electromagnetic waves are in the range of $10^5$ m to $10^{-15}$ m or frequency is in the range of $10^3$ Hz. to $10^{22}$ Hz.).

- ***Radio waves***: Frequency range: $10^3$Hz.- $10^9$ Hz (e.g. AM radio, FM radio, TV waves)
  Wave length range: $10^5$ m - 1 m
- ***Micro-waves***: Frequency range: $10^9$ Hz.- $10^{11}$ Hz (e.g. microwave oven, wi fi etc.)
  Wave length range: 1 m - 0.1 m
- ***Infra-red rays***: Frequency range: $10^{11}$ Hz - $10^{14}$Hz
  Wave length range: 0.1 m- $10^{-6}$ m
- ***Visible light***: Frequency range: $10^{14}$Hz - $10^{15}$Hz
  Wave length range: $10^{-6}$ m - $10^{-7}$ m
- ***Ultra-violet rays***: Frequency range: $10^{15}$Hz - $10^{16}$Hz
  Wave length range: $10^{-7}$ m - $10^{-8}$ m
- ***X-rays***: Frequency range: $10^{16}$Hz - $10^{19}$ Hz
  Wave length range: $10^{-8}$ m - $10^{-10}$ m
- ***γ-rays***: Frequency range: $10^{19}$Hz - $10^{22}$Hz
  Wave length range: $10^{-10}$ m - $10^{-13}$ m




## ***Electromagnetic spectrum***

- ***Note that*** in the entire electromagnetic spectrum, radio waves are **longest wave length** and of **smallest frequency** electromagnetic waves but gamma rays ($\gamma$-rays) are **shortest wave length** and of **highest frequency** electromagnetic waves. Since energy of waves is directly proportional to its frequency (or inversely proportional to its wavelength) therefore, gamma rays ($\gamma$-rays) are the **most energetic** electromagnetic waves (They have more energy than any other electromagnetic wave). On the other hand, radio waves are **least energy** electromagnetic waves.

# *Wave fronts*:

- The geometrical pattern (Or locus of all points) having same state of vibration (such as a crest) is known as a ***wave front***.
- A source of light produces wave fronts in all the three directions, hence light wave fronts are three dimensional. A point source of light produces spherical wave fronts.
- According to Huygens, any point on a wave front may be considered as a source of secondary spherical wave lets. If position of a wave front at a certain instant is known then its new position at a latter instant can be located by drawing a plane tangential to secondary wave lets.
- Any small portion of a spherical wave front at a large distance from its source will be plane in shape, hence it is known as a ***plane wave front***.

If a circular pointed rod is vertically dipped into still water ***circular wave fronts*** are produced on the surface of water. If the same rod is dipped in water length-wise then ***straight wave fronts*** are produced.

# *Ray of light*:

- Straight line which indicates the direction in which any given point on a light wave front travels is known as a ***ray of light***.

Ray of light is always ***perpendicular*** to the wave front. Hence in case of a spherical wave front rays spread out radially, but in case of a plane wave front rays are ***parallel*** to each other. Hence to get a beam of parallel rays we must take a small part of a spherical wave front at large distance from its source. In case of sun light rays are parallel.

# *Interference of light*:

***Q.No.1: What is interference of light?*** (***2010 Karachi Board***)

***Ans:*** When light waves having phase coherence superpose each other (they pass through the same region of space simultaneously) then at some points they **reinforce** and at other points they **cancel** each other's effect.

As a result of interference of light ***dark and bright fringes*** are obtained. ***Bright fringes*** are obtained at those points where light waves ***reinforce (or enhance)*** each other's effect, whereas ***dark fringes*** are obtained at those points where they ***cancel*** each other's effect.

- Interference of light is in accordance with the principle of ***superposition of waves***
- Interference of light shows that light ***behaves as waves*** i.e. Interference of light proves that light has ***wave nature***.

***Q.No.2*****: *What is phase coherence*?**

***Ans*****:** Waves are said to have phase coherence if they are **exactly similar** and are produced **exactly at the same time**. ***In other words***, interfering waves must have **same wave length**, **same frequency** and **same phase**.

***Q.No.3*****: *What is the condition for interference of waves*?**

***Ans*****:** The necessary condition for interference is that the interfering waves must have ***phase coherence***.

***Q.No.4*****: *What is constructive interference*?**

***Ans*****:** Waves are said to interfere constructively at those points where they ***reinforce*** each other's effect.

- In case of constructive interference of light, a ***bright fringe*** is produced.
- Constructive interference takes place at those points where **crest of one wave overlaps crest of the other** or **trough of one wave overlaps trough of the other wave**.
- As a result of constructive interference of waves, either a bigger crest (of larger amplitude) or a deeper trough is produced.
- Intensity of light at points of constructive interference is ***maximum***.

***Q.No.5*****: *What is destructive interference*?**

***Ans*****:** Waves are said to interfere destructively at those points where they ***cancel*** each other's effect.

In case of destructive interference of light, a ***dark fringe*** is produced.

- Destructive interference takes place at those points where **crest of one wave overlaps trough of the other wave**.
- As a result of destructive interference of waves, **neither crest nor a trough remains**.
- Intensity of light at points of destructive interference is ***minimum***.

***Q.No.6*****: *What does interference of light prove?***

***Ans*****:** interference of light proves that light travels in the form of ***waves***.

***Q.No.7*****: *What is meant by path difference*?**

***Ans*****:** It is the difference in distance covered by waves measured from their sources to the given point.

***Q.No.8*****: *What is the condition for constructive interference*?**

***Ans*****:** Condition for constructive interference to take place at a point is that the ***path difference*** between the interfering waves at that point must be an integral multiple of their wave length (i.e. path difference = 0, $\lambda$, 2 $\lambda$, 3 $\lambda$, ….).

$$\text{Path difference} = m\,\lambda$$

And ***phase difference*** between the waves must be an integral multiple of $2\pi$ radian (i.e. phase difference = 0, $\pi$ radian, 2 $\pi$ radian, 3 $\pi$ radian, ….).

$$\text{Phase difference} = 2m\pi \text{ radian}$$

Where m is a positive integer, m = 0, 1, 2, 3……..
and $\lambda$ is wave length of waves.

(When waves of same phase travel equal distance from their source, path difference = 0, either crests or troughs of the two waves simultaneously reach the given point, similarly if one wave covers more distance than the other wave such that the difference in distance covered by them, i.e. path difference is $\lambda$, again either crests or troughs from the two sources will reach simultaneously at the given point, so that in both these cases we get a bright fringe due to constructive interference).

- Constructive interference takes place at those points where crest from one source of light overlaps a crest from the other source, or a trough from one source overlaps a trough from the other source.

***Q.No.9*: What is the condition for destructive interference?**

***Ans*:** Condition for destructive interference to take place at a point is that the ***path difference*** between the interfering waves at that point must be an odd multiple of $\lambda/2$ (i.e. path difference = $\lambda/2$, $3\lambda/2$, $5\lambda/2$, ….).

$$\text{Path difference} = (m + \tfrac{1}{2})\lambda$$

And ***phase difference*** between the waves must be an odd multiple of $\pi/2$ radian (i.e. phase difference = $\pi/2$ radian, $3\pi/2$ radian, $5\pi/2$ radian, ….).

$$\text{Phase difference} = (m + \tfrac{1}{2})\pi \text{ radian}$$

Where m is an integer, m = 0, 1, 2, 3, ……..
and $\lambda$ is wave length of waves.

(When distance traveled by waves, from their sources, on reaching a given point is such that the path difference is $\lambda/2$, $3\lambda/2$, $5\lambda/2$ etc. or an odd multiple of $\lambda/2$, then a crest from a source will overlap a trough from the other. In such a case light from one source will cancel light from the other, no light remains. ***In other words***, a dark fringe is obtained).

- Destructive interference takes place at those points where crest from one source overlaps a trough from the other source.

***Note that*** the phenomenon of interference is the ***basic characteristic*** of wave motion It means that ***all types of waves*** (including mechanical or electromagnetic waves) under suitable conditions show i***nterference effects***.

***Q.No.10*: What happens if two bulbs are used to observe interference?**

***Ans*:** With two separate sources of light (e.g. light from two bulbs) no interference effect is produced, because light waves emitted by two different sources ***cannot have phase coherence***. The emission of light is a completely independent process. If one source of light produces a crest at a certain instant it is not necessary that the other source must also produce a crest at that instant.

# *Young's double slit experiment*:

***Q.No.1*: How did Thomas Young achieve the condition of phase coherence for light in his double slit experiment?**

***Ans*:** To produce interference effect of light, Thomas Young achieved the condition of phase coherence in his double slit experiment by using one source of monochromatic light. Light from this source was allowed to fall on a narrow slit whose width was comparable to wavelength of light. After passing through the slit light spreads out as spherical wave fronts. This single slit may be considered as a single coherent source of light. Wavefronts from this slit now pass through two more narrow slits at equal distance from the first slit. Light after passing through the two slits will have phase coherence as it was originally produced by one source.

Hence light emerging from two slits can be considered to be emitted from two sources of light having **phase coherence**.

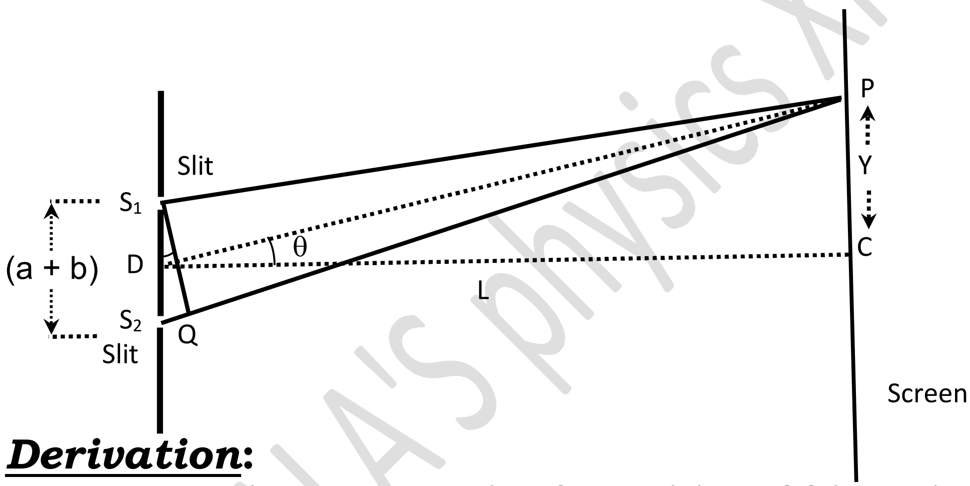
***Q.No.2*: What is monochromatic light?**

***Ans*:** Light of ***one wave length*** is called ***monochromatic light***. Actually, monochromatic light means light of one color (mono = one, chromatic = color). Practically light of one color may have slightly different shades of slightly different wavelengths. Hence in strict sense, for scientific work, light having one wavelength is taken as monochromatic.

***Q.No.3*: Describe Young's double slit experiment.**

***Ans.*** In Young's double slit experiment light from a monochromatic source after passing through a narrow slit, is allowed to pass through two parallel narrow slits separated by a small distance. Two beams of light emerging from the two slits ***will have phase coherence***, and will be in ***phase***, because they were originally produced by the same source. Hence the two beams of light will produce interfere effects. Due to their interference, bright and dark bands are produced on a screen placed at some distance "L" from the slits. These bands are called "***fringes***".

Bright fringes are produced at those points where light waves from the two slits interfere constructively, at these points intensity of light will be **maximum**. Whereas dark fringes are produced at those points where light waves from the two slits interfere destructively, at these points intensity of light will be **minimum**.




## ***Derivation*:**

***Q.No.4*: Derive an expression for position of fringes in Young's double slit experiment.**

***Ans.*** At any point on the screen light waves will interfere according to their path difference. Let the distance of a point "P" from the center C of the screen be "Y". To reach point "P" waves coming from slit "$S_2$" will cover longer distance than the waves coming from "$S_1$". The difference of distances covered by the two waves is called "***path difference***". We can see from the figure that the path difference between the waves coming to point "P" from $S_1$ and $S_2$ is "$S_2Q$". Triangles DPC and $S_1S_2Q$ are similar, therefore:

$$\frac{PC}{S_2Q} = \frac{DP}{S_1S_2}$$

But PC = Y (Distance of point P from the center of the screen).

$S_1S_2$ = d (Separation of slits).

Since point "P" is practically very close to the center "C" of the screen, In other words "Y" is practically very small as compared to "L". Hence:

DP ≈ DC = L (D is the center of slits)

$\therefore \quad \dfrac{Y}{S_2Q} = \dfrac{L}{d}$

$\therefore \quad \boxed{Y = S_2Q \times \dfrac{L}{d}}$

## Position of bright fringes:

***Q.No.5: Derive an expression for position of bright fringes in Young's double slit experiment.***

***Ans.*** For bright fringes, where constructive interference takes place, the path difference "$S_2Q$" must be an integral multiple of "$\lambda$", Therefore:

$\therefore \quad S_2Q = m\lambda$

Where m = 0, 1, 2, 3, …….. (a positive integer)

On taking path difference $S_2Q = m\lambda$, the above equation gives us distance "Y" of various bright fringes from center C of the screen.

$\therefore \quad \boxed{Y = m\lambda \dfrac{L}{d}}$ (For bright fringes)

If we take m = 0, the path difference between two beams of light coming from the two slits will be equal to zero, and light will interfere constructively forming a bright fringe. But the two beams of light will cover equal distances on reaching "C", hence the center "C" of the screen will always be ***bright***. (m = 0 corresponds to C).

Similarly, first, second, third, …….. etc. bright fringes will be formed above and below C at distances $\dfrac{\lambda L}{d}$, $2\dfrac{\lambda L}{d}$, $3\dfrac{\lambda L}{d}$, $4\dfrac{\lambda L}{d}$, ………. etc. from the center C of the screen.

## Position of dark fringes:

***Q.No.6: Derive an expression for position of dark fringes in Young's double slit experiment.***

***Ans.*** For dark fringes, where destructive interference takes place, the path difference "$S_2Q$" must be an odd integral multiple of "$\lambda/2$", Therefore:

$\therefore \quad S_2Q = (m + \frac{1}{2})\lambda$

Where m = 0, 1, 2, 3, …….. (a positive integer)

On taking path difference $S_2Q = (m + \frac{1}{2})\lambda$, the above equation gives us distance "Y" of various dark fringes from center C of the screen.

$\therefore \quad \boxed{Y = (m + \tfrac{1}{2})\lambda \dfrac{L}{d}}$ (For dark fringes)

First, second, third, …….. etc. dark fringes will be formed above and below C at distances $\dfrac{\lambda L}{2d}$, $\dfrac{3\lambda L}{2d}$, $\dfrac{5\lambda L}{2d}$, $\dfrac{7\lambda L}{2d}$, ………. etc. from the center C of the screen.

Between two bright fringes there will be a dark fringe.

***Note that*** for each value of "m" a bright and a dark fringe, at equal distance, is obtained above and below "C" whereas C itself will always be ***bright*** and it will correspond to zero path difference.

# ***Fringe spacing:***

***Q.No.7: What is fringe spacing? Derive an expression for fringe spacing.***

***Ans:*** It is the ***distance*** between two ***consecutive*** (side by side) ***bright*** or ***dark*** fringes.

Expression for fringe spacing can be obtained by subtracting the expressions for distances of two adjacent bright or dark fringes from center of the screen. Hence:

Distance of (say) 4th and 5th bright fringes from the center of screen is given by:

$$Y_4 = \frac{4\lambda L}{d} \qquad \text{and} \qquad Y_5 = \frac{5\lambda L}{d}$$

$$\therefore \qquad \text{Fringe spacing} = Y_5 - Y_4$$

$$= \frac{5\lambda L}{d} - \frac{4\lambda L}{d}$$

$$\boxed{\text{Fringe spacing } \Delta x = \frac{\lambda L}{d}}$$

In Young's double slit experiment, fringes are ***equally spaced*** (i.e. at equal distance from each other).

***Q.No.8: How can wave length of light 'λ' be determined by Young's double slit experiment?***

***Ans:*** Distance between any two adjacent bright or dark fringes (called fringe spacing) in Young's slit experiment is given by:

$$\text{Fringe spacing } \Delta x = \frac{\lambda L}{d}$$

Fringe spacing '$\Delta x$' can be measured accurately, distance between screen and the slits 'L' and the separation of slits 'd' are measurable, therefore, wave length 'λ' of light used can be calculate with high degree of accuracy. This is the reason why Young's double slit experiment is used to determine wave length of light.

***Q.No.9: What will happen if light used in Young's double slit experiment is NOT monochromatic?***

***Ans:*** If incident light used in Young's double slit experiment is not monochromatic i.e. it is of two or more different wave lengths (colors) then for each wave length a separate bright and a dark fringe is obtained for each value of integer "m" (for given L and d).

***In other words***, for each value of 'm', we will get a series of parallel lines, number of these colored lines for each 'm' depends upon the number of wavelengths present in the interfering light. It is, therefore, possible that a bright fringe of one wavelength for a given 'm' may overlap a dark fringe of another wavelength for some other value of 'm'. All these observations will take place according to the path difference for each wave length for each value of 'm'.

- Young's double slit experiment supports ***wave theory*** of light.

# ***Interference in thin films:***

***Q.No.1: What is phase reversal? When does it take place?***

***Ans.*** When a light wave traveling in a rarer medium (medium of lower refractive index) reflects from the surface of a denser medium (medium of higher refractive index), then its crest before reflection becomes a trough after reflection and vice versa. Under this condition the wave is said to have undergone ***phase reversal***. The reflected wave is $180^\circ$ out of phase as compared to the original wave.

**No phase reversal** will take place if a wave traveling in a medium of higher refractive index (a denser medium) reflects from the surface of a medium of lower refractive index (a rarer medium).

***Q.No.2: Give examples of interference in a thin film?***

***Ans. Example:*** When sun light falls on the surface of a very thin film made of a refracting medium, such as the surface of a ***soap solution bubble*** or ***surface of a thin layer of oil*** spread on the surface of water, colored patterns are formed. These colored patterns are ***due to the interference of light***.

In these cases, interference of light is between light waves reflected from the opposite surfaces of the soap bubble or oil film. Since sun light consists of seven colors, therefore, due to interference between waves of various wavelengths colored patterns are formed.

***Q.No.3: Is there a phase reversal in Young's double slit experiment?***

***Ans.*** No!

There is no phase reversal in Young's double slit experiment, because light waves travel in the same medium, there is no change of refractive index. Phase reversal takes place when light is reflected from the boundary of two media of different refractive index.

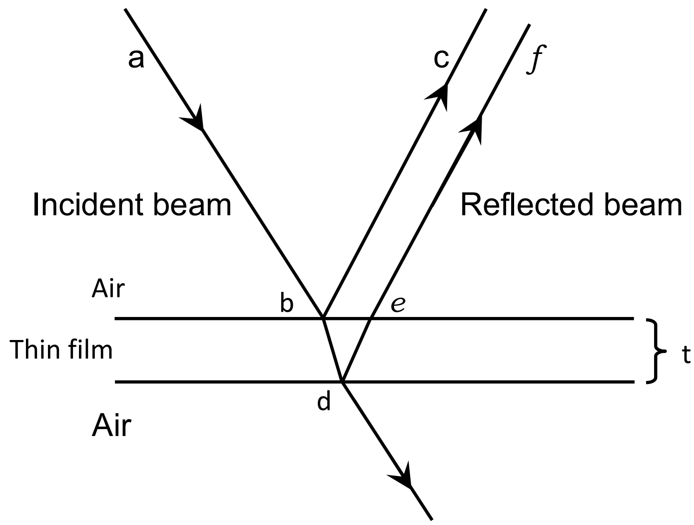
***Q.No.4: Describe interference of light in a thin film.***

***Ans. Explanation:***

Consider a beam of monochromatic light traveling in air and falling on a very thin film of a refracting medium of refractive index "$n$". This beam of light is partially reflected from the upper surface of the film and partially refracted into it. Since the reflected part "b c" was initially travelling in air (a medium of lower refractive index) and is reflected from the surface of a denser medium, such as the surface of soap solution bubble (a medium of higher refractive index), therefore, it will undergo ***phase reversal***, it's phase will change by $180^\circ$. It means that an ***incident crest*** after reflection from the surface of the film becomes a ***trough*** and vice versa.

The refracted part undergoes reflection from the lower surface of the film (at d) Since this part before reflection from the lower surface of the film was traveling in a denser medium e.g. soap solution (medium of higher refractive index), therefore, it does not undergo phase reversal.

The refracted part after reflection from the lower surface of the film emerges out as "$e\,f$" without phase reversal (i.e. crests as crests or troughs as troughs).




Wave length of light also <u>changes</u> in thin film interference. Hence when light of wave length "λ" travels from air (a rarer medium) to a denser medium of refractive index "$n$" its wave length changes (decreases) and is given by:

$$\lambda_n = \frac{\lambda}{n}$$

Parts "b c" and "$e\,f$" on reaching the eye will interfere according to their path difference.

***Q.No.5*: What is the condition for constructive interference in a thin film?**

***Ans*:** In case of thin film interference since one of the interfering parts has undergone phase reversal, (It means that one part will have a crest and the other part will have a trough), therefore, condition for constructive interference will have to be reversed. Hence condition for constructive interference in a thin film is:

$$\boxed{\text{Path difference} = (m + \tfrac{1}{2})\,\lambda_n}$$

Where m is a positive integer, m = 0, 1, 2, 3, ……..

and $\lambda_n$ is wave length of light in the refracting medium (e.g. soap solution film).

If "t" is thickness of the film, then for normal incidence (i.e. when light falls normally on the film) the path difference will be "2 t", hence condition for constructive interference will be:

$$2\,t = (m + \tfrac{1}{2})\,\lambda_n$$

OR

$$2\,t = (m + \tfrac{1}{2})\,\frac{\lambda}{n}$$

$$\therefore \quad \boxed{2\,n\,t = (m + \tfrac{1}{2})\,\lambda}$$

***Q.No.6*: What is the *condition for destructive interference in a thin film*?**

***Ans*:** Condition for destructive interference in a thin film is:

$$\boxed{\text{Path difference} = m\,\lambda_n}$$

Where m is a positive integer, m = 0, 1, 2, 3, ……..

and $\lambda_n$ is wave length of light in the refracting medium..

If "t" is thickness of the film, then for normal incidence the path difference will be "2 t", hence condition for destructive interference will be:

$$2\,t = m\,\lambda_n$$

OR

$$2\,t = m\,\frac{\lambda}{n}$$

$\therefore$

$$\boxed{2\,n\,t = m\,\lambda}$$

***Note that*** the conditions for constructive and destructive interference in a thin film has to be reversed only because of ***phase reversal*** in one of the interfering parts.

In Young's double slit experiment normal conditions for constructive or destructive interference apply. In that case, ***no phase reversal*** takes place.

# Newton's rings:

***Q.No.1*: What are Newton's rings?** (***Karachi Board 1994***)

***Ans.*** If a beam of monochromatic light is allowed to fall on a plano-convex lens placed on a plane glass plate then alternate bright and dark rings are produced, these rings are known as ***Newton's rings.***

Actually, air present between the plano-convex lens and the surface of plane glass acts as a refracting film of a variable thickness. Newton's rings are formed because of ***interference of light*** taking place between light beams reflected from the upper and lower surfaces of the air film present between lens and the plane glass.

The air film present between lens and the plane glass is circular and wedge shaped.

Centre of Newton's rings is always ***dark***; it corresponds to the point of contact of lens and the plane glass surface. Here path difference is ***zero***.

Formation of Newton's rings is due to ***thin film interference***.

Formation of Newton's rings also proves an important property of light …. ***Interference***, which is a basic characteristic of wave motion and shows that light travels in the form of ***waves***.

# Derivation:

***Q.No.2*:Show how can Newton's rings be used to find the radius of curvature of curved surface of a convex lens?**

***Ans.*** For any bright Newton's ring of radius "r" let "t" is the thickness of the corresponding air film and "R" is the radius of curvature of convex lens, then from the geometry of the following figure we have:

$$AB \times BC = r \times r$$

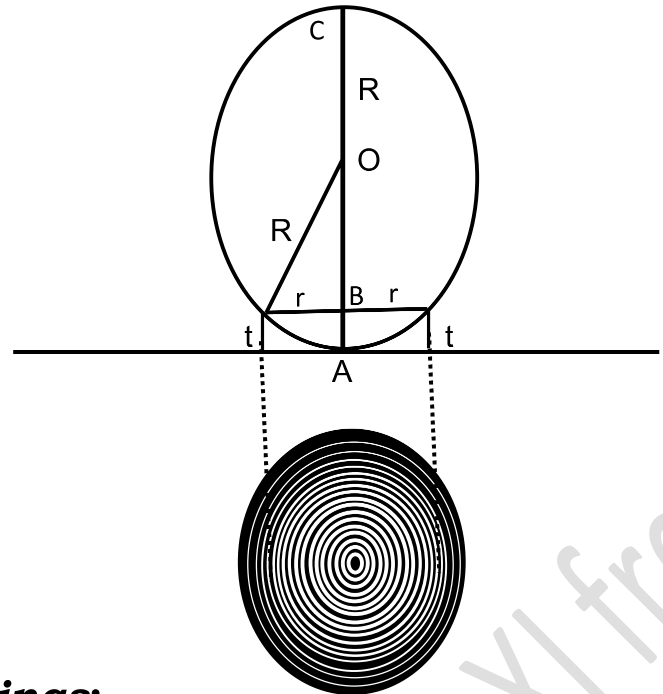
But $AB = t$ and $BC = (2\,R - t)$

$$\therefore \quad t\,(2\,R - t) = r^2$$

$$2\,R\,t - t^2 = r^2$$

Since the air film is very thin, therefore, "t" is very small, hence "$t^2$" is negligible. Neglecting "$t^2$" we get:

$$2\,R\,t = r^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (i)$$




## ***For bright rings*:**

The condition for constructive interference in a thin film or in other words , in this case for a bright Newton's ring is:

$$2\,n\,t = (m + \tfrac{1}{2})\,\lambda$$

In this case the thin film is made of air, whose refractive index $n$ =1,

$$\therefore \qquad 2\,t = (m + \tfrac{1}{2})\,\lambda \quad \ldots\ldots\ldots\ldots\ldots.. (ii)$$

On substituting equation (*ii*) in equation (*i*), we get:

$$r^2 = R\,(m + \tfrac{1}{2})\,\lambda$$

$$\therefore \qquad \boxed{r = \sqrt{R\,\lambda\,(m + \tfrac{1}{2})}} \quad \ldots\ldots\ldots\ldots\ldots..(iii)$$

When we put m = 0 in the above equation, we will get radius of the first bright ring, given by:

$$r_1 = = \sqrt{\frac{R\,\lambda}{2}}$$

Similarly for second, third, fourth etc. bright rings m = 1, 2, 3, ……….etc. respectively. Hence the radii of second, third, fourth etc. bright rings are given by:

$$r_2 = = \sqrt{\frac{3R\,\lambda}{2}} \qquad m = 1 \text{ (for second ring)}$$

$$r_3 = = \sqrt{\frac{5\,R\,\lambda}{2}} \qquad m = 2 \text{ (for third ring)}$$

$$r_4 = = \sqrt{\frac{7\,R\,\lambda}{2}} \qquad m = 3 \text{ (for fourth ring)}$$

$$r_5 = \sqrt{\frac{9\,R\,\lambda}{2}} \qquad m = 4 \text{ (for fifth ring) etc.}$$

If "N" is the number of bright rings, then $N = m + 1$ OR $m = N - 1$:
On substituting the expression for "m" in equation (*iii*) we will get radius of N th bright ring.

$$r_N = \sqrt{R\,\lambda\,(N - 1 + \tfrac{1}{2})}$$

Radii of bright rings: $$\boxed{r_N = \sqrt{R\,\lambda\,(N - \tfrac{1}{2})}}$$

This formula gives us radius of the N th bright ring, it can also be used to find the radius of curvature "R" of the convex lens accurately.

***Note that*** "m" is a positive integer i.e. 0, 1, 2, 3 ………… Whereas "N" is the number of Newton's bright ring.

## *For dark rings*:

The condition for destructive interference in a thin film, ***in other words***, in this case for a dark Newton's ring is: $2\,n\,t = m\,\lambda$ (m = 0, 1, 2, 3, 4 ….)

Since the thin film is made of air, whose refractive index $n = 1$,

$$\therefore \quad 2\,t = m\,\lambda \quad \ldots\ldots\ldots\ldots (iv)$$

On substituting equation (*iv*) in equation (*i*), we get:

$$r^2 = m\,R\,\lambda$$

Radii of dark rings: $$\boxed{r = \sqrt{m\,R\,\lambda}} \quad \ldots\ldots\ldots\ldots (v)$$

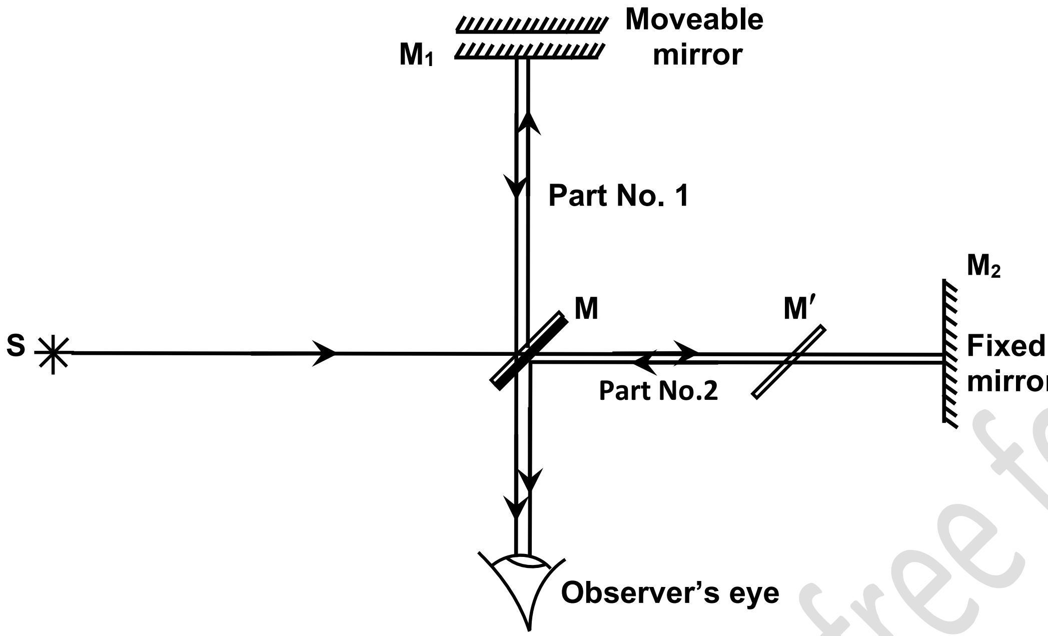
Where m = 0 for the central dark spot, which corresponds to the point of contact of lens and the plane glass plate, it is dark due to destructive interference.
Similarly, m = 1, 2, 3, …. for first, second, third ,……. dark rings.

# *Michelson's interferometer*:

## *Construction and working*:

In a Michelson's interferometer light from a monochromatic source "S" (Source of light may be extended, whereas in Young's double slit experiment fine beam of light is used) is allowed to fall on a semi silvered glass plate "M" (also called beam splitter) held at an angle of 45° to the beam of light. From this glass plate light is partially reflected and partially refracted through it. The reflected part No. 1 goes towards a highly polished plane mirror "$M_1$" which is held perpendicular to the reflected beam. After reflection from $M_1$ this part of light passes through the glass plate "M" and finally it enters the observer's eye.

The transmitted part No.2 after passing through another plane glass plate "M′ " which is not silvered and is of the same thickness as the semi silvered glass plate "M", falls on another highly polished plane mirror "$M_2$". Glass plate "M′ " is called compensating plate, it is used so that both parts of light pass through the same thickness of glass. The transmitted part after reflection from $M_2$ passes through M′ and is then reflected from M, also enters the eye. The two parts of light entering the observer's eye will have ***phase coherence***, because they were originally produced by the same source. Hence they will interfere, producing bright and dark fringes.




Plane mirror "$M_1$" is moveable and any small distance through which it is moved can be measured accurately. Mirror "$M_2$" is fixed.

## ***Derivation:***

Let the distances covered by part No.1 and No.2 is exactly equal when they enter the observer's eye. Hence path difference will be zero and the two parts will ***interfere constructively*** due to which a ***bright*** fringe appears at a reference point in the field of view of the observer.

Now if the moveable mirror "$M_1$" is moved through a distance equal to $\lambda/4$ then path difference between the two parts will be $\lambda/2$ due to which the ***bright*** fringe is replaced by a ***dark*** fringe. If the moveable mirror is further moved through a distance equal to $\lambda/4$ the path difference will become $\lambda$ and now the ***dark*** fringe will be replaced by a ***brigh***t one.

In this manner alternate bright and dark fringes will cross a reference point, each time "$M_1$" is moved through a distance $\lambda/4$.

***In other words***, if the moveable mirror is moved through $\lambda/2$, a bright fringe will be replaced by the next bright fringe. Hence if "m" bright fringes cross a reference point then the distance through which the moveable mirror is moved is given by:

$$d = m\frac{\lambda}{2}$$

If the distance "d" through which the moveable mirror is moved to observe "m" fringes crossing a reference point is known, then the wave length "$\lambda$" of the incident light can be calculated.

# ***Diffraction of light:***

***Q.No.1: What is diffraction of light?***

***Ans:*** The ***bending of light*** across the edges of an obstacle or an opening into the geometrical shadow is called ***diffraction of light***.

Diffraction of light takes place only when dimensions of the obstacle or the opening is ***comparable to the wave length*** of light.

After bending across the edges of an obstacle, diffracted waves interfere with each other on the screen, as a result of which alternate bright and dark fringes are obtained.

Distance between bright fringes obtained in diffraction pattern ***increases*** as the angle of diffraction is ***increased***.

Also, the intensity of light gradually ***decreases*** with the angle of diffraction.

***Note that*** diffraction of light is another phenomenon that supports ***wave theory of light*** (i.e. light travels in the form of waves). In this process light exhibits ***wave nature***.

## ***Difference between interference and diffraction of light:***

- The main difference between interference and diffraction is that, the fringes in interference are ***equally spaced*** (i.e. at equal distance from each other) but in diffraction pattern they ***are not equally spaced***, their ***separation increases*** with the angle of diffraction.
- In Interference bright fringes are of ***uniform intensity*** whereas bright fringes produced in diffraction are of ***non-uniform intensity***. Intensity of light gradually ***decreases*** with the angle of diffraction.

# ***Diffraction grating:***

***Q.No.2: What is a diffraction grating?***

***Ans:*** A diffraction grating consists of a transparent glass sheet on which large number of opaque parallel lines (equally spaced) very close to each other, are ruled (Usually several hundred lines per centimeter are ruled).

The transparent portion of the glass between any two adjacent opaque lines acts as a slit. Its width is comparable to the wave length of ordinary light.

Let "a" be the width of a slit and "b" be the width of an opaque line then, (a + b) is known as the "***grating element***".

***Q.No.3: How is grating element calculated?***

***Ans:*** The grating element is calculated ***by dividing length of the diffraction gating by the number of lines ruled on it.***

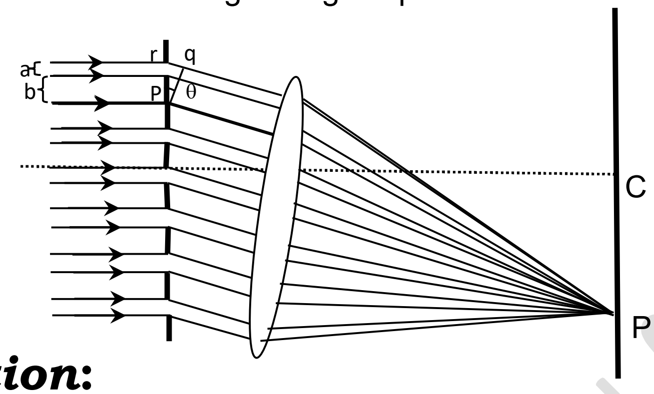
***For example,*** let a diffraction grating has 2000 lines/cm. then grating element for this grating is given by:

$$(a + b) = \frac{1\text{ cm.}}{2000} = 5\times10^{-4}\text{ cm.} = 5\times10^{-6}\text{ m}$$

## ***Working of diffraction grating:***

***Q.No.4: Show how diffraction grating can be used to find accurately the wave length of light used?***

***Ans:*** When a monochromatic beam of parallel rays falls normally on diffraction grating, they are diffracted in many directions. The parallel diffracted rays in a particular direction are allowed to pass through a convex lens of suitable focal length which focuses them on a screen at some point "P" where a bright fringe is produced.




## ***Derivation:***

For any two beams, diffracted from two adjacent slits, let the angle of diffraction be "θ". The path difference between the two diffracted beams at a point "P" is "r q", it must be an integral multiple of wave length "λ" so that a bright diffracted image is formed at "P".

In $\Delta$ p q r: $\sin\theta = \dfrac{r\,q}{p\,r}$

$\therefore \quad r\,q = p\,r \sin\theta$

But p r = (a + b) is the grating element and r q is the path difference.

$\therefore \quad r\,q = (a + b)\sin\theta$

For bright diffracted images the path difference "r q" must be an integral multiple of "λ".

$$\therefore \quad \boxed{m\lambda = (a + b)\sin\theta}$$

$$\text{Or} \quad \boxed{m\lambda = d\sin\theta}$$

Where m = 0, 1, 2, 3, …………… (an integer)

For m = 0 we will get zero $^{th}$ order un-diffracted image, at the center C.

But for m = 1, 2, 3, ….. we will get 1$^{st}$ order, 2$^{nd}$ order, and 3rd order etc. images for which:

$$\lambda = (a + b)\sin\theta$$
$$2\lambda = (a + b)\sin\theta$$
$$3\lambda = (a + b)\sin\theta \;\ldots\ldots\ldots\ldots\text{etc.}$$

Selecting a suitable value of "m" (order of image), and measuring the corresponding angle of diffraction "θ", the wave length of incident light "λ" can be calculated with the help of one of the above equations.

## ***Diffraction of X-rays through crystals:***

***Q.No.1: Why is a crystal needed for the diffraction of X-rays?***

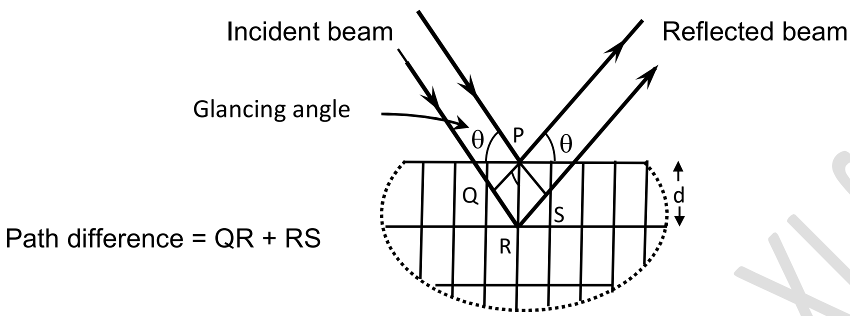
***Ans:*** Wave length of X-rays is ***very short*** as compared to the wave length of visible light. Therefore, they ***do not*** show diffraction effects by ordinary diffracting objects (such as diffraction grating, fine slits or sharp-edged objects like shaving blade etc.). Similarly for the same reason, interference pattern of X-rays by Young's double slit or by a thin soap solution film or thin oil film ***cannot*** be produced.

In certain crystals (such as rock salt), the atoms are arranged in a regular manner such that the planes thus formed are at a distance (about 2 Å - 5 Å) comparable to the wave length of X-rays. Hence X-rays can be diffracted with the help of a suitable crystal.

***Q.No.2*: Derive Brag's law**?

***Ans*:** Consider a beam of X-rays incident upon a crystal, beams of X-rays reflected from two adjacent planes of the crystal will interfere constructively when their path difference is an integral multiple of their wave length.

An X-ray beam is reflected from the upper plane and the second X-ray beam is reflected from the lower plane. These two beams will cover different distances before entering the eye (second beam will cover longer distance). Difference in distance covered, called ***path difference***, can be determined with help of the following diagram.




## Calculation of path difference:

In triangle PQR: $\sin\theta = \dfrac{QR}{PR}$

∴ $QR = PR \sin\theta$ But $PR = d$ (Distance between atomic planes)

∴ $QR = d \sin\theta$

Similarly: $RS = d \sin\theta$

But path difference between the two beams of X-rays = $QR + RS$

∴ Path difference = $d \sin\theta + d \sin\theta$

∴ **Path difference = $2 d \sin\theta$**

Hence path difference between two X-rays beams reflected from two adjacent lattice planes of the crystal is "$2 d \sin\theta$, where "d" is the distance between the two planes, and "θ" is the glancing angle. These beams of X-rays will interfere constructively if their path difference is an integral multiple of its wave length "λ". Therefore,

$$m\lambda = 2 d \sin\theta \qquad (m = 1, 2, 3, \ldots.)$$

- This relation is known as ***Brag's law***.
- The separation "d" of planes of the crystal can be calculated with the help of its density and atomic weight. Since "m" and "θ" can be measured accurately by experiment, the wave length of X-rays can be calculated.

## ***Section–A Multiple-Choice Questions MCQs*:**

***Q.No.1*:** If the wavelength of an electromagnetic wave is about the diameter of a cricket ball what type of radiation is it.

(a) X-rays (b) Ultraviolet.
(c) Radio waves. (d) Visible light.

***Q.No.2*:** Electromagnetic waves from an unknown source in space are found to be diffracted when passing through gaps of the order of $10^{-5}$ m. Which type of the waves are they most likely to be?

(a) Microwaves. (b) Ultraviolet.
(c) Radio waves. (d) Infra-red.

***Q.No.3*:** Huygen's conception of secondary waves:

(a) Helps us to find the focal length of a thick lens.
(b) is a geometrical method to find a wavefront
(c) is used to determine the velocity of light.
(d) is used to explain the polarization of light.

***Q.No.4*:** Interference fringes are produced using monochromatic light of same intensity from a double slit screen. If the intensity of light emerging from one of the slits is reduced, the effect on interference pattern will be:

(a) all the dark and bright fringes become brighter
(b) all the dark and bright fringes become darker
(c) Bright fringes become brighter and dark fringes become darker.
(d) Bright fringes become darker and dark fringes become brighter.

***Q.No.5*:** In Young's double slit experiment when the distance between slits and screen is doubled, while separation of slits is halved, then fringe width will be:

(a) 4 times. (b) ¼ times
(c) doubled. (d) unchanged.

***Q.No.6*:** A ray of light passes from air into water, striking the surface of water at an angle of incidence 45°. Which of these quantities change as the light enters water,

($i$) Wavelength ($ii$) frequency
($iii$) Speed of propagation
($iv$) direction of propagation.

(a) $i$ and $ii$ only. (b) $iii$ and $iv$ only.
(c) $i$, $iii$ and $iv$ only. (d) all of them.

***Q.No.7*:** A hill separates a television (TV) transmitter from a house. The transmitter cannot be seen from the house but still the TV in the house has good reception. What wave phenomena make it possible?

(a) Coherence of waves. (b) Diffraction of waves.
(c) Interference of waves.
(d) Refraction of waves.

***Q.No.8*:** Monochromatic light is incident on a diffraction grating and a pattern is observed. What effect is observed by replacing the grating with one that has more lines per millimeter?

(a) Number of maxima **decreases** with **decrease** in angle between first and second order maxima.
(b) Number of maxima **decreases** with **increase** in angle between first and second order maxima.
(c) Number of maxima **increases** with **decrease** in angle between first and second order maxima.
(d) Number of maxima **increases** with **increase** in angle between first and second order maxima.

***Q.No.9*:** Optically active substances are those substances which:

(a) produce polarized light.
(b) rotate the plane of polarized light.
(c) Produce double refraction.
(d) convert a plane polarized light into a circularly polarized light.

***Q.No.10*:** Plane polarized light is passed through a Polaroid. On viewing through the polaroid, we find that when Polaroid is given one complete rotation about the direction of light:

(a) The intensity of light gradually decreases to zero and remains at zero.
(b) The intensity of light gradually increases to maximum and remains at maximum.
(c) There is no change in intensity of light.
(d) The intensity of light varies such that it is twice maximum and twice zero.

**Q.No.11**: In Young's double slit experiment, the fringe spacing is: (***2004,14 Karachi Board***)

- $\frac{d\lambda}{L}$ • $\frac{\lambda L}{d}$ • $\frac{d}{\lambda L}$

**Q.No.12**: Which of the following phenomenon cannot be explained by wave theory?
(***2005, 2004 Karachi Board***)

- interference •diffraction •photoelectric effect

**Q.No.13**: The condition for the interference in a thin film is reversed because of:
(***2016, 15, 08, 04 Karachi Board***)

- small thickness.
- phase reversal. • refraction.

**Q.No.14**: Colors in thin film of soap are due to: (***2005 Karachi Board***)

- Refraction of light •diffraction of light
- Interference of light •Scattering of light

**Q.No.15**: To replace a bright fringe by the next bright fringe in a Michelson interferometer, the moveable mirror is moved through a distance equal to: (***2005 Karachi Board***)

- $\lambda$ • $\lambda/2$ • $\lambda/4$ • $2\lambda$

**Q.No.16**: Which property of light is used to determine the concentration of an optically active substance such as sugar?
(***2005 Karachi Board***)

- Interference. • Dispersion.
- Diffraction. • Polarization.

**Q.No.17**: If 2000 lines /cm. are ruled on a grating, its grating element is:
(***2006 Karachi Board***)

- $5\times10^{-4}$ m. • $5\times10^{-3}$ m
- $5\times10^{-6}$ m. • $5\times10^{-7}$ m

**Q.No.18**: The transverse nature of light was confirmed by: (***2008 Karachi Board***)

- Interference • Diffraction
- Polarization. • Dispersion

**Q.No.19**: Yellow light from a sodium lamp is used to form Newton's rings. The central spot in Newton's rings will be:(***2008 Karachi Board***)

- Yellow. • Bright.
- Dark. • Neither bright nor dark.

**Q.No.20**: In Young double slit experiment condition for constructive interference is that the path difference must be:

- An odd multiple of the half wavelength.
- An odd multiple of the whole wavelength.
- An integral multiple of the wavelength.
- An even multiple of the wavelength.

**Q.No.21**: According to Maxwell's theory, light travels in the form of:(***2012 Karachi Board***)

- Transverse wave. • Longitudinal wave
- Mechanical wave. • Electromagnetic wave.

**Q.No.22**:**77** Huygen's principle is used to:
(***2012 Karachi Board***)

- Determine the speed of light.
- Express polarization.• Locate the wave front.
- Find the refractive index.

**Q.No.23**: Monochromatic yellow light is unable to show: (***2013 Karachi Board***)

- Reflection. • Refraction.
- Dispersion. • Interference..

**Q.No.24**: A wave enters from one medium to another medium, no change occurs in its:
(***2013 Karachi Board***)

- Frequency •Wave length •Amplitude • Speed

**Q.No.25**: Diffraction of light is a special type of: (***2014, Karachi board***)

- Reflection. • Refraction.
- Interference. • Polarization.

**Q.No.26**: Newton's rings illustrate the phenomenon of: (***2015 Karachi Board***)

- Polarization. • Diffraction.
- Interference. • Dispersion.

**Q.No.27**: Polarization of light in tourmaline crystals takes place because of:
(***2015 Karachi Board***)

- Reflection. • Absorption.
- Refraction. • Collision.

**Q.No.28**: The number of lines per cm. of a diffraction grating is 4000. Its grating element is: (***2017 Karachi Board***)

- $2.5\times10^{-4}$ cm. • $2.5\times10^{-6}$ cm.
- $4\times10^{2}$ cm. • $4\times10^{5}$ cm.

**Q.No.29**: The experimental evidence of transverse nature of light is:
(***2018 Karachi Board***)

- diffraction. • interference.
- polarization. • dispersion.

***Q.No.30*:** When Newton's rings are observed by reflected light, the center of rings appears dark due to: (***2018 Karachi Board***)

- Phase reversal only.
- Path difference zero only.
- Intensity of light being maximum
- Phase reversal and path difference zero

***Q.No.31*:** The wave front of waves will be spherical when the rays of light are: (***2019 Karachi Board***)

- parallel.
- perpendicular.
- monochromatic.
- not parallel

***Q.No.32*:** When a transverse wave travelling through a rare medium is reflected from a dense medium, then phase change produced in it will be equal to: (***2019 Karachi Board***)

- 0°
- 90°
- 180°
- 360°

***Q.No.33*:** Bragg's law is: (***2022, 16 Karachi Board***)

- $2\,d \sin\theta = m\lambda$
- $2\,m\lambda = d\sin\theta$
- $d\sin\theta = m\lambda$
- $d\cos\theta = m\lambda$

***Q.No.34*:** The bending of light around a sharp obstacle is called: (***2022 Karachi Board***)

- Interference
- Polarization
- Refraction
- Diffraction

***Q.No.35*:** Huygen's principle is used to determine: (***2022 Karachi Board***)

- Speed of light.
- Position of wave front
- Polarization.
- Refractive index

***Q.No.36*:** Wave front near the point source is (***2022 Karachi Board***)

- Plane
- Spherical
- Conical
- cylindrical

***Q.No.37*:** Electromagnetic waves consist of oscillating electric and magnetic fields, both are: (***2011 Karachi Board***)

- Parallel to each other.
- Perpendicular to each other.
- Non-parallel to each other.
- Neither of these.

***Q.No.38*:** For constructive interference, path difference between two coherent waves must be: (***2025 Karachi Board***)

- $(m + \frac{1}{2})\lambda$.
- $\frac{1}{4} m\lambda$.
- $m\lambda$.
- $3\lambda/2$.

***Answers***:

(1) Radio waves.
(2) Infra-red.
(3) is a geometrical method to find a wavefront.
(4) all the dark and bright fringes become darker.
(5) 4 times.
(6) *i*, *iii* and *iv* only.
(7) Diffraction of waves.
(8) Number of maxima **increases** with **increase** in angle between first and second order maxima.
(9) rotate the plane of polarized light.
(10) The intensity of light gradually decreases to zero and remains at zero.
(11) $\lambda L / d$
(12) Photoelectric effect.
(13) phase reversal.
(14) Interference of light.
(15) $\lambda/2$.
(16) Polarization.
(17) $5\times10^{-6}$ m
(18) Polarization
(19) Dark.
(20) An integral multiple of the wavelength.
(21) Electromagnetic wave.
(22) Locate the wave front.
(23) Dispersion.
(24) Frequency.
(25) Interference.
(26) Interference.
(27) Absorption.
(28) $2.5\times10^{-4}$ cm.
(29) polarization.
(30) Path difference zero only
(31) not parallel.
(32) 180°
(33) $2\,d \sin\theta = m\,\lambda$
(34) Diffraction.
(35) Position of wave front.
(36) Spherical.
(37) Perpendicular to each other.
(38) $m\,\lambda$.

## ***Numericals***:

***Q.No.1***: **A monochromatic light of wavelength 6900 Å is used to illuminate two parallel slits on a screen that is 3.30 m away from the slits. Interference fringes are observed. Distance between adjacent bright fringes in the center of the pattern is 1.80 cm.**

**What is the distance between slits?**

***Data***:

Wave length of light $\lambda = 6900$ Å

(1Å $=10^{-10}$ m) $\lambda = 6900\times10^{-10}$ m

Distance of the screen $L = 3.30$ m

Distance between adjacent bright fringes (or fringe spacing) $\Delta x = 1.80$ cm. $= 0.018$ m

Distance between slits $d = ?$

***Solution***:

In Youngs double slit experiment fringe spacing is given by:

$$\Delta x = \frac{\lambda L}{d}$$

$$0.018 = \frac{6900\times10^{-10}\times 3.30}{d}$$

$$\therefore \quad d = \frac{6900\times10^{-10}\times 3.30}{0.018}$$

$$\boxed{d = 0.0001265 \text{ m} = 1.265\times10^{-4} \text{ m.}}$$

Distance between slits is ***1.265×10⁻⁴ m.***

***Q.No.2***: **Michelson interferometer is adjusted so that a bright fringe appears on the screen. As one of the mirrors is moved 25.8 micrometer, 92 bright fringes are counted on the screen.**

**What is the wavelength of light used in the interferometer?**

***Data***:

Wavelength of light used $\lambda = ?$

Distance moved $d = 25.8$ micrometer $= 25.8\times10^{-6}$ m

Number of fringes $m = 92$

***Solution***:

Distance through which moveable mirror of the interferometer is moved is given by:

$$d = m\frac{\lambda}{2}$$

$$\lambda = \frac{2d}{m}$$

$$\lambda = \frac{2\times 25.8\times10^{-6}}{92}$$

$$\boxed{\begin{aligned}\lambda &= 560\times10^{-9}\text{ m}\\ \lambda &= 560\text{ nm}\end{aligned}}$$

Wave length of light used is ***560×10⁻⁹ m.*** or ***560 nm.***

***Q.No.3***:

***Q.No.4***: **Newton's rings are formed by light of 400 nm wavelength. Determine the change in air film thickness between the third and sixth bright fringe. If the radius of curvature of the curved surface is 5.0 m**

**What is the radius of third bright fringe?**

***Data***:

Wave length of light $\lambda = 400$ nm $= 400\times10^{-9}$ m

Radius of curvature of the curved surface $R = 5.0$ m

Change in air film thickness between third and sixth bright rings $\Delta t = ?$

Radius of third bright ring $r_3 = ?$

***Solution***:

Condition for m^th^ bright Newton's ring (or for constructive interference):

$$2t = (m + \tfrac{1}{2})\lambda$$

**For third ring**: m= 2 for third ring

$$2t_3 = (2 + \tfrac{1}{2})\,400$$

$2t_3 = (5/2)\,400$ OR **$t_3 = 500$ nm.**

**For sixth ring**: m= 5 for third ring

$$2t_6 = (5 + \tfrac{1}{2})\,400$$

$2t_6 = (11/2)\,400$ OR **$t_6 = 1{,}100$ nm.**

$\therefore$ Change in air film thickness between third and sixth bright rings $\Delta t = t_6 - t_3$

$= 1100 - 500 =$ **600 nm**.

Change in air film thickness is **600 nm**.

Or **600×10⁻⁹ m.**

Radius of a bright ring can be found by:

$r^2 = R(N - \tfrac{1}{2})\lambda$ N = 3 for 3rd ring

$\therefore \quad r_3^2 = 5(3 - \frac{1}{2})\ 400 \times 10^{-9}$

$r_3^2 = 0.000005 \qquad r_3 = \sqrt{0.000005}$

$\therefore \quad$ **$r_3$ = 0.002236 m** OR **$r_3$ = 2.236 mm**

Radius of 3rd ring is **0.002236 m** OR **2.236 mm.**

***Q.No.5*: A soap film has an index of refraction n = 1.50. The film is viewed in reflected light.**

**(a) at a spot where the film thickness is 910.0 nm. Which wavelengths are missing in the reflected light?**

**(b) Which wavelengths are strongest in the visible light?**

***Data*:**

Refractive index of soap film $n = 1.50$
Film thickness $t = 910$ nm
(a) Missing wave lengths = ?
(b) Strongest wave lengths = ?

***Solution*:**

(a) For soap film interference (Thin film interference) for destructive interference or for dark fringes:

$2\,n\,t = m\,\lambda$

For m = 1

$2 \times 1.50 \times 910 = 1 \times \lambda$

$\therefore \quad$ **$\lambda$ = 2730 nm**

For m = 2

$2 \times 1.50 \times 910 = 2 \times \lambda$

$\therefore \quad$ **$\lambda$ = 1365 nm**

For m = 3

$2 \times 1.50 \times 910 = 3 \times \lambda$

$\therefore \quad$ **$\lambda$ = 910 nm**

For m = 4

$2 \times 1.50 \times 910 = 4 \times \lambda$

$\therefore \quad$ **$\lambda$ = 682.5 nm**

For m = 5

$2 \times 1.50 \times 910 = 5 \times \lambda$

$\therefore \quad$ **$\lambda$ = 546 nm**

For m = 6

$2 \times 1.50 \times 910 = 6 \times \lambda$

$\therefore \quad$ **$\lambda$ = 455 nm**

missing wavelengths are **682.5 nm, 546 nm** and **455 nm.**

(b) for constructive interference (or for strongest wavelengths)

$2\,n\,t = (m + \frac{1}{2})\,\lambda$

For m = 1

$2 \times 1.50 \times 910 = (1 + \frac{1}{2})\,\lambda$

$2730 = (3/2)\,\lambda$

$\therefore \quad$ **$\lambda$ = 1820 nm**

For m = 2

$2 \times 1.50 \times 910 = (2 + \frac{1}{2})\,\lambda$

$2730 = (5/2)\,\lambda$

$\therefore \quad$ **$\lambda$ = 1092 nm**

For m = 3

$2 \times 1.50 \times 910 = (3 + \frac{1}{2})\,\lambda$

$2730 = (7/2)\,\lambda$

$\therefore \quad$ **$\lambda$ = 780 nm**

For m = 4

$2 \times 1.50 \times 910 = (4 + \frac{1}{2})\,\lambda$

$2730 = (9/2)\,\lambda$

$\therefore \quad$ **$\lambda$ = 607 nm**

For m = 5

$2 \times 1.50 \times 910 = (5 + \frac{1}{2})\,\lambda$

$2730 = (11/2)\,\lambda$

$\therefore \quad$ **$\lambda$ = 496.4 nm**

For m = 6

$2 \times 1.50 \times 910 = (6 + \frac{1}{2})\,\lambda$

$2730 = (13/2)\,\lambda$

$\therefore \quad$ **$\lambda$ = 420 nm**

strongest wavelengths within the range of visible light are **780 nm, 607 nm, 496.4 nm and 420 nm.**

***Q.No.6,7,8*:**

See theory.

***Q.No.9*: The diffraction pattern from a single slit of width 0.020 mm is viewed on a screen. If the screen is 1.20 m from the slit and light of wavelength 430 nm is used. What is the width of central maximum?**

***Data***:

Width of slit d= 0.02mm=$0.02\times10^{-3}$ m

Distance of screen L = 1.20 m.

Wave length of light $\lambda$= 430 nm

= $430\times10^{-9}$ m

Width of central maxima (bright fringe) =?

***Solution***:

Distance of 1st dark fringe on left side of central maxima (central bright fringe) $x_1 = \lambda L/d$

$x_1 = \frac{430\times10^{-9}\times1.20}{0.02\times10^{-3}}$ = **0.0258 m**.

Since dark fringes are at equal distance on either side of central maxima, therefore, Distance of 1st dark fringe on right side of central maxima (central bright fringe) $x_2$ = **0.0258 m**.

Width of central maxima (central bright fringe) will be equal to the sum of distance of these two dark fringes = 0.0258 + 0.0258

= **0.0516 m** or **5.16 cm**.

***Q.No.10***: **A grating has exactly 8000 lines uniformly spaced over 2.54 cm and is illuminated by light from a mercury vapor discharge lamp. What is the expected angle for the third order maximum of the light of wavelength 546 nm.**

***Data***:

Order of image m = 3

Angle $\theta$ = ?

Wave length of light $\lambda$= 546 nm

= $546\times10^{-9}$ m

Number of lines = 8000

Length of grating = 2.54 cm.= 0.0254 m

Grating element (a+b)= $\frac{0.0254}{8000}$ =$3.175\times10^{-6}$m

***Solution***:

For a diffraction grating:

$(a + b)\,\text{Sin}\,\theta = m\,\lambda$

$\therefore\quad 3.175\times10^{-6}\times \text{Sin}\,\theta = 3\times546\times10^{-9}$

$\therefore\quad \text{Sin}\,\theta = \frac{3\times546\times10^{-9}}{3.175\times10^{-6}} = 0.5159$

$\therefore\quad \theta = \text{Sin}^{-1}\,0.5159$ = **31.06°**

Angle for third order maximum will be **31.06°**.

***Q.No.11***: **How many lines per centimeter are there in a grating which gives 1st order spectra at an angle of 30° when the wavelength of light is $6\times10^{-5}$ cm?**

***Data***:

Order of image m = 1

Angle $\theta$ = 30°

Wave length of light $\lambda$= $6\times10^{-5}$ cm.

(No need to convert $\lambda$ from cm. into m. as we are required to find No. of lines per cm.)

Number of lines/cm.= ?

***Solution***:

For a diffraction grating:

$(a + b)\,\text{Sin}\,\theta = m\,\lambda$

$(a + b)\,\text{Sin}\,30^\circ = 1\times6\times10^{-5}$

$(a + b)\times0.5 = 6\times10^{-5}$

$(a + b) = \frac{6\times10^{-5}}{0.5} = 12\times10^{-5}$ cm.

Number of lines = $\frac{1}{12\times10^{-5}}$ = **8,333 line/cm.**

There are **8,333 lines per centimeter**.

***Q.No.12***: **Light of wavelength 450 nm is incident on a diffraction grating on which 5000 lines/cm have been ruled. Determine:**

**(a) How many orders of spectra can be observed on either side of direct beam?**

**(b) Determine the angle corresponding to each spectrum.**

***Data***:

Wave length of light $\lambda$= 450 nm

= $450\times10^{-9}$ m.

Number of lines = 5000 lines/cm

$5000\times100$ lines/m.= 500000 lines/m.

Grating element (a + b) = $\frac{1}{500000}$ m

= $2\times10^{-6}$ m

(a) How many orders of spectra m =?

(b) Angle for each spectrum $\theta$ =?

***Solution***:

For a diffraction grating:

$(a + b)\,\text{Sin}\,\theta = m\,\lambda$

Maximum number of orders of spectra can be found by assuming "$\theta = 90^o$".

$\therefore \quad 2\times10^{-6} \times \sin 90^o = m \times 450\times10^{-9}$

(But $\sin 90^o = 1$)

$\therefore \quad m = \dfrac{2\times10^{-6}}{450\times10^{-9}} = 4$

(a) Orders of spectra that can be observed on either side of direct beam is **4**.
(**in other words** maximum four bright diffracted fringes can be observed)

(b) **Angle for m =1**:

$(a + b) \sin\theta = m\lambda$

$2\times10^{-6} \times \sin\theta = 1 \times 450\times10^{-9}$

$\sin\theta = \dfrac{1 \times 450\times10^{-9}}{2\times10^{-6}} = 0.225$

$\therefore \quad \theta = \sin^{-1} 0.225 = \mathbf{13^\circ}$

Angle for first order maximum will be **13**.

**Angle for m = 2**:

$\therefore \quad 2\times10^{-6} \times \sin\theta = 2 \times 450\times10^{-9}$

$\therefore \quad \sin\theta = \dfrac{2 \times 450\times10^{-9}}{2\times10^{-6}} = 0.45$

$\therefore \quad \theta = \sin^{-1} 0.45 = \mathbf{26.74^\circ}$

**Angle for m = 3**:

$\therefore \quad 2\times10^{-6} \times \sin\theta = 3 \times 450\times10^{-9}$

$\therefore \quad \sin\theta = \dfrac{3 \times 450\times10^{-9}}{2\times10^{-6}} = 0.675$

$\therefore \quad \theta = \sin^{-1} 0.675 = \mathbf{42.45^\circ}$

**Angle for m = 4**:

$\therefore \quad 2\times10^{-6} \times \sin\theta = 4 \times 450\times10^{-9}$

$\therefore \quad \sin\theta = \dfrac{4 \times 450\times10^{-9}}{2\times10^{-6}} = 0.9$

$\therefore \quad \theta = \sin^{-1} 0.9 = \mathbf{64.16^\circ}$.

***Q.No.13***: see theory.

***Q.No.14***: **A beam of X-rays of wavelength 0.071 nm is diffracted by a diffracting plane of rock salt with distance between the atomic planes 1.98 Å . Find the glancing angle for the second order diffraction.**

***Data***:

Wave length of X-rays $\lambda = 0.071$ nm
$= 0.071\times10^{-9}$ m

Order of diffraction $m = 2$

Glancing angle $\theta = ?$

Separation of planes $d = 1.98$ Å
$= 1.98\times10^{-10}$ m

***Solution***:

For X-ray diffraction from a crystal, according to Bragg's law:

$m\lambda = 2d\sin\theta$

For second order:

$2\times0.071\times10^{-9} = 2 \times 1.98\times10^{-10} \sin\theta$

$0.142\times10^{-9} = 3.96\times10^{-10} \sin\theta$

$\sin\theta = \dfrac{0.142\times10^{-9}}{3.96\times10^{-10}} = 0.3586$

$\theta = \sin^{-1} 0.3586 \qquad \boxed{\theta = 21.01^\circ}$

***Q.No.15***: **Unpolarized light passes through two polarizers in turn with polarization axes at 45° to each other. What is the fraction of the incident light intensity that is transmitted.**

***Data***:

Angle between polarization axes $\theta = 45^o$

Fraction of the incident light intensity transmitted $I = ?$

***Solution***:

While passing through first polarizer intensity of light reduces to **half**, whereas it reduces by a factor of $\cos^2\theta$ by the second polarizer.

Let "$I_o$" be the initial intensity of light incident upon first polarizer, then "$I_o/2$" will be the intensity of light incident upon the second polarizer.

Hence the fraction of initial intensity transmitted $= \dfrac{I_o}{2} \cos^2\theta$

$= \dfrac{I_o}{2} (\cos 45^o)^2 = \dfrac{I_o}{2} * 0.5 = 0.25\, I_o$

Net intensity of light transmitted will be **$0.25\, I_o$** which is **25%** of **initial intensity ($I_o$)** of unpolarized light.

***Q.No.16***: **If a diffraction grating produces a first order spectrum of light of wave length $6\times10^{-7}$ m at an angle of 20° from the normal. What is its spacing.**

***Data***:

Order of image $m = 1$

Wave length of light $\lambda = 6\times10^{-7}$m

Angular deviation $\theta = 20°$.

Spacing of lines =?

***Solution***:

For diffraction grating:

$$(a + b)\sin\theta = m\lambda$$

$$(a + b)\sin 20° = 1\times 6\times10^{-7}$$

$$(a + b) = \frac{1\times 6\times10^{-7}}{\sin 20°}$$

$$(a + b) = \frac{6\times10^{-7}}{0.342} = 1.7543\times10^{-6}\text{m}$$

Therefore, spacing of slits $= \frac{1}{(a + b)}$

$$= \frac{1}{1.7543\times10^{-6}} = \textbf{570033.57 lines/m}$$

OR spacing of slits= ***570 lines/mm***

***Q.No.17***: **How far apart are the diffracting planes in a NaCl crystal for which X-rays of wavelength 1.54 Å makes a glancing angle of 15°-54′ in the first order.**

***Data***:

Wavelength $\lambda = 1.54$ Å $=1.54\times10^{-10}$m

Glancing angle $\theta = 15°\text{-}54' = 15.9°$

Order of image $m = 1$.

Distance between planes $d = ?$

***Solution***:

According to Bragg's law:

$$m\lambda = 2d\sin\theta$$

$$1\times1.54\times10^{-10} = 2d\sin 15.9°$$

$$d = \frac{1.54\times10^{-10}}{2\sin 15.9°} = \frac{1.54\times10^{-10}}{2\times0.274}$$

$$\boxed{d = 2.81\times10^{-10}\text{ m}}$$

OR $\boxed{d = 2.81\text{ Å}}$

Distance between diffracting planes is ***2.81 Å.***

***Q.No.18***: **Two parallel slits are illuminated by light of two wave lengths one of which is $6.0\times10^{-7}$ m on a screen the fourth dark line of the known wave length coincides with the fifth bright line of the unknown wave length.**

**Calculate the unknown wave length.**

(***1992 Karachi Board***)

***Data***:

Known wave length $\lambda_1 = 6.0\times10^{-7}$ m

Unknown wave length $\lambda_2 = ?$

***Solution***:

Distance of bright fringes from the center of the screen is given by:

$$y = \frac{m\lambda L}{d}$$

Distance of fifth bright fringe of unknown wave length "$\lambda_2$" from the center of the screen will be:

$$y = \frac{5\lambda_2 L}{d}$$

Distance of fourth dark fringe of known wave length "$\lambda_1$" from the center of the screen will be:

$$y = \left(4 + \tfrac{1}{2}\right)\frac{\lambda_1 L}{d} = \frac{9\lambda_1 L}{2d}$$

Since fourth dark fringe of known wave length coincides with the fifth bright fringe of unknown wave length, therefore, both must be at equal distance from center of the screen.

$$\frac{5\lambda_2 L}{d} = \frac{9\lambda_1 L}{2d}$$

$$5\lambda_2 = \frac{9\lambda_1}{2}$$

$$\lambda_2 = \frac{9\lambda_1}{10} = \frac{9\times6.0\times10^{-7}}{10}$$

$$\boxed{\lambda_2 = 5.4\times10^{-7}\text{ m} \quad \lambda_2 = 5400\text{ Å}}$$

Unknown wave length is ***5.4 × 10⁻⁷m*** or ***5400 Å.***

***Q.No.19***: **Interference fringes were produced by two slits on a screen 0.8 m from them when light of wave length $5.8\times10^{-7}$ m was used. If the separation**

**between the first and the fifth bright fringe is 2.5 mm. Calculate the separation of the two slits.** (***1995 Karachi Board***)

***Data***:

Distance between slits and screen L =0.8m.

Wave length of light $\lambda = 5.8\times10^{-7}$ m

Distance between first and fifth bright fringe =2.5 mm=$2.5\times10^{-3}$ m

Separation of slits d = ?

***Solution***:

Distance of the first bright fringe from center of the screen is given by: $y = \dfrac{\lambda L}{d}$

Distance of the fifth bright fringe from center of the screen is given by: $y = \dfrac{5\lambda L}{d}$

But the distance between the two fringes is 2.5 mm

Therefore $\dfrac{5\lambda L}{d} - \dfrac{\lambda L}{d} = 2.5\times10^{-3}$

$$\frac{4\lambda L}{d} = 2.5\times10^{-3}$$

$$\frac{4\times5.8\times10^{-7}\times0.8}{d} = 2.5\times10^{-3}$$

$$d = \frac{4\times5.8\times10^{-7}\times0.8}{2.5\times10^{-3}}$$

$$\boxed{\begin{array}{c} d = 7.424\times10^{-4}\ m \\ d = 0.74\ mm \end{array}}$$

***Q.No.20***: **Red light falls normally on a diffraction grating ruled 4000 lines/cm. and the second order image is diffracted 34° from the normal.**

**Compute the wave length of light in angstroms.** (***1996 Karachi Board***)

***Data***:

Number of lines ruled = 4000 lines /cm

Grating element (a + b)

$= \dfrac{1}{4000}$ cm.= $2.5\times10^{-4}$ cm= $2.5\times10^{-6}$ m

Angular deviation $\theta = 34°$

Order of image m = 2

Wave length $\lambda$ =?

***Solution***:

For diffraction grating:

$$(a + b)\ \text{Sin}\ \theta = m\lambda$$

$$2.5\times10^{-6}\ \text{Sin}\ 34° = 2\lambda$$

$$2.5\times10^{-6}\times0.5992 = 2\lambda$$

$$\lambda = \frac{2.5\times10^{-6}\times0.5992}{2}$$

$$\boxed{\begin{array}{c} \lambda = 6.9899\times10^{-7}\ m \\ \lambda = 6989.9\ \text{Å} \end{array}}$$

***Q.No.21***: **How many fringes will pass a reference point if the moveable mirror of the Michelson's interferometer is moved by 0.08 mm? The wave length of light used is 5800 Å.** (***1998 Karachi Board***)

***Data***:

Wave length of light used $\lambda$ =5800 Å

$= 5800\times10^{-10}$ m

Distance through which the moveable mirror is moved d = 0.08 mm = $0.08\times10^{-3}$ m

Number of fringes m =?

***Solution***:

Distance through which moveable mirror of the interferometer is moved is given by:

$$d = m\frac{\lambda}{2}$$

$$m = \frac{2d}{\lambda}$$

$$m = \frac{2\times0.08\times10^{-3}}{5800\times10^{-10}} \qquad \boxed{m = 275}$$

***275 fringes*** will pass the reference point.

***Q.No.22***: **Interference fringes were produced by light coming from two slits 0.3 mm apart. If five fringes occupied 1.75 mm on a screen at 200 mm from the slits, find the wave length of light used.**

(***2000 Karachi Board***)

***Data***:

Separation of slits d = 0.3 mm

$= 0.3\times10^{-3}$m

Distance in which 5 fringes were formed $5\Delta x = 1.75$ mm.

Fringe spacing $\Delta x = 1.75/5$ mm

$= 0.35$mm $= 0.35\times10^{-3}$ m

Distance of the screen L=200mm=0.2m

Wave length of light $\lambda$ =?

***Solution*:**

$\Delta x = \frac{\lambda L}{d}$ $\lambda = \frac{\Delta x\, d}{L}$

$\lambda = \frac{0.35 \times 10^{-3} \times 0.3 \times 10^{-3}}{0.2}$

$\lambda = 5.25\times10^{-7}$ m **$\lambda$ = *5250* Å**

***Q.No.23*: How much should the moveable mirror of the Michelson's interferometer be moved in order to observe 400 fringes with reference to a point? The wavelength of the light used is 5890 Å.** (***2002 Karachi Board***)

***Data*:**

Distance moved d = ?
Number of fringes n = 400
Wave length light used $\lambda$ =5890 Å

***Solution*:**

Distance, through which moveable mirror of Michelson's interferometer is moved, is given by:

$d = n \frac{\lambda}{2} = \frac{400 \times 5890 \times 10^{-10}}{2}$.

**d = *1.178* × *10*$^{-4}$ *m = 0.1178 mm***

***Q.No.24*: If the radius of 12 $^{th}$ dark Newton's ring is 1 mm when light of wave length 5890 Å is used, what is the radius of curvature of the lower surface of the lens used?** (***2017, 03, 1990,94 Karachi Board***)

***Data*:**

Radius of the dark ring
$r_{12}$ =1 mm = 1 × 10 $^{-3}$ m
Wave length of light $\lambda$ = 5890 Å
= 5890 ×10 $^{-10}$ m
Radius of curvature of lens R =?
Dark ring No. m = 12

***Solution*:**

Radius of the m th dark ring is given by:

$r_N = \sqrt{m \lambda R}$

m = 0,1,2,3… (m = 0 for dark center)

Squaring both sides and putting the values we get:

$(1\times10^{-3})^2 = 12 \times 5890 \times10^{-10}$ R

$1 \times 10^{-6} = 12 \times 5890 \times 10^{-10}$ R

$R = \frac{1 \times 10^{-6}}{12 \times 5890 \times 10^{-10}}$

***R= 0.1414 m***

***Q.No.25*: X-rays of wave length 1.54 Å are diffracted by a crystal whose planes are 2.81 Å apart. Find the glancing angle for the first order.** (***2004 Karachi Board***)

***Data*:**

Wave length of X-rays $\lambda$ = 1.45 Å
Separation of planes d = 2.81 Å
Order m = 1
Glancing angle $\theta$ = ?

***Solution*:**

For X-ray diffraction from a crystal, according to Bragg's law:

m $\lambda$ = 2 d Sin $\theta$

(No need to convert Å into meters, as in this case "$\lambda$"and "d" divide each other, their unit cancel out).

1×1.54 = 2×2.81 Sin $\theta$ (For first order)

1.54 = 5.62 Sin $\theta$

Sin $\theta$ = $\frac{1.54}{5.62}$ = 0.2740

$\theta$ = Sin$^{-1}$ 0.2740 **$\theta$ = *15.90*°**

***Q.No.26*: Interference fringes were produced by two slits 0.25 mm apart on a screen 150 mm from the slits. If ten fringes occupy 3.275 mm, what is the wavelength of the light producing fringes?** (***2016, 08, 02 Karachi Board***)

***Data*:**

Separation of slits d = 0.25mm
= $0.25\times10^{-3}$m
Distance of screen L =150 mm = 0.15m
Distance occupied by 10 fringes
= 3.275 mm.
Fringe spacing $\Delta x = \frac{3.275}{10}$ mm
= 0.3275 mm = $0.3275\times10^{-3}$ m
Wave length of light used $\lambda$ = ?

***Solution*:**

In Young double slit experiment fringe spacing (distance between two consecutive fringes) is given by:

$$\Delta x = \frac{\lambda L}{d}$$

$$0.3275\times10^{-3} = \frac{\lambda \times 0.15}{0.25\times10^{-3}}$$

$$\therefore \quad \lambda = \frac{0.3275 \times 10^{-3} \times 0.25 \times 10^{-3}}{0.15}$$

$$\boxed{\lambda = 5.458 \times 10^{-7}\ m = 5458\ \text{Å}}$$

***Q.No.27*: If radius of 14th bright Newton's ring is 1 mm and the radius of curvature of the lens is 125 mm.**

**Calculate wavelength of light.**

(***2010, 1997 Karachi Board***)

***Data*:**

Radius of 14th ring $r_{14}$ =1mm= $1\times10^{-3}$ m

Ring number N = 14

Radius of curvature of lens R = 125 mm = $125 \times10^{-3}$ m.

Wave length of light $\lambda$= ?

***Solution*:**

Radius of Nth Newton's ring is given by:

$$r_N = \sqrt{R\lambda(N - \tfrac{1}{2})}$$

$$r_{14} = \sqrt{R\lambda(14 - \tfrac{1}{2})}$$

$$1\times10^{-3} = \sqrt{125\times10^{-3}\ \lambda\ (14 - \tfrac{1}{2})}$$

$$1\times10^{-3} = \sqrt{125\times10^{-3}\ \lambda\ (27/2)}$$

Squaring both sides we get:

$$(1\times10^{-3})^2 = 125\times10^{-3}\ \lambda\ (27/2)$$

$$\lambda = \frac{(1\times10^{-3})^2}{125\times10^{-3}\ (27/2)}$$

$$\boxed{\lambda = 5.926 \times 10^{-7}\ m. \quad \lambda = 5926\ \text{Å}}$$

***Q.No.28*:In a double slit experiment, eight fringes occupy 2.62 mm on screen 145 mm away from the slits. The wave length of light is 545 nm. Find the slit separation.**

(***2012 Karachi Board***)

***Data*:**

No. of fringes = 8

Distance occupied = 2.62 mm.

Fringe spacing $\Delta x = \frac{2.62}{8}$ mm

$\Delta x$ = 0.3275 mm= $3.275 \times10^{-4}$ m

Wave length $\lambda$ = 545 nm = $545\times10^{-9}$m.

Distance of screen L = 145 mm= $145\times10^{-3}$ m.

Slit separation d = ?

***Solution*:**

Fringe spacing is given by:

$$\Delta x = \frac{\lambda L}{d} \qquad d = \frac{\lambda L}{\Delta x}$$

$$d = \frac{545\times10^{-9} \times 145\times10^{-3}}{3.275\times10^{-4}}$$

$$d = \frac{7.9025\times10^{-8}}{3.275\times10^{-4}}$$

$$\boxed{d = 2.413\times10^{-4}\ m \quad d = 0.2413\ mm.}$$

***Q.No.29*: Green light of wave length 5400Å. is diffracted by grating having 2000 lines/cm.**

**Compute the angular deviation of the third order image.**

(***2014, 05, 1999 Karachi Board***)

***Data*:**

Wave length $\lambda$=5400 Å = $5400\times10^{-10}$ m

Number of lines/cm = 2000

Grating element (a + b)

$= \frac{1}{2000}$ cm.= $5\times10^{-4}$cm= $5\times10^{-6}$ m

Angular deviation $\theta$ = ?

Order of image m = 3

***Solution*:**

$$(a + b) \sin\theta = m\lambda$$

$$5\times10^{-6} \sin\theta = 3\times 5400\times10^{-10}$$

$$\sin\theta = \frac{3\times 5400\times10^{-10}}{5\times10^{-6}}$$

$$\sin\theta = \frac{16200\times10^{-10}}{5\times10^{-6}} = 0.3240$$

$$\theta = \sin^{-1} 0.3240 \qquad \boxed{\theta = 18.91^\circ}$$

***Q.No.30*: A parallel beam of X-rays is diffracted by rock salt crystals. The first order maximum being obtained when the glancing angle of incidence is 6 degree and 5 minutes, the distance between the atomic planes of crystal is $2.81\times10^{-10}$ m.**

**Calculate the wave length of radiation.**

(***2015, 2009 Karachi Board***)

***Data*:**

Order m = 1

Glancing angle $\theta$ = 6° 5 minutes

= 6.083°

Separation of planes $d = 2.81\times10^{-10}$m

Wave length of X-rays $\lambda$ =?

***Solution*:**

According to Brag's law:

$m\lambda = 2d\sin\theta$

$\therefore \quad 1\times\lambda = 2\times2.81\times10^{-10}\times\sin 6.083^\circ$

$\therefore \quad \lambda = 2\times2.8\times10^{-10}\times0.106$

$\therefore \quad \boxed{\lambda = 5.956\times10^{-11}\ m}$

$\boxed{\lambda = 0.5956\ \text{Å}}$

🗣 Wave length of X-rays is **$5.956\times10^{-11}$m** or **0.5956 Å**.

***Q.No.31*: A diffraction grating produces 3rd order spectrum of light of wavelength 7000 Å at an angle of 30° from the normal.**

**What is the grating element?**

**Calculate the number of lines of grating per cm.**

(***2019, 17, 11, 97, 03, 01,1991 Karachi Board***)

***Data*:**

Angular deviation $\theta = 30^\circ$

Order of image $m = 3$

Wave length of light $\lambda$ =7000 Å= $7\times10^{-7}$ m.

Grating element (a + b) =?

Number of lines per cm. =?

***Solution*:**

For diffraction grating:

$(a + b)\sin\theta = m\lambda$

$(a + b)\sin 30^\circ = 3\times7\times10^{-7}$

$(a + b)\times0.5 = 21\times10^{-7}$

$(a + b) = \dfrac{21\times10^{-7}}{0.5}$

$\boxed{(a + b) = 42\times10^{-7}\text{m.} = 4.2\times10^{-4}\text{cm}}$

Number of lines per cm. $= \dfrac{1}{(a + b)}$

Number of lines per cm. $= \dfrac{1}{4.2\times10^{-4}}$

Number of lines per cm. = 2380

🗣Given grating has ***2380 lines per cm***.

***Q.No.32*: 271 fringes are passed through a reference point when the moveable mirror of Michelson's interferometer is moved by 0.08mm**

**Find the wavelength of light used in Å.**

(***2019, 2002 Karachi Board***)

***Data*:**

Number of fringes passing reference point

$n = 271$

Moveable mirror moved by d = 0.08 mm

$= 8\times10^{-5}$ m

Wave length of light used $\lambda$ = ?

***Solution*:**

For Michelson interferometer:

$d = n\dfrac{\lambda}{2}$

$8\times10^{-5} = 271\dfrac{\lambda}{2}$

$\lambda = \dfrac{2\times8\times10^{-5}}{271}$

$\boxed{\lambda = 5.9\times10^{-7}\ m}$ OR $\boxed{\lambda = 5900\ \text{Å}}$

***Do you know that*** **……**

**Speed of light in air or vacuum is very high about 300,000,000 m/s or 300,000 km/s (three crore kilometers in one second) (actually it is about $2.98 \times 10^8$ m/s). Travelling with such a high speed you can circle 7½ times around the earth in just one second. The distance to the sun can be covered in about slightly over 8 minutes. Traveling with speed of light from earth will take about 200,000 years to cross our galaxy "Milky way", and galaxy ANDROMEDA nearest to our galaxy, can be reached in 2 million years.**

**Actually, all electromagnetic waves (light waves are also electromagnetic), in vacuum can travel with such a high speed.**

**According to Einstein's special theory of relativity, mass of material objects starts increasing as its speed approaches speed of light.**

***Unit.14**.....* ✍

# Communication

## *Communication*:

Communication is the exchange of information between two or more bodies.
Modern telecommunication may be **wired** or **wireless**.
Massages are transmitted using electromagnetic waves by an antenna (**transmitter antenna**) at one end and are received by another antenna at the other end. Message may be voice message, video, voice message etc.

## *Wave length of waves* ($\lambda$):

It is the distance between two consecutive crests or troughs of a wave.
S.I unit is **m**.

## *Frequency of waves* ($f$):

It is the number of waves passing through a point in one second.
S.I unit is **Herz Hz**.

## *Relation between f and $\lambda$*:

Wave length "λ" of a wave is inversely proportional to its frequency "$f$". (higher the frequency shorter is the wave length and vice versa)

$$\lambda \propto \frac{1}{f}$$

$$\lambda f = \text{speed v of a wave}$$

Since all electromagnetic waves, including radio waves, travel with speed of light "c", therefore,

$$\boxed{\lambda f = c} \qquad (c = 3 \times 10^{8} \text{ m/s, actually it is } 2.98 \times 10^{8} \text{ m/s})$$

Energy "E" of a wave depends upon its frequency "$f$". High frequency waves are more **energetic** (High frequency and therefore, high energy waves are of **short** wave length).

***Q.No.1**: What are fiber optics?*

## *Fiber optics*:

A fiber optic cable consists of a large number of very thin glass or plastic strands bundled together inside a strong insulating pipe.
A single fiber can carry thousands of telephone calls at the same time.

**Working:** First of all any signal in the form of electromagnetic waves, to be transmitted through an optic fiber, is converted into **coded light signals**. These light signals are coded with information to be transmitted. These coded light signals travel through the fiber cable undergoing **total internal reflection** whenever it touches sides of the cable, thus always remaining in the middle. Since these are now in the form of light signals they travel with a very high speed (close to speed of light).

There is **minimum energy loss** in signals to be transmitted in fiber cables as compared to copper wires and the method of transmission is **cost effective** also.

## ***Radio waves*:**

Radio waves are the **longest wave length**, **lowest frequency** and hence of **lowest energy** electromagnetic waves. Since radio waves are lowest energy waves therefore, they are non-ionizing electromagnetic waves.

In radio communication radio waves are emitted in all directions by **transmitter antenna**. These transmitted radio waves are received by another antenna called **receiver antenna**.

Length of receiving antenna must be about one fourth ($\lambda/4$) of wave length "$\lambda$" of signal wave.

Data signal waves, such as radio waves, TV waves, sound, music etc. are of few kHZ. Frequency, ***in other words***, they are basically long wave length signals. Their wave length is usually such that at the receiving side a very long receiving antenna will be required, which is not practical. Beside such signals can not be sent efficiently over long distances. Hence before transmission these signals are **modulated.** (see modulation)

## ***Microwaves*:**

**Microwaves** are **electromagnetic waves** having frequency range of 1GHz to 300 GHz. These electromagnetic waves due to their short wave length, higher frequency (more energy) are widely used for transmitting signals over long distances even through space. These signals can not pass through or around large solid obstacles, such as mountains elc. Because of their relatively short wave length they can be received by small sized antenna.

They are widely used in radar systems.

## ***Satellite communication*:**

Satellites revolving around the earth at suitable heights, are used for long distance communication. The path at a suitable height around the earth where the satellite revolves is called its **orbit**. Three satellites placed at different locations, in suitable orbit separated from each other by a suitable distance can cover most populated part of our beautiful little earth.

Satellites are used to keep track of weather conditions, for communication, for civil and military use, for global mobile communication etc. Now a days GPS (Global Positioning System) has become a simple daily life application of satellite communication.

***Q.No.2*: *What are data or signal waves?***

## ***Data or signal waves*:**

Waves which carry voice messages, music, speeches, videos, in general any type of data are called ***signal waves*** or ***data signals***.

Data or signal waves are usually of low frequency hence of long wavelength.

## ***Type of data signals*:**

Data signals are basically of two types:

(1) **Analog signals**. Analog signals are basically in the form of electromagnetic waves.

(2) **Digital signals**. Digital signals are basically in the form Binary digits (which means that there are two digits). Hence binary system needs only two digits 0 and1, 1 bit is equal to either of these two numbers (0 or 1), and any data can be represented by the combination of these two numbers. Whereas 1 byte = 8 bits. (it means that 1 byte will have 00001111).

Digital signals are used in modern devices such as computers, mobile phones, internet, Wi Fi, satellite communication etc.

## ***Basic memory units*:**

Memory units are the standard units to calculate the size of data.

Binary system is used in a computer. In binary system there are two units either 0 or 1, each of them is called a **bit**,

Smallest unit in computer memory is a **bit (b)**,

8 bits make a **Byte (B).** Hence.

1 Byte (B) = 8 bits (b)
1 Kilo Byte (KB) = 1024 Bytes
1 Megabyte (MB) = 1024 KB
I Giga byte (GB) = 1024 MB
1 Terabytes (TB) = 1024 GB
1 Petabytes (PB) = 1024 TB

These memory units tell you about size of memory or the size of data storing capacity of a computer device.

***Q.No.3*: What are carrier waves?**

## ***Carrier waves*:**

Waves that carry **signal or data waves** over long distances are called **carrier waves**. Carrier waves carry signal (data) waves after a process called **modulation**.

Frequency of carrier waves is much **higher** than the frequency of signal waves. Since energy of a wave depends upon its frequency therefore, carrier waves are **high energy waves** and, therefore, they can travel long distances with a very small loss of energy. Besides they also **reduce** size of receiving antenna.

***Q.No.4*: What is modulation?**

## ***Modulation*:**

The process of super-imposing the data signal over the carrier wave to change its **amplitude**, **frequency** or **phase** is called **modulation**.

Modulation changes a particular characteristic of a carrier wave while keeping the other characteristics constat.

Modulation can be done with both analog and digital information signals.

***Q.No.5*: Why is modulation needed?**

## ***Need for modulation*:**

- **To transmit the signal over long distances**. Signel waves are low frequency waves. Low frequency signals cannot be transmitted over long distances. In modulated signal high frequency carrier wave (having more energy) caries the data signal to long distance.
- **To reduce the size of receiving antenna**. Since data signal is of low frequency, therefore, its wavelength will be long. To receive a long wavelength signal longmreceiving antenna is needed (practically not feasible). In modulated signal carrier wave is of high frequency and small wavelength therefore, small sized receiving antenna will be needed.
- **Avoid mixing of signals**.

## ***Types of modulation*:**

Depending upon the modification basically there are three types of modulations:

(1) Amplitude modulation (AM).

(2) Frequency modulation (FM).

(3) Phase modulation.

- In **amplitude modulation** only the amplitude of the carrier wave is changed, whereas its frequency and phase remain unchanged.
- In **frequency modulation** only the frequency of the carrier wave is changed, whereas its amplitude and phase remain unchanged.
- In **phase modulation** only phase of the carrier wave is changed, whereas its amplitude and frequency remain unchanged.

***Q.No.6*: What is amplitude modulation?**

## ***Amplitude modulation (AM):***

The process of super-imposing the data signal over the carrier wave to change the amplitude is called **amplitude modulation (AM)**.

In amplitude modulation (AM), amplitude of carrier wave is changed corresponding to the data signal wave.

Carrier wave after modulation is called **modulated wave**.

Amplitude of the carrier wave at the crest of modulated wave is strongest and weakest at the trough of modulated wave.

Frequency of modulated wave remains equal to the **frequency of signal** wave only its **amplitude changes**.

***Q.No.7*: What are the advantages/disadvantages of amplitude modulation?**

## ***Advantages/disadvantages of AM- modulation:***

- Frequency range of AM is **much lower** than the frequency range of FM, due to this reason AM covers **larger area**. This is the reason why most news stations use AM, so that any news can reach lager population living is large area.
- AM signals are **easily affected** by noise because noise affects amplitude, as a result of which we hear static.

***Q.No.8*: What is frequency modulation?**

## ***Frequency modulation (FM- modulation):***

The process of super-imposing the data signal over the carrier wave to change its frequency is called **frequency modulation (FM)**.

In frequency modulation (FM), frequency of carrier wave is changed according to frequency of data signal wave.

Frequency of the carrier wave at the crest of modulated wave is highest and lowest at the trough of modulated wave.

Amplitude of modulated wave remains equal to the amplitude of signal wave only the **frequency changes**.

***Q.No.9*: What are the advantages/disadvantages of frequency modulation?**

## ***Advantages/disadvantages of FM- modulation:***

- FM frequency range is **much higher**, therefore it can carry **more date** than AM.
- Effect of interference (such as of noise) on FM signal is **very small** as a result of which there is **no static effect** on FM broadcast.
- Due to the above two reasons FM radio has better **sound quality**.

***Q.No.10*: What are the advantages of transmission in digital form over analog form?**

Transmission in digital form has many advantages over transmission in analog form, such as it has less noise and distortion, it has maximum security, digital signals can be stored and retrieved more accurately etc.

## ***Section–A Multiple-Choice Questions MCQs:***

***Q.No.1***: In radio and television broadcast, the information signal is in the form of:
(a) analog signal. (b) digital signal.
(c) Both analog and digital signals.
(d) neither analog nor digital signal.

***Q.No.2***: A communication channel consists of:
(a) Transmission line only.
(b) Optical fiber only.
(c) Free space only.
(d) All of them.

***Q.No.3***: Voltage signal generated by a microphone is:
(a) Digital in nature. (b) Analog in nature.
(c) Hybrid in nature.
(d) Consists of bits and bytes.

***Q.No.4***: As compared to sound waves frequency of radio waves is:
(a) Higher. (b) Equal.
(c) Lower. (d) may be higher or lower.

***Q.No.5***: The process of superimposing signal frequency on the carrier wave is known as:
(a) Transmission. (b) Detection.
(c) Reception. (d) Modulation.

***Q.No.6***: What is the frequency range of signals that can be transmitted in case of twisted pair of wires.
(a)10MHz. to 15MHz. (b)5 MHz. to 10MHz.
(c)100MHz. to 5MHz. (d)10 Hz. to 100Hz.

***Q.No.7***: The maintenance of a satellite's orbital position is called:
(a)Maintenance keeping. (b)Station keeping.
(c) Station maintenance. (d) altitude control.

***Q.No.8***: Process of mapping the sampled analog voltage values to discrete voltage levels is called:
(a) Sampling. (b) Sampling frequency.
(c) Quantizing. (d) Encoding.

***Q.No.9***: AM is used for broadcasting because:
(a) It requires less transmitting power compared with other systems.
(b) It is more noise immune than other modulation systems.
(c) No other modulation can provide the necessary bandwidth faithful transmission.
(d) Its use avoids receiver complexity.

***Q.No.10***: Data in compact disc is stored in the form of:
(a) Analog signal. (b) Digital signal.
(c) Noise. (d) Both (a) and (b).

***Q.No.11***: The process of superposing signal frequency onto carrier wave is called:
(***2025 Karachi Board***)
- Modulation.
- Detection.
- Transmission.
- Reception.

***Answers*:**

(1) analog signal.
(2) All of them.
(3) Analog in nature.
(4) Higher.
(5) Modulation.
(6) 100MHz. to 5MHz.
(7) Station keeping.
(8) Quantizing.
(9) No other modulation can provide the necessary bandwidth faithful transmission
(10) Digital signal.
(11) Modulation.



## ***A piece of advice to students*:**

Dear students your examination paper comprises of a number of ***numericals/problems***. For high scoring rate ***solve numericals*** from text book as well as from last several year's board papers. Try to ***understand*** the theoretical background of each. Do real good practice. Do not take any chance. Hard work with systematic approach, courage and determination always pays.
***College years are a turning point of your life!!!***